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بعض التعديلات الجديدة في الطرق Kibria و Dorugade: تطبيق لبيانات الناتج المحلي
الإجمالي التركي

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المخلص:

ان الخطية المتعددة هي مشكلة هامة في العديد من تحليل الانحدار الخطي. انحدار ريدج هو واحدة من أكثر الطرق شيوعا للتغلب على هذه المشكلة وهناك العديد من معاملات ريدج المقترحة في هذا المجال. في هذه الدراسة، نقترح بعض التعديلات الجديدة لاختيار معامل ريدج وتم استخدام محاكاة مونت كارلو لتقييم المعاملات. أيضا، التحيزات من المقدرات تؤخذ بعين الاعتبار. يستخدم الخطأ التربيعي (MSE) لمقارنة أداء المقدرات المقترحة مع ما تم نشره من قبل الباحثين الآخرين. ووفقا للنتائج، كل المقدرات المقترحة (OLS) تعلق على مقدر الأقل تربيع العادي.



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ORIGINAL ARTICLE

Some new modifications of Kibria’s and Dorugade’s methods: An application to Turkish GDP data



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Abstract In multiple linear regression analysis, multicollinearity is an important problem. Ridge regression is one of the most commonly used methods to overcome this problem. There are many proposed ridge parameters in the literature. In this paper, we propose some new modifications to choose the ridge parameter. A Monte Carlo simulation is used to evaluate parameters. Also, biases of the estimators are considered. The mean squared error is used to compare the performance of the proposed estimators with others in the literature. According to the results, all the proposed estimators are superior to ordinary least squared estimator (OLS).

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1. Introduction

Consider the following standard linear regression model

$$Y = X\beta + \varepsilon \tag{1.1}$$

where Y is an $n \times 1$ vector of dependent variable, X is a design matrix of order $n \times p$ where p is the number of explanatory variables, β is a $p \times 1$ vector of coefficients and ε is the error vector of order $n \times 1$ distributed as $N(0, \sigma^2 I_n)$. Ordinary least squared (OLS) method is the most common method of estimating β and the OLS estimator of β is given as follows

$$\hat{\beta} = (X'X)^{-1}X'Y \tag{1.2}$$

In some situations, the matrix $X'X$ has almost zero eigenvalues meaning the explanatory variables are correlated. This leads to a large variance and so large mean squared error (MSE). Thus one may not reach a reliable solution for β . This is the commonly faced problem called multicollinearity. There are various methods to solve this problem. The ridge regression is one of the most popular methods proposed by Hoerl and Kennard (1970a,b).

In ridge regression, adding a small positive number $k(k > 0)$ called ridge parameter to the diagonal elements of the matrix $X'X$, we obtain the following ridge estimator

$$\hat{\beta}_{RR} = (X'X + kI_p)^{-1}X'Y, \quad k > 0 \tag{1.3}$$

The MSEs of the OLS estimator and the ridge estimator $\hat{\beta}_{RR}$ are as follows respectively,

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$$\text{MSE}(\hat{\beta}) = \sigma^2 \sum_{i=1}^p \frac{1}{\lambda_i} \quad (1.4)$$

$$\begin{aligned} \text{MSE}(\hat{\beta}_{RR}) &= \text{Var}(\hat{\beta}_{RR}) + [\text{Bias}(\hat{\beta}_{RR})]^2 \\ &= \sigma^2 \sum_{i=1}^p \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \beta' (X'X + kI_p)^{-2} \beta \end{aligned} \quad (1.5)$$

where λ_i 's are eigenvalues of the matrix $X'X$ and σ^2 is the error variance.

Hoerl and Kennard, 1970b showed the properties of this function in detail. They concluded that the total variance decreases and the squared bias increases as k increases. The variance function is monotonically decreasing and the squared bias function is monotonically increasing. Thus, there is the probability that some k exists such that the MSE for $\hat{\beta}_{RR}$ is less than MSE for the usual $\hat{\beta}$ (Hoerl and Kennard, 1970ab).

We know that k is estimated from the observed data. There are many papers proposing different ridge parameters in the literature. In recent papers, these parameters have been compared with the one proposed by Hoerl et al., 1975 and each other. After Hoerl and Kennard, 1970b, many researchers studied this area and proposed different estimates of the ridge parameter. Some of them are McDonald and Galarneau (1975), Lawless and Wang (1976), Saleh and Kibria (1993), Liu and Gao (2011), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi et al. (2006), Adnan et al. (2006), Yan (2008), Yan and Zhao (2009), Muniz and Kibria (2009), Mansson et al. (2010), Al-Hassan (2010), Muniz et al. (2012), Asar et al. (2014) and Dorugade (2014).

The purpose of this article is to study much of the parameters in the literature and propose some new ones and also make a comparison between them by conducting a Monte Carlo experiment. The comparison criterion is based on the mean squared properties.

The article is organized as follows. In Section 2, we present the methodology of different estimators and give some new estimators. A Monte Carlo simulation has been provided in Section 3. Results of the simulation are discussed in Section 4. In Section 5, an application of the estimators is given. Finally, we give a summary and conclusion.

2. Model and estimators

Firstly we write the general model (1.1) in canonical form. Suppose that there exists an orthogonal matrix D we apply a transformation such that

$$D(X'X)D' = \Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_p) \quad (2.1)$$

where D is a $p \times p$ orthogonal matrix and $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p$. If we substitute $Z = XD$ and $\alpha = D'\beta$ in the model (1.1), then the model may be rewritten as

$$Y = Z\alpha + \varepsilon \quad (2.2)$$

where $Z'Z = \Lambda$.

Thus, the ridge estimator of α becomes $\hat{\alpha}_{RR} = (Z'Z + kI_p)^{-1}Z'Y$. It is stated in Hoerl and Kennard, 1970a that the value of k minimizing the $\text{MSE}(\hat{\alpha}_{RR})$ is

$$k_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}. \quad (2.3)$$

As seen in the formula (2.3), k depends on the unknown parameters σ^2 and α . Hence we use the estimators $\hat{\sigma}^2$ and $\hat{\alpha}$ due to Hoerl and Kennard, 1970b and get

$$k_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}. \quad (2.4)$$

2.1. Proposed estimators

In this section, we review some of the ridge estimators suggested earlier and propose some new ones. The list of estimators with which we will compare ours is given below:

$$(1) k_1 = k_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{\max}^2} \quad (\text{Hoerl \& Kennard, 1970a}) \quad (2.5)$$

where $\hat{\alpha}_{\max}$ is the maximum element of $\hat{\alpha}$.

$$(2) k_2 = \frac{p\hat{\sigma}^2}{\sum_{i=1}^p \lambda_i \hat{\alpha}_i^2}, i = 1, 2, \dots, p \quad (\text{Lawless \& Wang, 1976}) \quad (2.6)$$

which is proposed from the Bayesian point of view.

$$(3) k_3 = \text{median}\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right), i = 1, 2, \dots, p \quad (\text{Kibria, 2003}) \quad (2.7)$$

which is the median of $\hat{k}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$.

$$(4) k_4 = \frac{2\hat{\sigma}^2}{\lambda_{\max}(\prod_{i=1}^p \hat{\alpha}_i^2)^{1/p}}, i = 1, 2, \dots, p \quad (\text{Dorugade, 2014}) \quad (2.8)$$

which is the geometric mean of $\hat{k}_i = \frac{2\hat{\sigma}^2}{\lambda_{\max} \hat{\alpha}_i^2}$.

$$(5) k_5 = \frac{2p}{\lambda_{\max} \sum_{i=1}^p \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}, i = 1, 2, \dots, p \quad (\text{Dorugade, 2014}) \quad (2.9)$$

which is the harmonic mean of $\hat{k}_i = \frac{2\hat{\sigma}^2}{\lambda_{\max} \hat{\alpha}_i^2}$.

A sufficient condition that $\text{MSE}(\hat{\alpha}_{RR}) < \text{MSE}(\hat{\alpha})$ is given by Hoerl and Kennard (1970a,b) such that $k < k_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{\max}^2}$. A quick survey shows us that some of the existing ridge parameters are smaller than k_{HK} . However, if we try the estimators larger than k_{HK} , we observe that one can also have better estimators in sense of MSE.

In the figure given by Hoerl and Kennard (1970a,b), it is obvious that the first derivative of the function $\text{MSE}(\hat{\alpha}_{RR})$ is negative when the value of k_{HK} is used as the biasing parameter. Therefore, any estimator satisfying $0 < k < k_{HK}$ gives us a negative derivative. However, if we examine the intersection point of the variance and the squared bias functions, we see that it is absolutely greater than k_{HK} . Thus, one can find estimators such that the first derivative of the $\text{MSE}(\hat{\alpha}_{RR})$ function is positive and being greater than k_{HK} . There are greater estimators than k_{HK} in the literature, for example see Alkhamisi and Shukur, 2007 for the estimators k_{NAS} and k_{AS} .

It should also be pointed out that the optimal selection process of the parameter k in ridge regression cannot be truly provided from the theoretical point of view. Actually, this is an open problem to researchers. Thus we suggest some estimators which are modifications of $k_K = \frac{1}{p} \sum_{i=1}^p \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$ proposed in Kibria, 2003 $k_D = \frac{2p}{\lambda_{\max} \sum_{i=1}^p \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}$ and $k_D = \frac{2p}{\lambda_{\max} \sum_{i=1}^p \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}$ proposed in Dorugade, 2014. We apply some transformations and we fol-

Table 1 The AMSE as a function of σ^2 .

n	20				50				100			
$p = 4, \rho = 0.75$												
σ^2	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0
k_1	0.8018	4.3489	7.5183	39.0185	0.7438	3.7920	6.8246	27.9983	0.7029	2.9245	6.1673	23.5762
k_2	0.7727	4.7676	8.3778	44.2431	0.7133	4.0857	7.4317	28.0331	0.6664	3.0179	6.4671	23.1509
k_3	0.7846	5.0495	9.1799	48.9552	0.7428	4.3901	8.1411	31.7462	0.6884	3.3637	7.1451	27.2164
k_4	0.7167	4.4135	8.1842	44.2272	0.6709	3.9050	7.2784	28.5271	0.6226	3.0754	6.3872	25.1134
k_5	0.7088	3.6649	6.0314	29.9480	0.6477	3.2296	5.5757	22.1414	0.6069	2.4728	5.0739	18.7134
k_{N1}	0.6910	3.6451	6.0659	30.2548	0.6325	3.2132	5.5900	22.0714	0.5918	2.4698	5.0680	18.7983
k_{N2}	0.7298	3.7024	6.0289	29.7302	0.6691	3.2641	5.5984	22.4143	0.6299	2.5016	5.1228	18.8006
k_{N3}	0.7245	3.6846	6.0369	29.7471	0.6706	3.2559	5.6112	22.5370	0.6343	2.5223	5.1435	19.0303
k_{N4}	0.6862	4.3086	7.3445	36.7735	0.6309	3.7810	6.7037	25.8954	0.5828	2.8875	5.9905	22.0626
OLS	1.4625	8.6222	14.2958	71.7297	1.2800	7.3098	12.6382	50.2923	1.1755	5.4638	11.4942	42.1464
$p = 4, \rho = 0.85$												
σ^2	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0
k_1	2.5925	7.9881	12.6128	102.2997	1.4107	5.3567	11.8893	79.6890	1.2014	4.4455	10.2231	60.0734
k_2	2.8936	10.1650	15.3204	137.0792	1.4823	6.1005	14.3277	103.9478	1.2273	4.8987	11.7653	74.9186
k_3	2.9571	10.6945	15.7596	137.3515	1.5931	6.3183	14.4882	98.3121	1.2976	5.1854	12.1973	76.2568
k_4	2.6016	9.4165	13.5265	121.9175	1.4319	5.4256	12.2743	84.8187	1.1766	4.5210	10.4937	66.7519
k_5	2.2860	6.0829	9.6697	72.4183	1.2541	4.4396	9.4002	60.2166	1.0848	3.6822	7.9935	45.1846
k_{N1}	2.2572	6.1846	9.7280	73.4788	1.2377	4.4184	9.3862	59.9767	1.0650	3.6675	7.9889	45.3840
k_{N2}	2.2968	5.9923	9.6551	71.7758	1.2685	4.4704	9.4386	60.4528	1.1005	3.7128	8.0291	45.1771
k_{N3}	2.2387	6.0057	9.6579	72.5659	1.2449	4.4367	9.3929	59.9748	1.0752	3.6966	8.0014	45.1750
k_{N4}	2.6230	7.9527	12.1402	96.0898	1.4050	5.3006	11.4860	73.5664	1.1539	4.3835	9.7517	56.7623
OLS	5.3294	14.4634	23.1875	178.5352	2.7670	10.1480	21.6767	139.8967	2.2929	8.1819	18.1937	104.3610
$p = 4, \rho = 0.95$												
σ^2	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0
k_1	4.4843	17.6719	43.2137	285.2298	3.6713	15.2385	39.8144	211.9142	3.0023	13.8284	27.6675	187.9357
k_2	5.4332	24.3150	62.0304	428.1496	4.3521	20.3761	57.6365	317.5391	3.5398	18.2148	37.9291	276.4823
k_3	5.2084	21.7618	53.5358	369.9291	4.2803	18.1883	51.3576	277.0999	3.6190	16.7532	34.8457	241.7900
k_4	4.4122	17.5500	43.9138	326.1944	3.6350	14.6845	42.0855	242.7515	3.1331	13.6560	28.8147	212.4845
k_5	3.8087	13.6617	32.3764	205.8630	3.1348	11.9416	29.4144	153.8375	2.5679	10.7092	20.8151	137.6582
k_{N1}	3.7673	13.5906	32.2286	206.1835	3.1006	11.8395	29.4070	153.9551	2.5506	10.6571	20.7991	137.6976
k_{N2}	3.8173	13.6882	32.4253	205.9081	3.1437	11.9751	29.4385	153.9099	2.5752	10.7345	20.8414	137.7477
k_{N3}	3.7398	13.6022	32.3259	209.1952	3.0774	11.7948	29.6282	155.4830	2.5457	10.6581	20.8693	138.7788
k_{N4}	4.4834	16.7420	39.8726	263.3648	3.6897	14.3511	37.4322	196.8083	3.0924	13.1481	26.1427	174.9628
OLS	9.0396	32.5136	78.5206	514.2492	7.1454	27.9423	68.4895	361.1289	5.7255	24.6632	48.0141	321.5286
$p = 8, \rho = 0.75$												
σ^2	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0
k_1	2.2324	7.9806	20.7148	98.3940	1.7880	7.4095	15.8289	89.4041	1.7462	7.2615	13.0052	82.4673
k_2	2.2673	8.1524	25.6294	122.2006	1.8878	7.8641	18.0645	108.6631	1.7441	7.6161	13.9494	97.8043
k_3	2.6692	8.7870	27.0009	125.1359	2.2413	8.5847	19.2593	111.2952	1.9512	8.2726	15.0922	100.5385
k_4	2.3364	7.6846	23.7623	107.8760	2.0024	7.5203	16.8049	96.3113	1.7414	7.3143	13.2117	86.9273
k_5	2.1026	7.7269	17.9763	85.8827	1.6169	7.0094	14.4084	79.5819	1.6307	6.8876	12.0654	74.4692
k_{N1}	2.0667	7.5343	17.9416	84.9353	1.6063	6.8763	14.2413	78.6360	1.6036	6.7574	11.8623	73.4395
k_{N2}	2.0581	7.4499	17.9446	84.9656	1.6090	6.8610	14.2226	78.3751	1.5976	6.7238	11.8579	73.1182
k_{N3}	2.1998	8.1117	18.5642	90.3647	1.6988	7.4657	15.0726	82.3965	1.7116	7.2623	12.8917	77.3164
k_{N4}	2.3118	7.9974	22.2105	101.7014	1.8764	7.6855	16.6246	92.7855	1.7125	7.4613	13.3351	85.0792
OLS	4.2967	15.5416	38.8491	185.4425	3.1334	13.8163	28.8404	159.1808	2.9932	13.1563	23.3809	145.3175
$p = 8, \rho = 0.85$												
σ^2	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0
k_1	3.1444	21.4056	54.3085	202.0053	2.5682	17.7387	36.0596	156.8762	2.4620	10.8207	25.1449	119.7675
k_2	3.3571	28.3796	75.8952	284.6723	2.6303	22.1753	47.4561	211.9725	2.5101	12.2457	30.9778	154.2272
k_3	3.8979	29.1936	72.4889	275.8329	2.9825	22.5715	46.6600	201.8615	2.7762	12.8455	31.2573	150.6953
k_4	3.3834	25.8376	59.1447	238.7088	2.6130	19.3626	39.2020	172.1409	2.4645	11.0033	26.5278	128.3152
k_5	2.9414	17.5096	47.5644	170.9078	2.4781	15.6459	31.5818	139.3364	2.3925	10.2494	22.8063	108.1431
k_{N1}	2.8966	17.6051	46.9861	170.3857	2.4293	15.5167	31.2769	137.6626	2.3448	10.0590	22.5121	106.6644
k_{N2}	2.8683	17.6876	46.4738	170.4134	2.3926	15.4309	31.0563	136.3521	2.3049	9.9242	22.3054	105.6248
k_{N3}	2.9750	17.4701	46.9108	171.0634	2.5064	15.6132	31.4888	139.1297	2.4100	10.4023	22.9707	109.0800
k_{N4}	3.3558	23.5494	57.9280	219.3679	2.6441	19.0878	38.4865	165.6201	2.4915	11.3337	26.5494	125.2557
OLS	6.0621	39.0492	101.3570	375.4153	4.6611	31.7765	63.3548	279.4298	4.3699	19.6167	44.6007	209.6424

(continued on next page)

Table 1 (continued)

n	20				50				100			
$p = 8, \rho = 0.95$												
σ^2	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0	0.1	0.5	1.0	5.0
k_1	10.1517	62.3589	256.3495	510.1337	8.3145	43.598	114.5677	443.13	7.0777	38.516	72.2927	441.6436
k_2	12.4626	91.64	394.4763	797.0015	10.055	61.718	175.5814	679.2201	8.1842	53.2412	104.6281	677.2535
k_3	12.9914	84.9505	367.1545	668.6289	10.5799	55.2194	150.651	586.6407	8.6537	50.1042	92.8478	598.2785
k_4	10.7853	69.2146	320.9709	554.7026	9.163	44.3726	122.6003	497.4623	7.4245	41.7137	76.8874	516.3498
k_5	9.3234	53.168	195.5032	464.0262	7.4015	40.5554	103.2034	388.1869	6.6582	33.5896	65.7234	376.9323
k_{N1}	9.1688	52.6306	196.4644	455.3866	7.3301	39.7718	101.5887	383.3861	6.5494	33.2534	64.7785	374.3269
k_{N2}	8.9755	52.1332	199.8805	443.7877	7.2614	38.6766	99.4564	378.0306	6.4089	32.921	63.6095	372.6178
k_{N3}	8.9577	52.1043	201.9508	441.7225	7.2644	38.5061	98.9504	377.8803	6.4152	32.9412	63.6302	372.6184
k_{N4}	10.9554	67.0785	283.3606	534.8243	9.0699	45.7474	121.9183	472.5697	7.5963	41.3113	76.988	481.2463

low Khalaf and Shukur (2005) and Alkhamisi and Shukur (2007) in order to get some estimators being greater than k_{HK} and having better performances. The first two estimators are smaller than k_{HK} and others are greater than it.

The following are our proposed estimators:

$$(1) k_{N1} = \frac{\sqrt{5}p}{\lambda_{\max}} \frac{\hat{\sigma}^2}{\sum_{i=1}^p \hat{\alpha}_i^2} \tag{2.10}$$

We suggest the modification by multiplying $\frac{\lambda_{\max}}{\sqrt{5}}$ to the denominator of (2.4). Thus the suggested estimator is $\frac{\sqrt{5}\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$. This is an estimator having a denominator greater than that of Hoerl and Kennard, 1970a. Thus, we can write $\frac{\hat{\alpha}_i^2}{\lambda_{\max}\hat{\alpha}_i^2} \geq \frac{\sqrt{5}\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}, i = 1, 2, \dots, p$. Finally, we use harmonic mean function and get the new estimator given in Eq. (2.10).

$$(2) k_{N2} = \frac{p}{\sqrt{\lambda_{\max}}} \frac{\hat{\sigma}^2}{\sum_{i=1}^p \hat{\alpha}_i^2} \tag{2.11}$$

Similar to the above discussion, we multiply the denominator of (2.4) by λ_{\max} and we get $\frac{\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$. Again, we have $\frac{\hat{\alpha}_i^2}{\lambda_{\max}\hat{\alpha}_i^2} \geq \frac{\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$ showing that this new estimator is clearly smaller than (2.4). Taking the harmonic mean, we finally get the new estimator given in (2.11).

$$(3) k_{N3} = \frac{2p}{\sum_{i=1}^p (\lambda_i^{1/4})} \frac{\hat{\sigma}^2}{\sum_{i=1}^p \hat{\alpha}_i^2} \tag{2.12}$$

$$(4) k_{N4} = \frac{2p}{\sqrt{\sum_{i=1}^p \lambda_i}} \frac{\hat{\sigma}^2}{\sum_{i=1}^p \hat{\alpha}_i^2} \tag{2.13}$$

We have $\sum_{i=1}^p \lambda_i > p$ because the matrix $X'X$ is in the correlation form. Thus, the new proposed estimators k_{N3} and k_{N4} are definitely greater than k_{HK} . All the above parameters will be compared by a Monte Carlo simulation and the whole process is explained in Section 3.

3. The Monte Carlo simulation

In this section, a Monte Carlo simulation has been conducted to compare the performances of the estimators. There are two criteria used to design a good Monte Carlo simulation. One of them is to specify what factors are expected to affect the properties of the estimators and the other is to determine the crite-

riion of judgment. We decided that the effective factors are the data size n , the number of explanatory variables p , the correlation between the explanatory variables ρ and the variance of error terms σ^2 . Mean squared error (MSE) will be the criterion to compare the performances of the estimators. In the simulation, we examined the average MSE (AMSE) of the ridge parameters. Now, we give details of the study.

The mean squared error of the ridge estimator $\hat{\beta}_R$ is

$$\begin{aligned} \text{MSE}(\hat{\beta}_R) &= \text{Var}(\hat{\beta}_R) + [\text{Bias}(\hat{\beta}_R)]^2 \\ &= \sigma^2 \sum_{i=1}^p \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^p \frac{\hat{\alpha}_i^2}{(\lambda_i + k)^2} \end{aligned} \tag{3.1}$$

Although we reviewed 41 different estimators for estimating the ridge parameter k , we finally consider k_1, k_2, k_3, k_4, k_5 from the literature and new proposed k_{N1}, k_{N2}, k_{N3} and k_{N4} of them.

The true model $Y = X\beta + \varepsilon$ is considered with independent $\varepsilon \sim N(0, \sigma^2)$ and β is chosen such that $\beta'\beta = 1$ since Newhouse and Oman, 1971 stated that if β is taken to be the eigenvector of the largest eigenvalue of the matrix $X'X$ then the MSE is minimized.

To generate the explanatory variables, we used the following commonly used process:

$x_{ji} = (1 - \rho^2)^{1/2} z_{ji} + \rho z_{jp}, j = 1, 2, \dots, n$ and $i = 1, 2, \dots, p$ where ρ^2 represents the correlation between the explanatory variables and z_{ij} 's are independent, random numbers following the standard normal distribution. Also, the dependent variable Y is generated by

$Y_j = \beta_1 x_{j1} + \beta_2 x_{j2} + \dots + \beta_p x_{jp} + \varepsilon_j, j = 1, 2, \dots, n$ where ε_j 's are independent normal pseudorandom numbers with zero mean and variance σ^2 .

Here, we consider the cases $n = 20, 50, 100; \rho = 0.75, 0.85, 0.95; p = 4, 8$ and $\sigma^2 = 0.1, 0.5, 1.0, 5.0$. After generating the explanatory variables X and the dependent variable Y , we standardized both of them so that $X'X$ and $X'Y$ are in the correlation form.

For the values of n, p, ρ and σ^2 , the experiment was repeated 10,000 times by generating the error terms in the Eq. (1.1). After this procedure, for each replicate $\text{MSE}_{OLS}, \text{MSE}_{RR}$ and the average mean squared error (AMSE) for each estimator are calculated for each of the values (n, p, ρ, σ^2) such that

$$\text{AMSE}(\hat{\alpha}) = \frac{1}{10000} \sum_{r=1}^{10000} \text{MSE}(\hat{\alpha}) \tag{3.2}$$

Table 2 The AMSE as a function of ρ .

σ^2	0.1	0.5				1.0				5.0			
<i>n</i> = 20, <i>p</i> = 4													
ρ	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	
k_1	0.8018	2.5925	4.4843	4.3489	7.9881	17.6719	7.5183	12.6128	43.2137	39.0185	102.2997	285.2298	
k_2	0.7727	2.8936	5.4332	4.7676	10.1650	24.3150	8.3778	15.3204	62.0304	44.2431	137.0792	428.1496	
k_3	0.7846	2.9571	5.2084	5.0495	10.6945	21.7618	9.1799	15.7596	53.5358	48.9552	137.3515	369.9291	
k_4	0.7167	2.6016	4.4122	4.4135	9.4165	17.5500	8.1842	13.5265	43.9138	44.2272	121.9175	326.1944	
k_5	0.7088	2.2860	3.8087	3.6649	6.0829	13.6617	6.0314	9.6697	32.3764	29.9480	72.4183	205.8630	
k_{N1}	0.6910	2.2572	3.7673	3.6451	6.1846	13.5906	6.0659	9.7280	32.2286	30.2548	73.4788	206.1835	
k_{N2}	0.7298	2.2968	3.8173	3.7024	5.9923	13.6882	6.0289	9.6551	32.4253	29.7302	71.7758	205.9081	
k_{N3}	0.7245	2.2387	3.7398	3.6846	6.0057	13.6022	6.0369	9.6579	32.3259	29.7471	72.5659	209.1952	
k_{N4}	0.6862	2.6230	4.4834	4.3086	7.9527	16.7420	7.3445	12.1402	39.8726	36.7735	96.0898	263.3648	
OLS	1.4625	5.3294	9.0396	8.6222	14.4634	32.5136	14.2958	23.1875	78.5206	71.7297	178.5352	514.2492	
<i>n</i> = 50, <i>p</i> = 4													
ρ	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	
k_1	0.7438	1.4107	3.6713	3.7920	5.3567	15.2385	6.8246	11.8893	39.8144	27.9983	79.6890	211.9142	
k_2	0.7133	1.4823	4.3521	4.0857	6.1005	20.3761	7.4317	14.3277	57.6365	28.0331	103.9478	317.5391	
k_3	0.7428	1.5931	4.2803	4.3901	6.3183	18.1883	8.1411	14.4882	51.3576	31.7462	98.3121	277.0999	
k_4	0.6709	1.4319	3.6350	3.9050	5.4256	14.6845	7.2784	12.2743	42.0855	28.5271	84.8187	242.7515	
k_5	0.6477	1.2541	3.1348	3.2296	4.4396	11.9416	5.5757	9.4002	29.4144	22.1414	60.2166	153.8375	
k_{N1}	0.6325	1.2377	3.1006	3.2132	4.4184	11.8395	5.5900	9.3862	29.4070	22.0714	59.9767	153.9551	
k_{N2}	0.6691	1.2685	3.1437	3.2641	4.4704	11.9751	5.5984	9.4386	29.4385	22.4143	60.4528	153.9099	
k_{N3}	0.6706	1.2449	3.0774	3.2559	4.4367	11.7948	5.6112	9.3929	29.6282	22.5370	59.9748	155.4830	
k_{N4}	0.6309	1.4050	3.6897	3.7810	5.3006	14.3511	6.7037	11.4860	37.4322	25.8954	73.5664	196.8083	
OLS	1.2800	2.7670	7.1454	7.3098	10.1480	27.9423	12.6382	21.6767	68.4895	50.2923	139.8967	361.1289	
<i>n</i> = 100, <i>p</i> = 4													
ρ	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	
k_1	0.7029	1.2014	3.0023	2.9245	4.4455	13.8284	6.1673	10.2231	27.6675	23.5762	60.0734	187.9357	
k_2	0.6664	1.2273	3.5398	3.0179	4.8987	18.2148	6.4671	11.7653	37.9291	23.1509	74.9186	276.4823	
k_3	0.6884	1.2976	3.6190	3.3637	5.1854	16.7532	7.1451	12.1973	34.8457	27.2164	76.2568	241.7900	
k_4	0.6226	1.1766	3.1331	3.0754	4.5210	13.6560	6.3872	10.4937	28.8147	25.1134	66.7519	212.4845	
k_5	0.6069	1.0848	2.5679	2.4728	3.6822	10.7092	5.0739	7.9935	20.8151	18.7134	45.1846	137.6582	
k_{N1}	0.5918	1.0650	2.5506	2.4698	3.6675	10.6571	5.0680	7.9889	20.7991	18.7983	45.3840	137.6976	
k_{N2}	0.6299	1.1005	2.5752	2.5016	3.7128	10.7345	5.1228	8.0291	20.8414	18.8006	45.1771	137.7477	
k_{N3}	0.6343	1.0752	2.5457	2.5223	3.6966	10.6581	5.1435	8.0014	20.8693	19.0303	45.1750	138.7788	
k_{N4}	0.5828	1.1539	3.0924	2.8875	4.3835	13.1481	5.9905	9.7517	26.1427	22.0626	56.7623	174.9628	
OLS	1.1755	2.2929	5.7255	5.4638	8.1819	24.6632	11.4942	18.1937	48.0141	42.1464	104.3610	321.5286	
<i>n</i> = 20, <i>p</i> = 8													
ρ	0.75	0.85	0.95	0.75	0.85	0.95	0.75	0.85	0.95	0.75	0.85	0.95	
k_1	2.2324	3.1444	10.1517	7.9806	21.4056	62.3589	20.7148	54.3085	256.3495	98.3940	202.0053	510.1337	
k_2	2.2673	3.3571	12.4626	8.1524	28.3796	91.6400	25.6294	75.8952	394.4763	122.2006	284.6723	797.0015	
k_3	2.6692	3.8979	12.9914	8.7870	29.1936	84.9505	27.0009	72.4889	367.1545	125.1359	275.8329	668.6289	
k_4	2.3364	3.3834	10.7853	7.6846	25.8376	69.2146	23.7623	59.1447	320.9709	107.8760	238.7088	554.7026	
k_5	2.1026	2.9414	9.3234	7.7269	17.5096	53.1680	17.9763	47.5644	195.5032	85.8827	170.9078	464.0262	
k_{N1}	2.0667	2.8966	9.1688	7.5343	17.6051	52.6306	17.9416	46.9861	196.4644	84.9353	170.3857	455.3866	
k_{N2}	2.0581	2.8683	8.9755	7.4499	17.6876	52.1332	17.9446	46.4738	199.8805	84.9656	170.4134	443.7877	
k_{N3}	2.1998	2.9750	8.9577	8.1117	17.4701	52.1043	18.5642	46.9108	201.9508	90.3647	171.0634	441.7225	
k_{N4}	2.3118	3.3558	10.9554	7.9974	23.5494	67.0785	22.2105	57.9280	283.3606	101.7014	219.3679	534.8243	
OLS	4.2967	6.0621	19.5364	15.5416	39.0492	116.0329	38.8491	101.3570	453.3480	185.4425	375.4153	961.3745	
<i>n</i> = 50, <i>p</i> = 8													
ρ	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	
k_1	1.7880	2.5682	8.3145	7.4095	17.7387	43.5980	15.8289	36.0596	114.5677	89.4041	156.8762	443.1300	
k_2	1.8878	2.6303	10.0550	7.8641	22.1753	61.7180	18.0645	47.4561	175.5814	108.6631	211.9725	679.2201	
k_3	2.2413	2.9825	10.5799	8.5847	22.5715	55.2194	19.2593	46.6600	150.6510	111.2952	201.8615	586.6407	
k_4	2.0024	2.6130	9.1630	7.5203	19.3626	44.3726	16.8049	39.2020	122.6003	96.3113	172.1409	497.4623	
k_5	1.6169	2.4781	7.4015	7.0094	15.6459	40.5554	14.4084	31.5818	103.2034	79.5819	139.3364	388.1869	
k_{N1}	1.6063	2.4293	7.3301	6.8763	15.5167	39.7718	14.2413	31.2769	101.5887	78.6360	137.6626	383.3861	
k_{N2}	1.6090	2.3926	7.2614	6.8610	15.4309	38.6766	14.2226	31.0563	99.4564	78.3751	136.3521	378.0306	
k_{N3}	1.6988	2.5064	7.2644	7.4657	15.6132	38.5061	15.0726	31.4888	98.9504	82.3965	139.1297	377.8803	
k_{N4}	1.8764	2.6441	9.0699	7.6855	19.0878	45.7474	16.6246	38.4865	121.9183	92.7855	165.6201	472.5697	
OLS	3.1334	4.6611	14.9055	13.8163	31.7765	78.4528	28.8404	63.3548	201.7302	159.1808	279.4298	780.6791	

(continued on next page)

Table 2 (continued)

σ^2	0.1		0.5		1.0		5.0					
$n = 100, p = 8$												
ρ	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75	0.75	0.85	0.95	0.75
k_1	1.7462	2.4620	7.0777	7.2615	10.8207	38.5160	13.0052	25.1449	72.2927	82.4673	119.7675	441.6436
k_2	1.7441	2.5101	8.1842	7.6161	12.2457	53.2412	13.9494	30.9778	104.6281	97.8043	154.2272	677.2535
k_3	1.9512	2.7762	8.6537	8.2726	12.8455	50.1042	15.0922	31.2573	92.8478	100.5385	150.6953	598.2785
k_4	1.7414	2.4645	7.4245	7.3143	11.0033	41.7137	13.2117	26.5278	76.8874	86.9273	128.3152	516.3498
k_5	1.6307	2.3925	6.6582	6.8876	10.2494	33.5896	12.0654	22.8063	65.7234	74.4692	108.1431	376.9323
k_{N1}	1.6036	2.3448	6.5494	6.7574	10.0590	33.2534	11.8623	22.5121	64.7785	73.4395	106.6644	374.3269
k_{N2}	1.5976	2.3049	6.4089	6.7238	9.9242	32.9210	11.8579	22.3054	63.6095	73.1182	105.6248	372.6178
k_{N3}	1.7116	2.4100	6.4152	7.2623	10.4023	32.9412	12.8917	22.9707	63.6302	77.3164	109.0800	372.6184
k_{N4}	1.7125	2.4915	7.5963	7.4613	11.3337	41.3113	13.3351	26.5494	76.9880	85.0792	125.2557	481.2463
OLS	2.9932	4.3699	12.6329	13.1563	19.6167	66.3018	23.3809	44.6007	127.7137	145.3175	209.6424	758.4431

and results are given in Tables 1–3. We also computed biases of the ridge parameters and reported results in Figs. 1–12.

4. Results of the simulation

According to the results of the simulation, we get the following Tables 1–3 which show the average mean squared error (AMSE) values for different numbers of observation, number of explanatory variables, variances and the correlation values. We also give some of our important findings in terms of figures especially for some of the cases in which n or ρ changes when the others are fixed. Additionally, we give the comparison of biases in terms of figures for similar consideration. We did not give the tables of biases since they are too large.

4.1. Comparison of the estimators according to the AMSEs

4.1.1. Comparison according to the variances σ^2

In Table 1, we have given the average mean squared error values of the estimators as a function of the variances. We can see the change of AMSEs according to the variances of the errors (σ^2). It is obvious that when σ^2 increases, the AMSE of the estimators increases. For all of the cases, AMSE of the OLS estimator is larger than the AMSE of the new proposed ridge estimators. In most of the cases, the estimators k_{N1} , k_{N2} , k_{N3} , k_{N4} dominate the estimators k_1 , k_2 and k_3 . However, the performance of the proposed estimators k_{N2} , k_{N3} and k_{N4} (at least one of them) are the best in all cases.

For given values $n = 20$, $p = 4$, $\rho = 0.75$ and $n = 20$, $p = 8$, $\rho = 0.75$, the performances of the estimators are given in Figs. 1 and 2 respectively. We can see from these figures that as σ^2 changes from 0.1 to 5.0, the AMSE values of the estimators increase. The number of explanatory variables p has a great effect on multicollinearity. If there are more variables correlated in the model, the effect of collinearity increases. In these figures, there is a change in the number of explanatory variables. In the case of $p = 8$, the AMSEs are larger than the former case. Actually, changing $p = 4$ to $p = 8$, fixing n, σ^2 and ρ , we see that there is an increase in the AMSEs in all cases.

4.1.2. Comparison according to the correlation ρ

In Table 2, we have given the AMSE values as a function of the correlation ρ . If we fix n and p , we generally see that the

AMSE values increase when the correlation increases. The performances of the estimators k_{N1} , k_{N2} , k_{N3} and k_{N4} are better than the other estimators. For given values $n = 20$, $p = 4$, $\sigma^2 = 0.1$ and $n = 20$, $p = 4$, $\sigma^2 = 5.0$, performances of the estimators are given in Figs. 3 and 4 respectively.

According to these figures, for smaller values of σ^2 , the change in the correlation gives a small increase in the AMSE values. For each combination of the sample size n and the number of variables p , the smaller the correlation, the smaller the AMSE values. However, the change in the correlation gives a large increase in AMSE values when jumping from $\sigma^2 = 0.1$ to $\sigma^2 = 5.0$. In all situations the OLS estimator has a larger AMSE compared to all the ridge estimators.

4.1.3. Comparison according to the sample size n

In Table 3, we have given the AMSE values of the estimators as a function of the sample size n . If p and ρ are fixed, we generally see that the AMSE values decrease when the data size n increases. The performances of the estimators k_{N1} , k_{N2} , k_{N3} are again better than the rest of the estimators. Sometimes k_5 dominates one of k_{N1} , k_{N2} , k_{N3} but not all of them. For given values $p = 8$ and $\rho = 0.85$ the performances of the estimators for $\sigma^2 = 0.1$ and $\sigma^2 = 1.0$ are given in Figs. 5 and 6 respectively. We can say that there is a big amount of increase in the AMSE when jumping from $\sigma^2 = 0.1$ to $\sigma^2 = 1.0$. We did not include the line of AMSE values of the OLS estimator in the graph because if it is included, the scale becomes very large so that the difference between the estimators could not be seen from the figures.

It is obvious from Figs. 5 and 6 that k_{N3} is the best estimator for the given case and the AMSE decreases when the sample size increases. In general, when $p = 4$, k_{N1} has the best performance and if $p = 8$, then k_{N3} has the best performance for all cases.

4.2. Comparison of the estimators according to the biases

In this simulation study, we also considered biases of estimators. We know that some of the researchers need small biased estimators while the others only need estimators having small MSE. In this section, we compare biases of some selected estimators having least biases in the simulation. We only provide some graphs and make our comments using them.

Table 3 The AMSE as a function of n.

σ^2	0.1			0.5			1.0			5.0		
$\rho = 0.75, p = 4$												
n	20	50	100	20	50	100	20	50	100	20	50	100
k_1	0.8018	0.7438	0.7029	4.3489	3.7920	2.9245	7.5183	6.8246	6.1673	39.0185	27.9983	23.5762
k_2	0.7727	0.7133	0.6664	4.7676	4.0857	3.0179	8.3778	7.4317	6.4671	44.2431	28.0331	23.1509
k_3	0.7846	0.7428	0.6884	5.0495	4.3901	3.3637	9.1799	8.1411	7.1451	48.9552	31.7462	27.2164
k_4	0.7167	0.6709	0.6226	4.4135	3.9050	3.0754	8.1842	7.2784	6.3872	44.2272	28.5271	25.1134
k_5	0.7088	0.6477	0.6069	3.6649	3.2296	2.4728	6.0314	5.5757	5.0739	29.9480	22.1414	18.7134
k_{N1}	0.6910	0.6325	0.5918	3.6451	3.2132	2.4698	6.0659	5.5900	5.0680	30.2548	22.0714	18.7983
k_{N2}	0.7298	0.6691	0.6299	3.7024	3.2641	2.5016	6.0289	5.5984	5.1228	29.7302	22.4143	18.8006
k_{N3}	0.7245	0.6706	0.6343	3.6846	3.2559	2.5223	6.0369	5.6112	5.1435	29.7471	22.5370	19.0303
k_{N4}	0.6862	0.6309	0.5828	4.3086	3.7810	2.8875	7.3445	6.7037	5.9905	36.7735	25.8954	22.0626
OLS	1.4625	1.2800	1.1755	8.6222	7.3098	5.4638	14.2958	12.6382	11.4942	71.7297	50.2923	42.1464
$\rho = 0.85, p = 4$												
n	20	50	100	20	50	100	20	50	100	20	50	100
k_1	2.5925	1.4107	1.2014	7.9881	5.3567	4.4455	12.6128	11.8893	10.2231	102.2997	79.6890	60.0734
k_2	2.8936	1.4823	1.2273	10.1650	6.1005	4.8987	15.3204	14.3277	11.7653	137.0792	103.9478	74.9186
k_3	2.9571	1.5931	1.2976	10.6945	6.3183	5.1854	15.7596	14.4882	12.1973	137.3515	98.3121	76.2568
k_4	2.6016	1.4319	1.1766	9.4165	5.4256	4.5210	13.5265	12.2743	10.4937	121.9175	84.8187	66.7519
k_5	2.2860	1.2541	1.0848	6.0829	4.4396	3.6822	9.6697	9.4002	7.9935	72.4183	60.2166	45.1846
k_{N1}	2.2572	1.2377	1.0650	6.1846	4.4184	3.6675	9.7280	9.3862	7.9889	73.4788	59.9767	45.3840
k_{N2}	2.2968	1.2685	1.1005	5.9923	4.4704	3.7128	9.6551	9.4386	8.0291	71.7758	60.4528	45.1771
k_{N3}	2.2387	1.2449	1.0752	6.0057	4.4367	3.6966	9.6579	9.3929	8.0014	72.5659	59.9748	45.1750
k_{N4}	2.6230	1.4050	1.1539	7.9527	5.3006	4.3835	12.1402	11.4860	9.7517	96.0898	73.5664	56.7623
OLS	5.3294	2.7670	2.2929	14.4634	10.1480	8.1819	23.1875	21.6767	18.1937	178.5352	139.8967	104.3610
$\rho = 0.95, p = 4$												
n	20	50	100	20	50	100	20	50	100	20	50	100
k_1	4.4843	3.6713	3.0023	17.6719	15.2385	13.8284	43.2137	39.8144	27.6675	285.2298	211.9142	187.9357
k_2	5.4332	4.3521	3.5398	24.3150	20.3761	18.2148	62.0304	57.6365	37.9291	428.1496	317.5391	276.4823
k_3	5.2084	4.2803	3.6190	21.7618	18.1883	16.7532	53.5358	51.3576	34.8457	369.9291	277.0999	241.7900
k_4	4.4122	3.6350	3.1331	17.5500	14.6845	13.6560	43.9138	42.0855	28.8147	326.1944	242.7515	212.4845
k_5	3.8087	3.1348	2.5679	13.6617	11.9416	10.7092	32.3764	29.4144	20.8151	205.8630	153.8375	137.6582
k_{N1}	3.7673	3.1006	2.5506	13.5906	11.8395	10.6571	32.2286	29.4070	20.7991	206.1835	153.9551	137.6976
k_{N2}	3.8173	3.1437	2.5752	13.6882	11.9751	10.7345	32.4253	29.4385	20.8414	205.9081	153.9099	137.7477
k_{N3}	3.7398	3.0774	2.5457	13.6022	11.7948	10.6581	32.3259	29.6282	20.8693	209.1952	155.4830	138.7788
k_{N4}	4.4834	3.6897	3.0924	16.7420	14.3511	13.1481	39.8726	37.4322	26.1427	263.3648	196.8083	174.9628
OLS	9.0396	7.1454	5.7255	32.5136	27.9423	24.6632	78.5206	68.4895	48.0141	514.2492	361.1289	321.5286
$\rho = 0.75, p = 8$												
n	20	50	100	20	50	100	20	50	100	20	50	100
k_1	2.2324	1.7880	1.7462	7.9806	7.4095	7.2615	20.7148	15.8289	13.0052	98.3940	89.4041	82.4673
k_2	2.2673	1.8878	1.7441	8.1524	7.8641	7.6161	25.6294	18.0645	13.9494	122.2006	108.6631	97.8043
k_3	2.6692	2.2413	1.9512	8.7870	8.5847	8.2726	27.0009	19.2593	15.0922	125.1359	111.2952	100.5385
k_4	2.3364	2.0024	1.7414	7.6846	7.5203	7.3143	23.7623	16.8049	13.2117	107.8760	96.3113	86.9273
k_5	2.1026	1.6169	1.6307	7.7269	7.0094	6.8876	17.9763	14.4084	12.0654	85.8827	79.5819	74.4692
k_{N1}	2.0667	1.6063	1.6036	7.5343	6.8763	6.7574	17.9416	14.2413	11.8623	84.9353	78.6360	73.4395
k_{N2}	2.0581	1.6090	1.5976	7.4499	6.8610	6.7238	17.9446	14.2226	11.8579	84.9656	78.3751	73.1182
k_{N3}	2.1998	1.6988	1.7116	8.1117	7.4657	7.2623	18.5642	15.0726	12.8917	90.3647	82.3965	77.3164
k_{N4}	2.3118	1.8764	1.7125	7.9974	7.6855	7.4613	22.2105	16.6246	13.3351	101.7014	92.7855	85.0792
OLS	4.2967	3.1334	2.9932	15.5416	13.8163	13.1563	38.8491	28.8404	23.3809	185.4425	159.1808	145.3175
$\rho = 0.85, p = 8$												
n	20	50	100	20	50	100	20	50	100	20	50	100
k_1	3.1444	2.5682	2.4620	21.4056	17.7387	10.8207	54.3085	36.0596	25.1449	202.0053	156.8762	119.7675
k_2	3.3571	2.6303	2.5101	28.3796	22.1753	12.2457	75.8952	47.4561	30.9778	284.6723	211.9725	154.2272
k_3	3.8979	2.9825	2.7762	29.1936	22.5715	12.8455	72.4889	46.6600	31.2573	275.8329	201.8615	150.6953
k_4	3.3834	2.6130	2.4645	25.8376	19.3626	11.0033	59.1447	39.2020	26.5278	238.7088	172.1409	128.3152
k_5	2.9414	2.4781	2.3925	17.5096	15.6459	10.2494	47.5644	31.5818	22.8063	170.9078	139.3364	108.1431
k_{N1}	2.8966	2.4293	2.3448	17.6051	15.5167	10.0590	46.9861	31.2769	22.5121	170.3857	137.6626	106.6644
k_{N2}	2.8683	2.3926	2.3049	17.6876	15.4309	9.9242	46.4738	31.0563	22.3054	170.4134	136.3521	105.6248
k_{N3}	2.9750	2.5064	2.4100	17.4701	15.6132	10.4023	46.9108	31.4888	22.9707	171.0634	139.1297	109.0800
k_{N4}	3.3558	2.6441	2.4915	23.5494	19.0878	11.3337	57.9280	38.4865	26.5494	219.3679	165.6201	125.2557
OLS	6.0621	4.6611	4.3699	39.0492	31.7765	19.6167	101.3570	63.3548	44.6007	375.4153	279.4298	209.6424

(continued on next page)

Table 3 (continued)

σ^2	0.1			0.5			1.0			5.0		
$\rho = 0.95, p = 8$												
n	20	50	100	20	50	100	20	50	100	20	50	100
k_1	10.1517	8.3145	7.0777	62.3589	43.5980	38.5160	256.3495	114.5677	72.2927	510.1337	443.1300	441.6436
k_2	12.4626	10.0550	8.1842	91.6400	61.7180	53.2412	394.4763	175.5814	104.6281	797.0015	679.2201	677.2535
k_3	12.9914	10.5799	8.6537	84.9505	55.2194	50.1042	367.1545	150.6510	92.8478	668.6289	586.6407	598.2785
k_4	10.7853	9.1630	7.4245	69.2146	44.3726	41.7137	320.9709	122.6003	76.8874	554.7026	497.4623	516.3498
k_5	9.3234	7.4015	6.6582	53.1680	40.5554	33.5896	195.5032	103.2034	65.7234	464.0262	388.1869	376.9323
k_{N1}	9.1688	7.3301	6.5494	52.6306	39.7718	33.2534	196.4644	101.5887	64.7785	455.3866	383.3861	374.3269
k_{N2}	8.9755	7.2614	6.4089	52.1332	38.6766	32.9210	199.8805	99.4564	63.6095	443.7877	378.0306	372.6178
k_{N3}	8.9577	7.2644	6.4152	52.1043	38.5061	32.9412	201.9508	98.9504	63.6302	441.7225	377.8803	372.6184
k_{N4}	10.9554	9.0699	7.5963	67.0785	45.7474	41.3113	283.3606	121.9183	76.9880	534.8243	472.5697	481.2463
OLS	19.5364	14.9055	12.6329	116.0329	78.4528	66.3018	453.3480	201.7302	127.7137	961.3745	780.6791	758.4431

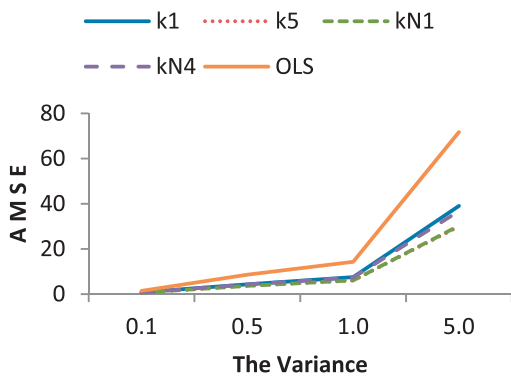


Figure 1 $n = 20, p = 4, \rho = 0.75$.

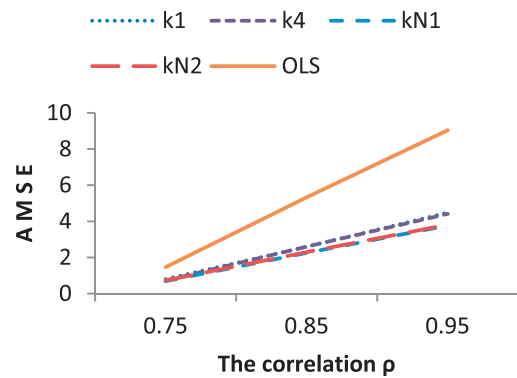


Figure 3 $n = 20, p = 4, \sigma^2 = 0.1$.

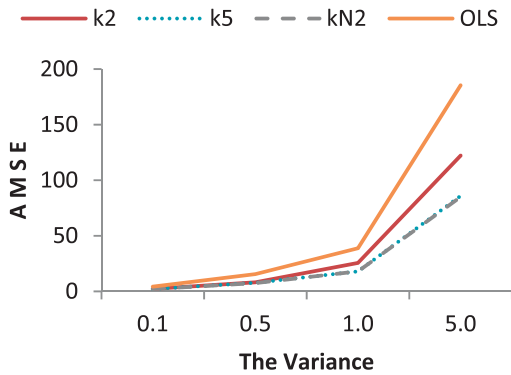


Figure 2 $n = 20, p = 8, \rho = 0.75$.

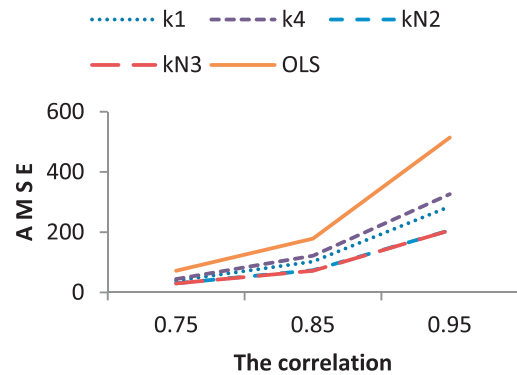


Figure 4 $n = 20, p = 4, \sigma^2 = 5.0$.

4.2.1. Comparison according to the variance σ^2

In the previous section, we see that an increase in the variance of the errors σ^2 makes an increase in the mean squared error. Similarly, there is an increase in biases when we increase the variances of errors. k_1, k_5, k_{N1} and k_{N3} are the selected estimators to be compared. For given cases $n = 20, p = 4, \rho = 0.75$ and $n = 20, p = 8, \rho = 0.75$, biases of the selected estimators are given in Figs. 7 and 8 respectively.

From these figures, we see that increasing the variance gives an increase in biases. k_{N1} has a better performance i.e. it has a

small bias among the estimators k_1, k_5, k_{N1} and k_{N3} for the given cases. It is obvious that if we increase the number of variable p from 4 to 8, then there is a small increase in the bias values fixing n and ρ . This is valid for all similar cases.

4.2.2. Comparison according to the correlation ρ

When the correlation ρ increases, biases of estimators increase. In most cases, the estimators k_{N1} and k_{N3} have the least biases. Especially when $p = 8, k_5$ is better than k_{N3} but k_{N1} is again the best estimator. For given values $n = 20, p = 4, \sigma^2 = 0.1$

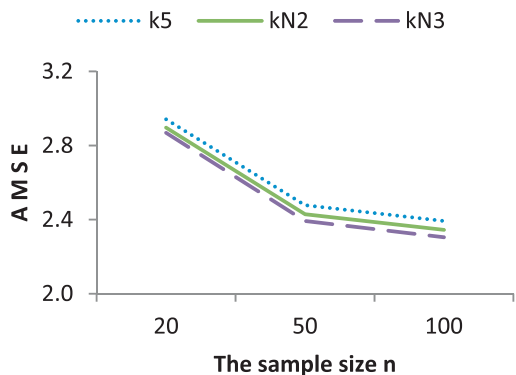


Figure 5 $p = 8, \rho = 0.85, \sigma^2 = 0.1.$

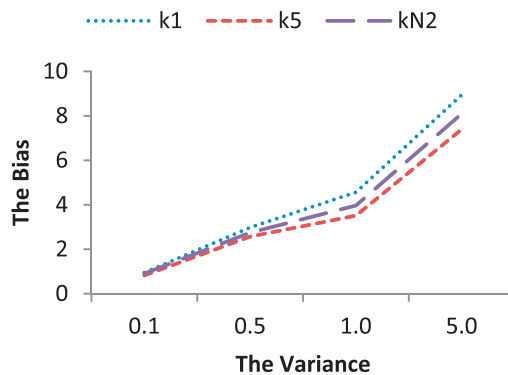


Figure 8 $n = 20, p = 8, \rho = 0.85.$

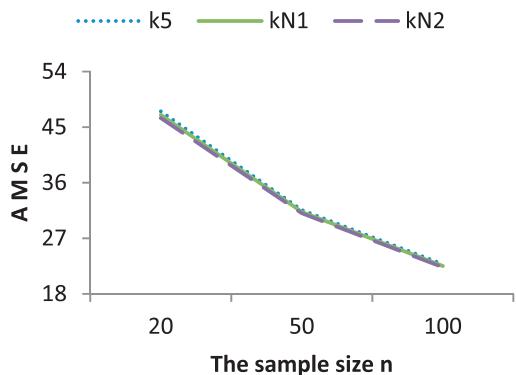


Figure 6 $p = 8, \rho = 0.85, \sigma^2 = 1.0.$

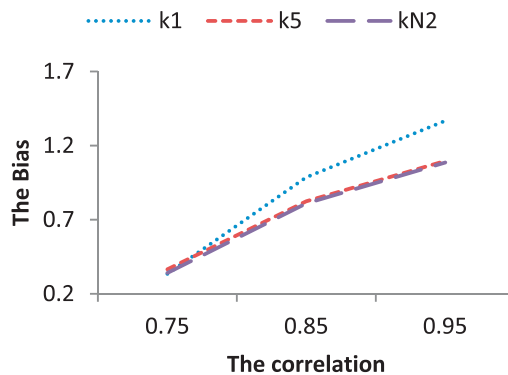


Figure 9 $n = 20, p = 4, \sigma^2 = 0.1.$

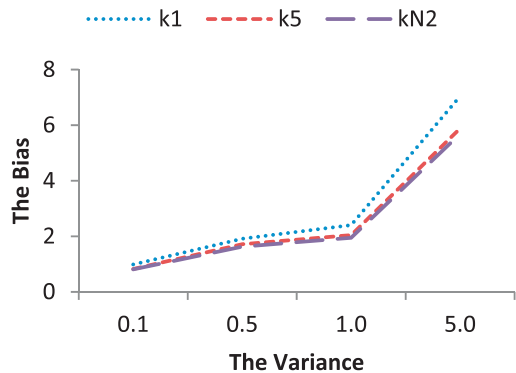


Figure 7 $n = 20, p = 4, \rho = 0.85.$

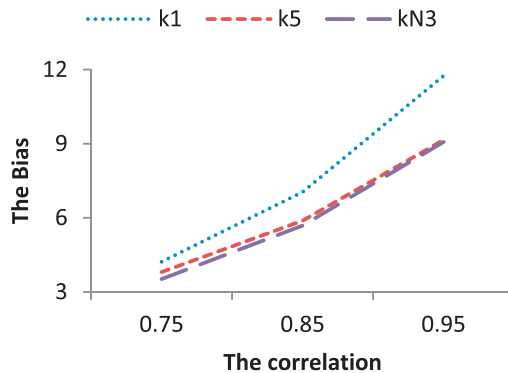


Figure 10 $n = 20, p = 4, \sigma^2 = 5.0.$

and $n = 20, p = 4, \sigma^2 = 5$, biases of the estimators are provided in Figs. 9 and 10. It is obvious from these figures that having a small variance namely $\sigma^2 = 0.1$, an increase in the correlation gives a small increase in biases. However the increase in biases is larger when $\sigma^2 = 5$, approximately ten times larger than the former case.

4.2.3. Comparison according to the sample size n

From previous sections, we see that the AMSE decreases as the sample size n increases. Similarly, biases of the selected estimators decrease as the sample size increases. For any combination of ρ, σ^2 and p , we observe that the bias decreases as n increases.

For given values $p = 4, \rho = 0.95, \sigma^2 = 0.1$ and $p = 4, \rho = 0.95, \sigma^2 = 5$, we have given the following Figs. 11 and 12. According to these figures, k_{N1} and k_{N3} have better performances than the other estimators. In most of the situations k_{N1} has the least bias. In some cases, k_5 is better than k_{N3} especially when $p = 8$, but it does not have a better performance than k_{N1} .

5. A real data application

To illustrate the findings of the paper, real life data have been analyzed in this section. The data are obtained from official

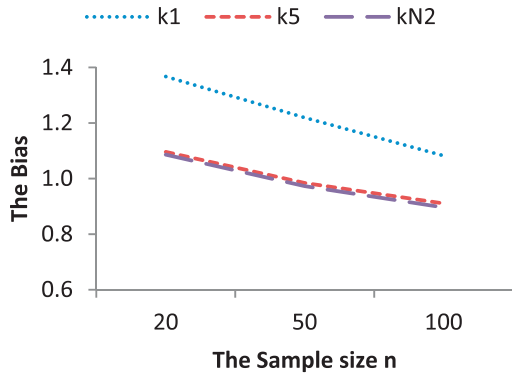


Figure 11 $p = 4, \rho = 0.95, \sigma^2 = 0.1$.

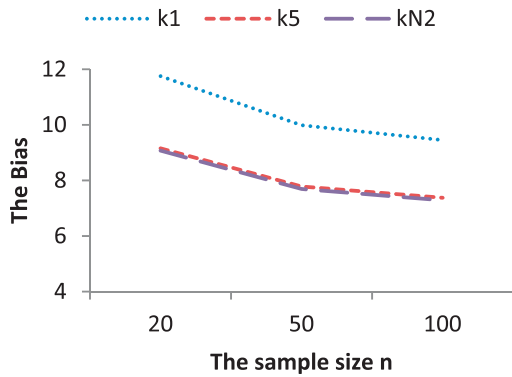


Figure 12 $p = 8, \rho = 0.85, \sigma^2 = 5.0$.

web site of the Turkish Statistical Institute (see <http://www.turkstat.gov.tr/>). The characteristic of the data is as follows:

Wide lands and underground sources are not the only wealth of countries. Some indicators are calculated to put forth the exact power of countries. In order to compare the wealth of nations, one of the most important indicators is the Gross Domestic Products (GDP). We have modeled GDP by cost components (at 1987 and 1998 prices) between the years 1968 and 2008, closely concerning the economy of Turkey in parallel to the growth and trends in the world. We have explained GDP using some parameters in the axes of foreign trade and production by multiple linear regression.

The dependent variable is the GDP of Turkey. The eight explanatory variables are the following respectively: X_1 : export, X_2 : import, X_3 : energy production, X_4 : number of establishments in manufacturing industry, X_5 : number of

Table 5 MSE values of the estimators in the application.

k	k values	MSE	Variance	Sq. Bias	R^2	PRESS
k_1	0.0022	0.1551	0.0804	0.0747	0.9880	0.0137
k_2	0.0023	0.1564	0.0785	0.0780	0.9879	0.0137
k_3	0.0098	0.2726	0.0230	0.2496	0.9816	0.0146
k_4	0.0026	0.1615	0.0723	0.0892	0.9876	0.0137
k_5	0.0008	0.1495	0.1312	0.0184	0.9895	0.0134
k_{N1}	0.0009	0.1480	0.1262	0.0219	0.9894	0.0135
k_{N2}	0.0011	0.1464	0.1178	0.0285	0.9892	0.0135
k_{N3}	0.0010	0.1471	0.1224	0.0247	0.9893	0.0135
k_{N4}	0.0018	0.1498	0.0902	0.0596	0.9884	0.0136
OLS	0.0000	0.1916	0.1916	0.0000	0.9906	0.0134

employees in manufacturing industry, X_6 : wheat production, X_7 : milk production and finally X_8 : meat production.

We have seen that the model has a multicollinearity problem since the condition number is $\kappa = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} = 49.1128 > 30$ which shows severe multicollinearity. We have given the correlation matrix of the GDP data in Table 4. One can see from that table that there are high correlations among the explanatory variables. The MSE values of the given estimators are provided in Table 5. It can be seen from Table 5 that k_{N1} , k_{N2} and k_{N3} have less MSE than the others, especially k_{N2} which has the best performance in the sense of MSE. Moreover, if we look at the determination coefficients and PRESS statistics of each model, it can be said that using these biased estimators makes no significant change in the model predictability. Thus, we advise to use the new defined estimators rather than the others.

6. Conclusion

In this paper, we reviewed some new modified ridge parameters and the ones proposed earlier. At first, we explained the multicollinearity and gave necessary information about methodology of the ridge regression. We introduced, secondly, ten ridge estimators half of which were proposed earlier and the other half were our proposals. Then, we compared the parameters according to their performance evaluating the average mean squared errors and also biases. The simulation study was performed for different combinations of the variances of the error terms (σ^2), the numbers of explanatory variables (p), the numbers of observations (n) and different correlation coefficients between the predictors (ρ). We found that our proposals are better than the ones proposed by k_1 : Hoerl and Kennard (1970a), k_2 : Lawless and Wang (1976), k_3 : Kibria

Table 4 The correlation matrix of the GDP data.

X_i	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
X_1	1.0000	0.9971	0.8961	0.9063	0.9456	0.4113	0.6804	0.4468
X_2	0.9971	1.0000	0.8978	0.8977	0.9399	0.4201	0.6793	0.4437
X_3	0.8961	0.8978	1.0000	0.8199	0.8902	0.5373	0.7435	0.4946
X_4	0.9063	0.8977	0.8199	1.0000	0.9643	0.3249	0.5092	0.2572
X_5	0.9456	0.9399	0.8902	0.9643	1.0000	0.5201	0.6869	0.4622
X_6	0.4113	0.4201	0.5373	0.3249	0.5201	1.0000	0.7583	0.6950
X_7	0.6804	0.6793	0.7435	0.5092	0.6869	0.7583	1.0000	0.8756
X_8	0.4468	0.4437	0.4946	0.2572	0.4622	0.6950	0.8756	1.0000

(2003), k_4 and k_5 : Dorugade (2014) according to their AMSE and bias performance. Finally, we conclude that our estimators are satisfactory over the multicollinearity problem and among our estimators k_{N4} has the best performance. The estimator k_{N1} has the least bias in most situations. However dealing with real data, the case may differ. Therefore we highly recommend researchers not to use just one ridge estimator to overcome their problem and not to decide without further study.

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