



A Stochastic Model for The Inventory Management of Antiretroviral Drugs: A Case Study

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Received March 26, 2017, Revised June 21, 2017, Accepted July 29, 2017, Published November 1, 2017

Abstract: The Anti Retroviral Treatment (ART) programme of Our Lady of Apostle Hospital Akwanga, Nigeria is faced with uncertain demands for antiretroviral (ARV) drugs and as a result of these, the management encounter stock-outs which is a major threat to the lives of the Patients. This work seeks to provide a solution to this problem via a stochastic modeling of the inventory process. This modeling approach is employed because of the stochastic nature of the demand for the antiretroviral drugs. The model determines for various service levels, the Economic order quantity (EOQ), the Optimal Re-order Point (ROP) and the optimal size of the Buffer stock. It further determine the cost of holding the Buffer stock in inventory ($C(B)$), the total cost per unit time of the ordering quantity ($TCU(y)$) and the total cost per unit time of inventory ($TCU(y+B)$) for each antiretroviral (ARV) drug. The result of this study is envisaged to provide relevant information that will assist the management of the ART programme in taking decisions that will ensure no stock-out of the drugs. The authors emphasize that unlike most models that only incorporate buffer stock size, this model deem it necessary to determine the size of a buffer stock as well as determine the cost of holding it in inventory. This is because, reducing the probability of stock-outs does increase the size of buffer stock and consequently, the cost of holding the buffer stock in inventory. The study recommends that the holding cost of buffer stock be included in the total cost of inventory and the model be used in selecting an optimal buffer stock size with a manageable cost and appreciable service level.

Keywords: Stochastic, Probabilized, Inventory, Buffer stock.

1. INTRODUCTION

The World Health Organization (WHO) defines access to medicine as a priority for citizens. It needs to be available at all times in adequate amounts, in appropriate dosage, quality and at an affordable price for individuals and communities ([8], [15]).

Access to life-saving medicines such as antiretroviral (ARV) drugs is increasingly allowing more people to live with Human Immunodeficiency Virus/Acquire Immune Deficiency Syndrome [14]. Based on projected HIV estimates for the year 2014, about 3,391,546 people are living with HIV in Nigeria out of which it is estimated that 227,518 (male 103,917 and female 123,601) people are newly infected and a total of 174,253 died from AIDS related cases which is lower than the year 2013 were 210,838 people died of HIV/AIDS. It is also estimated that a total of 1,665,403 people (1,454,565 adults and 210,838 children) require anti-retroviral (ARV) in the year 2014 [10].

A number of factors have been described to contribute to treatment discontinuity in an anti-retroviral treatment (ART) and one of the major factors includes the stock out of drugs [2]. Successful ART depends on lifelong patient adherence to prescribed ARV drug regimens and maintenance of a full supply of ARV drugs at ART sites. A reliable and uninterrupted supply of quality ARV drugs is absolutely critical given that more than 90 to 95 percent adherence to ART is required for the treatment regimens to be effective over the long term.

Stock outs of antiretroviral (ARV) drugs can cause unplanned treatment interruptions. Treatment interruptions affect treatment efficacy and could compromise treatment effectiveness thereby leading to substitution or switching of ART regimens to guarantee efficacy. Increased complications, could lead to hospitalizations and death. On the other hand, regular and uninterrupted supplies of antiretroviral (ARV) drugs will considerably decrease treatment



discontinuity and therefore, increasingly allow more people to live with HIV/AIDS. Hence, there is great need for a continuous and regular supply of antiretroviral drugs to people living with HIV/AIDS.

Unfortunately, ART programmes are faced with shortage of ARVs due to drug supply management malfunction due to poor inventory control. For an ART scale-up programme, [14] stated that inventory control should be one of the priority interventions for strengthening inventory management at the health facility level and that inventory management is a key step to avoiding stock outs and ensuring a continuous supply of antiretroviral drugs (ARVs).

Inventory serves as a buffer against uncertain and fluctuating usage and keeps the supply of items available in case the items are needed by the organization or its customers. While inventory serves this important and essential role, the expense associated with financing and maintaining inventories is a substantial part of the cost of doing business [1]. Most managers don't like inventories because they are like money placed in a drawer, asset tied up in investments that are not producing any return and in fact, incurring a borrowing cost.

Uncertainty plays a role in most inventory management situations such that the retail merchant wants enough to satisfy customer demands, but ordering too much increases the holding cost and the risk of losses through obsolescence or spoilage. An order too small increases the risk of lost sales and unsatisfied customers.

Inventory modeling comes handy in solving the aforementioned problems as it balances the cost of capital resulting from holding too much inventory against the penalty cost resulting from inventory shortage thereby ensuring smooth operation of the business [12].

Thus a good inventory system must seek to answer these two (2) questions:

- i. How much should be ordered when the inventory for the item is replenished?
- ii. When should the inventory for a given item be replenished?

One of such inventory models is the probabilized inventory model; a type of stochastic model for inventory management that is easy to solve and incorporates the Buffer Stock. The problem faced with the use of this model which this paper has been able to address is the "necessary evil syndrome" i.e. higher safety stock (Buffer Stock) than the required can block capital and increase operational stocks whereas low or no safety stock can lead to lost sales and customer dissatisfaction [7]. With respect to this work, low or no safety stock can lead to shortage of the antiretroviral drugs and eventually, the death of the patient. Hence, there is need for Buffer stock optimization which the probabilized inventory model has the flexibility to provide.

Some related works that include the Probabilized EOQ model and most especially incorporate safety stock include those of [9], [3], [4], [6], [5] and [11], [13]. The rest of the paper is sectioned as follows; Methodology, Results, Discussion, Conclusion and Recommendation.

2. METHODOLOGY

This section focuses on the source of data, the mathematical background of the Probabilized EOQ Model, estimate of model parameters sensitivity analysis and the use of software.

2.1 Source of Data

Quantitative data about the pattern of demand of antiretroviral (ARV) drugs were sourced from the existing records of Our Lady of Apostle Hospital Akwanga (OLA) pharmacy. It includes the antiretroviral drugs consumed in the hospital from January 2014 to May 2016.

2.2 The Classic and the Probabilized Economic-Order-Quantity (EOQ) Models

2.2.1 Classic EOQ Model

The inventory model employed in this work is the Probabilized Economic-Order Quantity (EOQ) Model. It was adopted because of the probabilistic nature of the demand for the antiretroviral drugs (see coefficient of variation for each drug in table 2). Since the model is an off-shoot of the Classic EOQ Model, we first present the Classic EOQ below.

The classic EOQ model involves constant-rate demand with instantaneous order replenishment and no shortage. The model defines;



y = Order quantity (number of units)

D = Demand rate (units per unit time)

t_0 = Ordering cycle length (time units)

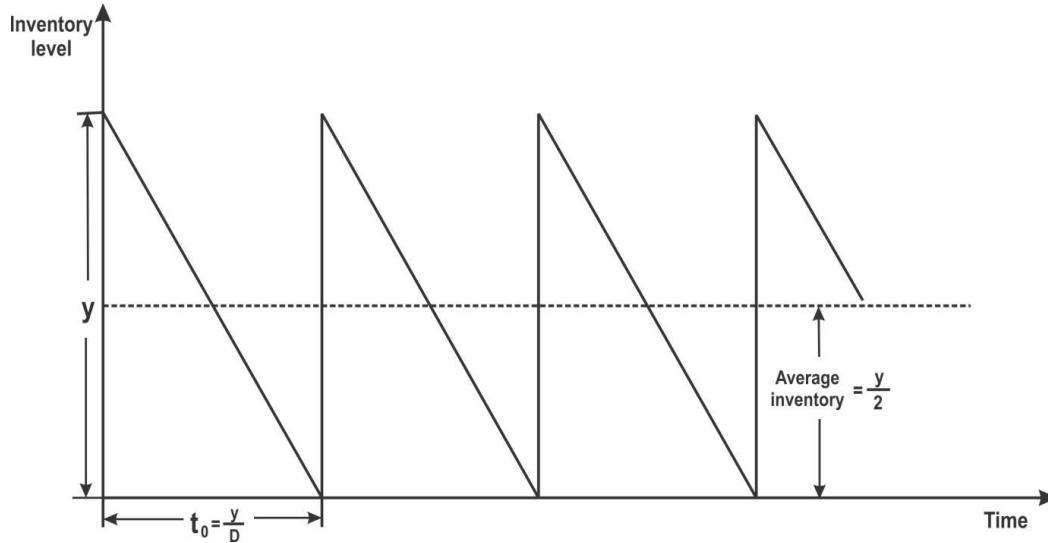


Figure 1. Inventory pattern of the classic EOQ model

The inventory level follows the pattern depicted in figure 1. An order of size y units is placed and received instantaneously when the inventory reaches zero level. The stock is then depleted uniformly at the constant demand D . The ordering cycle for this pattern is

$$t_0 = \frac{y}{D} \text{ time units}$$

The model requires two cost parameters.

K = Setup cost associated with the placement of an order (naira per order)

h = Holding cost (naira per inventory unit per unit time)

Given that the average inventory level is $\frac{y}{2}$, the total cost per unit time [TCU(y)] is thus computed as

TCU (y) = Setup cost per unit time + Holding cost per unit time

$$= \frac{\text{Setup cost} + \text{Holding cost per cycle } t_0}{t_0}$$

$$= \frac{K + h(\frac{y}{2})t_0}{t_0}$$

$$= \frac{K}{(\frac{y}{D})} + h(\frac{y}{2}) \tag{1}$$

The optimum value of the order quantity y is determined by minimizing TCU(y) with respect to y . Assuming y is continuous, a necessary condition for finding the optimal value of y is

$$\frac{dTCU(y)}{dy} = -\frac{KD}{y^2} + \frac{h}{2} = 0 \tag{2}$$

The condition is also sufficient because TCU(y) is convex.

The solution of the equation yields the EOQ (y^*) as

$$y^* = \sqrt{\frac{2KD}{h}} \tag{3}$$

Thus, the optimum inventory policy for the proposed model is



$$\text{Order } y^* = \sqrt{\frac{2KD}{h}} \text{ units every } t_0^* = \frac{y^*}{D} \text{ time units} \quad (4)$$

Actually, a new order need not be received at the instant it is ordered. Instead, a positive lead time, L , may occur between the placement and the receipt of an order as figure 2, demonstrates. In this case, the reorder point occurs when the inventory level drops to LD units.

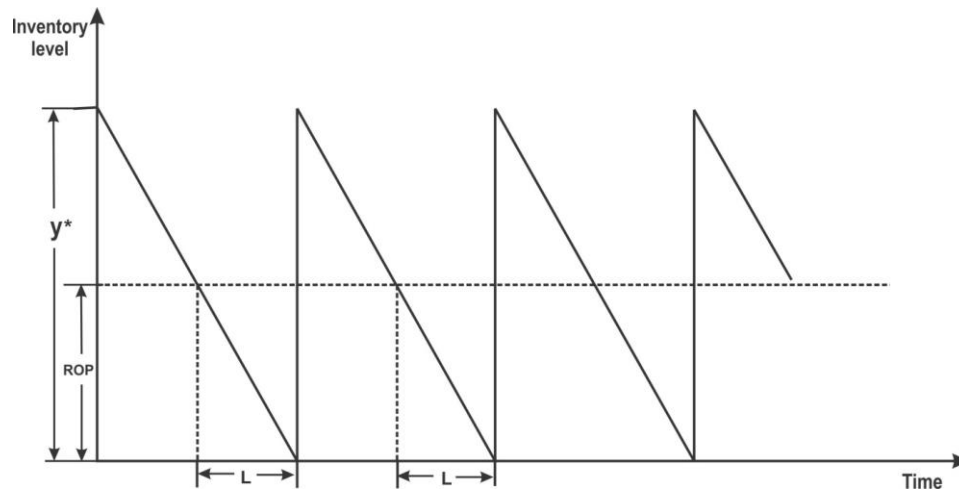


Figure 2. Reorder point in the classic EOQ model

This model assumes that the lead time L is less than the cycle length t_0^* , which may not be the case in general. To account for this situation, we define the effective lead time as

$$L_e = L - nt_0^* \quad (5)$$

where n is the largest integer not exceeding $\frac{L}{t_0^*}$. This result is justified because after n cycles of t_0^* each, the inventory situation acts as if the interval between placing an order and receiving another is L_e . Thus, the reorder point occurs at $L_e D$ units, and the inventory policy can be restated as:

Order the quantity y^* whenever the inventory level drops to $L_e D$ units [12].

As earlier mentioned, the probabilitized version of the Classic EOQ model is being used due to the stochastic or probabilistic nature of the demand for ARV drugs. Details of this model are presented below.

2.2.2 Probabilitized EOQ Model

The probabilitized EOQ model reflects the probabilistic nature of demand by using an approximation that superimposes a constant buffer stock on the inventory level throughout the entire planning horizon. The size of the buffer is determined such that the probability of running out of stock during lead time does not exceed a pre-specified value [12], [13]. It is important to mention that the model in its present state does not reflect the cost of holding the buffer stock in inventory.

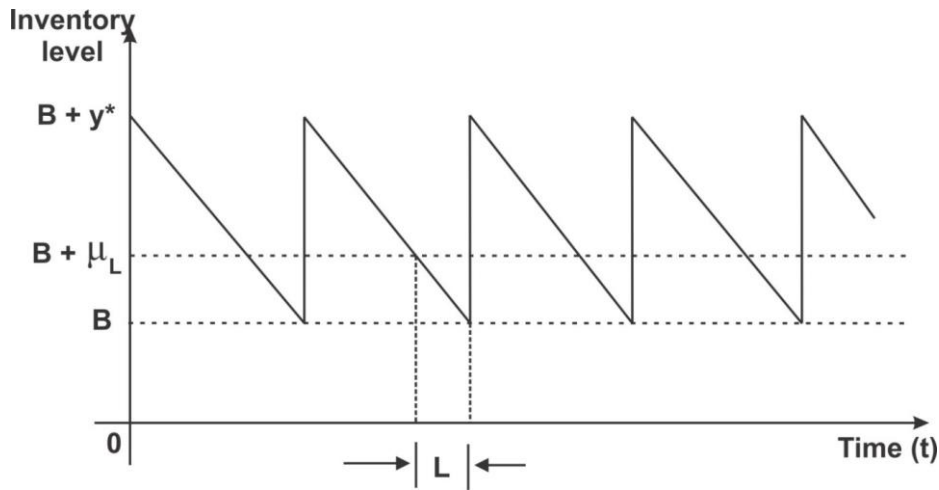


Figure 3. Probabilitized EOQ model

2.2.3 Model Description and Equations

The inventory policy of this model with buffer B , calls for ordering a quantity y^* whenever the inventory level drops to a point (ROP).

2.2.4 Assumptions and Notations

The following assumptions and notations will be adopted in deriving the inventory model.

2.2.5 Assumptions

The main assumption of the model is that the demand, during lead time L is normally distributed with mean μ_L and standard deviation that is Normal $(0,1)$.

2.2.6 Notations

Let

- L = Lead time (time between placing and receiving an order)
- x_L = Random variable representing demand during lead time
- μ_L = Average demand during lead time
- σ_L = Standard deviation of demand during lead time
- B = Buffer stock size
- $C(B)$ = Cost of buffer stock
- α = Maximum allowable probability of running out of stock during lead time

The probability statement used to determine B can be written as

$$p\{x_L \geq B + \mu_L\} \leq \alpha \tag{6}$$

We can convert x_L into a standard random variable (i.e Normal $(0,1)$) using the following substitution;

$$Z = \frac{x_L - \mu_L}{\sigma_L}$$

We set;

$$p\{Z \geq k_\alpha\} = \alpha$$

Hence, the buffer size must satisfy

$$B \geq \sigma_L k_\alpha \tag{7}$$

The demand during the lead time L (e.g. per day, week or month) usually is described by a probability density function from which the distribution of demand during L can be determined. Given that the demand per unit time is



normal with mean D and standard deviation σ , the mean and standard deviation, μ_L and σ_L of the demand during lead time, L , are computed as:

$$\mu_L = DL \quad (8)$$

$$\sigma_L = \sqrt{\sigma^2 L} \quad (9)$$

Thus, the optimal inventory policy of this model with buffer B , calls for ordering a quantity y^* whenever the inventory level drops to a point (ROP) as earlier stated.

where,

$$ROP = B + \mu_L \quad (10)$$

We proceed to establish in this work, that the total cost per unit time of the order quantity ($TCU(y)$) does not include the cost of the buffer stock [$C(B)$].

That is;

$$TCU(y + B) = TCU(y) \quad (11)$$

and as such, the buffer cost $C(B)$ must be separately added to $TCU(y)$ in order to obtain the total cost per unit time of inventory.

That is;

$$TCU(y + B) = TCU(y) + C(B) \quad (12)$$

Where,

$$C(B) = Bh, \quad B = \text{Buffer stock size and } h = \text{holding cost per unit time.}$$

We establish (10) as follows;

$$TCU(y) = \text{Setup cost per unit time} + \text{Holding cost per unit time}$$

From figure 2, the average inventory level is $\frac{y+B}{2}$, and considering $TCU(y + B)$ per cycle t_0 we have;

$$\begin{aligned} TCU(y + B) &= \frac{\text{Setup cost per cycle} + \text{holding cost } t_0}{t_0} \\ &= \frac{K + h \left(\frac{y+B}{2} \right) t_0}{t_0} \\ &= \frac{K}{t_0} + h \left(\frac{y+B}{2} \right) \\ &= \frac{K}{y/D} + h \left(\frac{y+B}{2} \right) \\ &= \frac{KD}{y} + h \left(\frac{y+B}{2} \right) \\ &= \frac{KD}{y} + \frac{hy}{2} + \frac{Bh}{2} \end{aligned}$$

$$\frac{dTCU(y + B)}{dy} = -\frac{KD}{y^2} + \frac{h}{2} = \frac{dTCU(y)}{dy}$$

Thus (10) is established.

This is practically true because the total cost per unit time of y ($TCU(y)$) (where y is the EOQ) does not include the fixed cost of buffer stock in inventory. To this end, (11) holds for $TCU(y + B)$.



2.3 Estimate of Model Parameters

For holding cost, the cost components include cost of electricity from the Power Company, cost of generator (Diesel) usage, cost of generator services/maintenance/repairs and the cost store room maintenance and repairs. The setup/ordering cost of antiretroviral drugs is a charge fixed by the hospital management for ARV drug replenishments. The distribution of the holding and ordering costs estimates are shown in tables 3 and 4.

2.4 Sensitivity Analysis

Sensitivity analysis was performed to determine the response of the Economic Order Quantity (EOQ), Buffer stock (B), Reorder Point (ROP), Cost of Buffer stock (C(B)), Total Cost per unit time of y (TCU(y)) and the Total cost per unit time of inventory (TCU(y + B)) to changes in the service levels.

2.5 Use of Software

The Statistical Package for Social Science (SPSS) version 21 was used to compute the descriptive statistics on the drugs consumed and for goodness of fit test for their respective demand distribution. Microsoft Excel (2007) was also used to perform sensitivity analysis on the study and to plot graphs.

3. RESULT

This section tabulates and displays the results of the study. These includes the descriptive statistics of demand for each drug per unit time, the goodness of fit summary for normality test and the estimates of the holding cost per month and the ordering cost per unit. It also present for each antiretroviral (ARV) drug, the results of the 100%, 99%, 97.5% and 95% service levels for the Economic order quantity (EOQ), the Optimal Re-order Point (ROP) and the optimal size of the Buffer stock. It further presents the cost of holding the Buffer stock in inventory (C (B)), and the total cost per unit time of inventory (TCU(y+B)).

TABLE 1. GOODNESS OF FIT SUMMARY FOR NORMALITY TEST

S/no	Drugs	K-S Statistics/ Parameters	P. value	Decision
1	3TC/TDF 600MG - PEPFAR Tablet(s)	N(37.634, 16.24)	0.83	Accept H_0
2	3TC/TDF/EFV 1200MG - FDC Tablet(s)	N(349.448, 84.59)	0.991	Accept H_0
3	3TC30/AZT60/NVP50 - 3FDC Tablet(s)	N(52.731, 11.24)	0.698	Accept H_0
4	3TC30/AZT60 - 2FDC Tablet(s)	N(2.155, 1.21)	0.91	Accept H_0
5	Cotrimoxizole 120MG Tablet(s)	N(12.938, 6.57)	0.507	Accept H_0
6	ABC600/3TC300 - FDC Tablet(s)	N(3.397, 1.31)	0.21	Accept H_0
7	ALV 250MG - PEPFAR Tablet(s)	N(63.128, 17.84)	0.548	Accept H_0
8	CBV/NVP650 - 3FDC Tablet(s)	N(139.662, 41.89)	0.945	Accept H_0
9	ATAZANAVIR/r 400MG Tablet(s)	N(11.393, 3.79)	0.387	Accept H_0
10	CBV 450MG - PEPFAR Tablet(s)	N(38.762, 10.25)	0.768	Accept H_0
11	ALV 125MG - PEPFAR Tablet(s)	N(1.593, 1.32)	0.852	Accept H_0
12	ABC/3TC 90MG Tablet(s)	N(2.314, 1.49)	0.98	Accept H_0
13	EFV 600MG - PEPFER Tablet(s)	N(11.052, 10.23)	0.352	Accept H_0
14	INH 300MG Tablet(s)	N(9.786, 10.8)	0.148	Accept H_0
15	NEVIRAPINE 10MG/ML OS-P Bottle(s)	N(23.59, 28.64)	0.059	Accept H_0
16	NEVIRAPINE 200MG - PEPFAR Capsule(s)	N(2.976, 3.03)	0.349	Accept H_0
17	Triomune Baby Tablet(s)	N(2.017, 2.39)	0.056	Accept H_0
18	TRUVADA 500MG - PEPFAR Tablet(s)	N(3.334, 2.76)	0.191	Accept H_0
19	ZIDOVUDINE 300MG - PEPFAR Tablet(s)	N(1.048, 1.13)	0.279	Accept H_0

$\alpha = 0.05$



TABLE 2. DESCRIPTIVE STATISTICS FOR DEMAND FOR EACH DRUG

S/no	Drug	Mean (D)	Std. Deviation	Coef. of Variation (V)	Lead time (L)	DLT
1	3TC/TDF 600MG - PEPFAR Tablet(s)	173.33	62.66	36	5	37.6
2	3TC/TDF/EFV 1200MG - FDC Tablet(s)	1598.66	182.38	11	5	349.5
3	3TC30/AZT60/NVP50 - 3FDC Tablet(s)	243.78	30.26	12	5	52.7
4	ALV 250MG - PEPFAR Tablet(s)	290.16	56.06	19	5	63.1
5	CBV/NVP650 - 3FDC Tablet(s)	644.91	152.03	24	5	139.7
6	3TC30/AZT60 - 2FDC Tablet(s)	10.23	6.02	59	5	2.2
7	Cotrimoxizole 120MG Tablet(s)	58.86	26.73	45	5	12.9
8	ABC600/3TC300 - FDC Tablet(s)	15.40	4.42	29	5	3.4
9	ATAZANAVIR/r 400MG Tablet(s)	51.75	10.75	21	5	11.4
10	CBV 450MG - PEPFAR Tablet(s)	176.96	24.21	14	5	38.8
11	ALV 125MG - PEPFAR Tablet(s)	7.21	5.40	75	5	1.6
12	ABC/3TC 90MG Tablet(s)	10.41	5.85	56	5	2.3
13	EFV 600MG - PEPFER Tablet(s)	49.91	42.37	85	5	11.1
14	INH 300MG Tablet(s)	44.25	49.38	112	5	9.8
15	NEVIRAPINE 10MG/ML OS-P Bottle(s)	109.45	128.64	118	5	23.6
16	NEVIRAPINE 200MG Capsule(s)	13.86	14.88	107	5	3.0
17	Triomune Baby Tablet(s)	5.51	9.42	171	5	2.0
18	TRUVADA 500MG - PEPFAR Tablet(s)	14.87	11.58	78	5	3.3
19	ZIDOVUDINE 300MG - PEPFAR Tablet(s)	5.04	6.26	124	5	1.1

Total expected drugs in the store per month = 3530.53 bottles/Packs

DLT: Demand during lead time

TABLE 3. ESTIMATION OF HOLDING COST (H) PER MONTH

Components	Hospital	Store room (6%)
Electricity bill (AEDC)	₦8,646	₦518.76
Generator Diesel (40KVA)	₦7,000	₦420
Generator services/Maintenance	₦3,000	₦180
Store room maintenance & repairs	-	₦2,000
Holding Cost (h)		₦3118.76

TABLE 4. SUMMARY OF COST PARAMETERS AND LEAD TIME

Parameters	Costs/Units/month
Setup cost(k)	₦1500 per order per month
Holding cost(h)	₦0.88 per unit per month
Lead time (L)	0.17 month



TABLE 5. A DISTRIBUTION OF INVENTORY PERFORMANCE MEASURES FOR EACH DRUG AT VARIOUS SERVICE LEVELS.

Drug	Service level (%)	EOQ	ROP	Cost of Optimal Buffer, C(B) (Naira)	Total TCU (y+B) (Naira)	Optimal Buffer Size (B)
1	95	769	72	37.4	713.9	43
	97.5	769	80	44.6	721.1	51
	99	769	90	52.9	729.4	60
	100	769	144	100.4	776.9	114
2	95	2335	396	108.9	2163.3	124
	97.5	2335	419	129.7	2184.1	174
	99	2335	447	153.9	2208.3	175
	100	2335	604	292.3	2346.7	332
3	95	912	62	18.1	820.3	21
	97.5	912	66	21.5	823.7	25
	99	912	71	25.5	827.7	29
	100	912	97	48.5	850.7	55
4	95	995	87	33.5	908.7	38
	97.5	995	66	39.9	915.1	25
	99	995	103	47.3	922.5	54
	100	995	151	89.8	965	102
5	95	1483	213	90.7	1395.5	103
	97.5	1483	233	108.1	1412.9	123
	99	1483	255	128.2	1433	146
	100	1483	387	243.6	1548.4	277
6	95	187	6	90.7	167.9	4
	97.5	187	7	108.1	168.6	5
	99	187	8	128.2	169.4	6
	100	187	13	243.6	174	11
7	95	448	28	16.0	410.2	18
	97.5	448	32	19	413.2	22
	99	448	36	22.6	416.8	26
	100	448	59	42.8	437	49
8	95	229	6	2.6	204.2	3
	97.5	229	6	3.1	204.7	4
	99	229	7	3.7	205.3	4
	100	229	11	7.1	208.7	8
9	95	420	16	6.4	376	7
	97.5	420	18	7.6	377.2	9
	99	420	19	9.1	378.7	10
	100	420	28	17.2	386.8	20
10	95	777	47	14.5	698	16
	97.5	777	50	17.2	700.7	20
	99	777	53	20.4	703.9	23
	100	777	74	38.8	722.3	44
11	95	157	5	3.2	141.2	4
	97.5	157	6	3.8	141.8	4
	99	157	6	4.6	142.6	5
	100	157	11	8.7	146.7	10
12	95	188	6	3.5	169.3	4
	97.5	188	7	4.2	170	5
	99	188	7	4.9	170.7	6
	100	188	12	9.4	175.2	11
13	95	413	37	25.3	388.3	29
	97.5	413	43	30.1	393.1	34
	99	413	49	35.8	398.8	41
	100	413	86	67.9	430.9	77
14	95	388	41	29.5	371.3	34
	97.5	388	47	35.1	376.9	40
	99	388	86	41.7	383.5	47
	100	388	98	79	420.8	90
15	95	611	106	76.8	614.3	87
	97.5	611	123	91.5	629	104
	99	611	142	108.5	646	123
	100	611	253	206.2	743.7	234
16	95	217	13	8.9	200.2	10
	97.5	217	14	10.6	201.9	12
	99	217	17	12.6	203.9	14
	100	217	30	23.8	215.1	27



17	95	137	7	5.6	126.2	6
	97.5	137	9	6.7	127.3	8
	99	137	10	7.9	128.5	9
	100	137	18	15.1	135.7	17
18	95	225	9	6.9	205	8
	97.5	225	12	8.3	206.4	9
	99	225	14	9.8	207.9	11
	100	225	24	18.6	216.7	21
19	95	131	5	3.7	119.1	4
	97.5	131	6	4.5	119.9	5
	99	131	7	5.3	120.7	6
	100	131	12	10	125.4	11

1. Match drug serial no. to drug name on table 1
2. Cost of optimal buffer and TCU(y + B) are in naira.

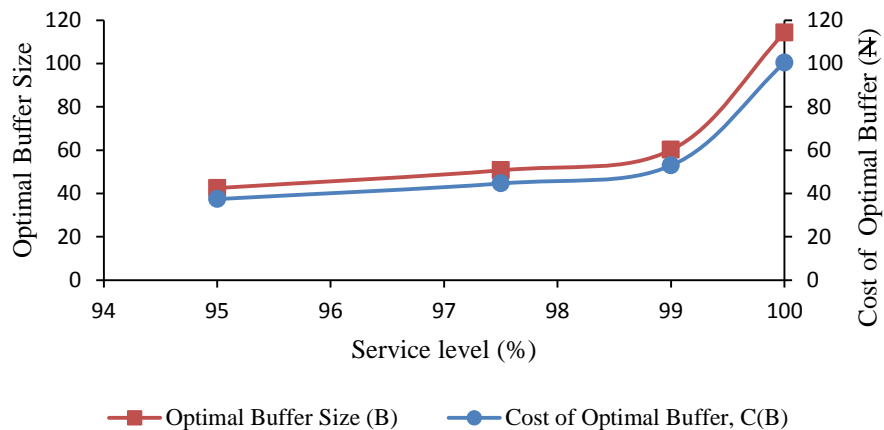


Figure 1. Optimal values of Buffer Size and Cost of optimal Buffer size against the service level for drug 1

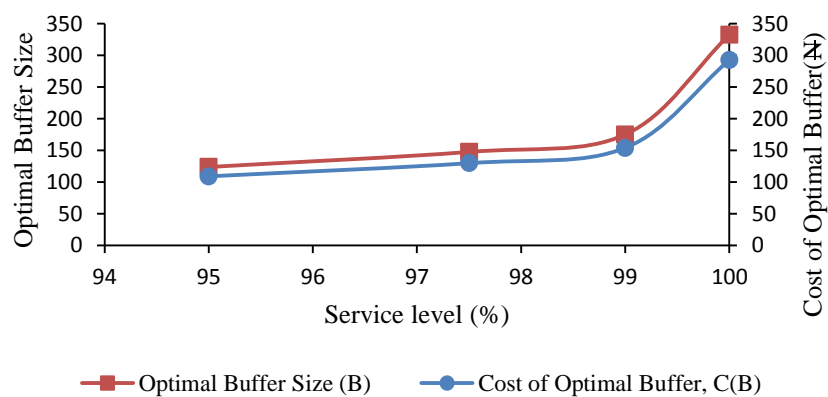


Figure 2. Optimal values of Buffer Size and Cost of optimal Buffer size against the service level for drug 2

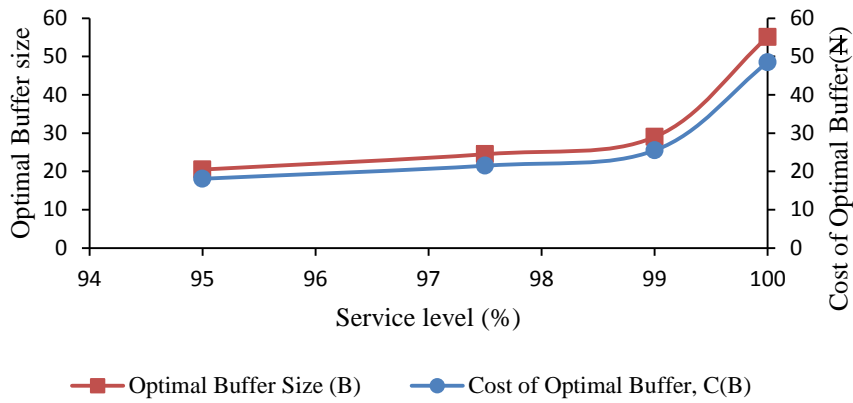


Figure3. Optimal values of Buffer Size and Cost of optimal Buffer size against the service level for drug 3.

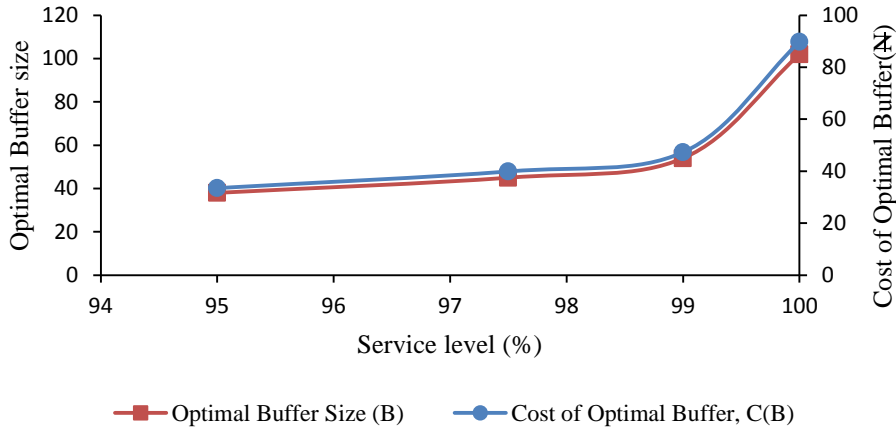


Figure 4. Optimal values of Buffer Size and Cost of optimal Buffer size against the service level for drug 4

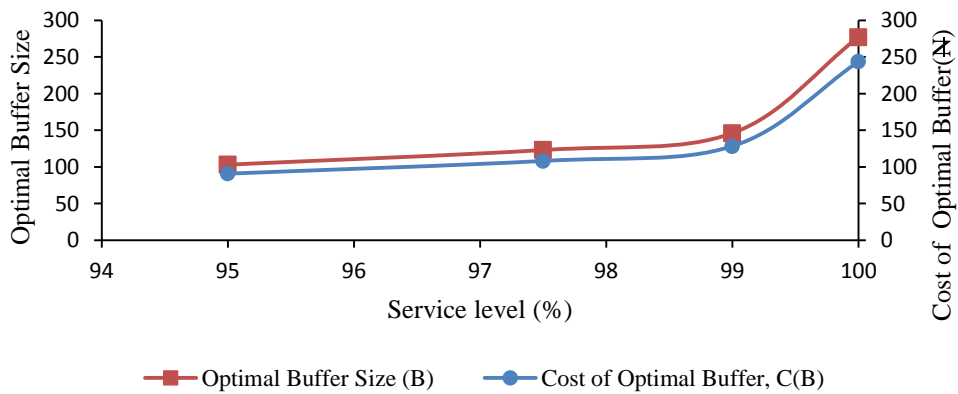


Figure 5. Optimal values of Buffer Size and Cost of optimal Buffer size against the service level for drug 5



4. DISCUSSION

4.1 Introduction

This entire section covers the discussion on the results of the descriptive statistics of demand for each ARV drug, the goodness of fit summary for normality test, the estimate of model parameters and results of the inventory performance measures at each service level

4.2 Discussion on the Goodness of fit summary for normality test

Goodness of fit test for normality was carried out on the demands for each ARV drug during lead time. It was ascertained that the demand for each drug, fit the normal distribution.

Table 2 shows the distribution fits and the parameter details.

4.3 Discussion on the descriptive statistics for demand for each drug

From table 1, it is observed that the standard deviation of the quantity of ARV drugs demanded per unit time is high. This indicates that the demand is highly variable. This translate into high coefficient of the variation therefore, the demand can be considered probabilistic or stochastic in nature hence the need for a stochastic inventory model such as the Probabilitized Economic Order Quantity (EOQ) model in this study.

4.4 Discussion on the estimates of the model parameters

The estimates for the lead time, setup cost and holding cost in Table 3 and 4, were based on the hospital's experts' opinion. A close interaction with the hospital's electrician and the ARV drug store keeper, reveals that the hospital's policy for ARV drug replenishment takes an average lead time of 5days and the hospital spends ₦1, 500.00 per month to order for ARV drugs, ₦8,646 per month on electricity bill, ₦7,000.00 per month on generator diesel, ₦3,000 per month on generator services/ maintenance and ₦2,000.00 per month on the ARV drugs store maintenance. Also, using the previous ADEC energy consumption rate by the hospital and the energy the ARV drug store room is likely to consume, the experts were able to estimate that the ARV drug store consumes 6% of the hospitals cost on electricity bill, generator diesel and generator services and maintenance. Using these findings, the holding cost of drugs was determined in table 3.

5.5 Discussion on the results of the inventory performance measures at various service levels.

Table 5 shows, the computed values of the Economic Order Quantity (EOQ), optimal Buffer stock (B), Reorder Point (ROP), Cost of optimal Buffer stock (C(B)), and the Total cost per unit time of inventory (TCU(y + B)) for each ARV at the 100%, 99%, 97.5% and 95% service levels.

The table 5 reveals that the Economic Order Quantity (EOQ) is not a function of the probability of running out of stock (α) as well as the service level ($1 - \alpha$). Hence, it remains constant as seen in table 5. On the other hand, reorder point (ROP) and the optimal Buffer stocks (B) are functions of the probability of stock outs (see equations 3.3 and 3.6). It can also be seen in table 5 that both the Reorder point (ROP) and optimal Buffer stock (B) decreases with increase in the stock out probabilities (i.e. both ROP and B increases as service level increases).

It is further revealed in table 5 and figures 1-5, that an increase in the service level (%) or a decrease in the probability of stock out would lead to an increase in the optimal Buffer size (B) and consequently, an increase in the optimal cost of Buffer (C(B)). So, if the management wants to reduce the holding cost of buffer stock (B), they will always find themselves in the dilemma of decreasing buffer stock (B) with the associated risk of increasing the probability of stock out. The way out of this dilemma is the use of the results in table 5 in selecting an optimal buffer stock size at a manageable cost and appreciable service level. It is important to mention that the expiry dates of the ARVs must be considered in the inventory in fact, the First-Expiry-First-Out discipline must be applied during dispensing.



6. CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

From the results of the study, the following conclusions were drawn;

- i. That a Stochastic Model has been formulated for the inventory management of Antiretroviral (ARV) drugs in Our Lady of Apostle Hospital Akwanga (OLA) Nassarawa State, Nigeria.
- ii. That the Economic Order Quantity (EOQ) is not a function of the probability of running out of stock (α) while the Reorder point (ROP) and Buffer stock (B) are functions of the probability of running out of stocks.
- iii. That the Total cost per unit time of y (TCU(y)) is independent of the holding cost of Buffer stock (C(B)) and thus, the holding cost of Buffer (C(B)) must be modeled separately and then added to the TCU(y) as the total cost per unit time of inventory.
- iv. That the size of Buffer stock and cost of buffer stock increases as the service levels increases.

6.2 Recommendations

- i. This model should be used in the inventory management and of Antiretroviral (ARV) drugs in Our Lady of Apostle Hospital Akwanga (OLA), Nassarawa State, Nigeria.
- ii. The expiry dates of the buffer stock must be considered alongside the cost of buffer. To this end, the First-Expiry-First-Out discipline should be applied during dispensing.
- iii. The application of renewal theory is recommended for future research.

REFERENCE

- [1] Bartmann, D. and Beckmann, M. J. (1992). Inventory Control Models and Methods. Springer-Verlag Berlin Heidelberg. 270pp.
- [2] Cameron, A., Ewen, M., Mantel-Teeuwisse, A. K., Leufkens, H. G. M., Laing, R. O. (2012). Switching from Original Brand Medicines to Generic Equivalents in Selected Developing Countries: How Much Could Be Saved? *Value in Health*. 15(5): 664-673.
- [3] Chopre, S., Reinhardt, G., and Dada, M. (2004). The Effect of Lead Time Uncertainty on Safety Stock. *Decision Sciences*, 35: 1 – 24.
- [4] Fergany, H. A. (2005). Periodic Review Probabilistic Multi-Item Inventory System with Zero Lead Time under Constraints and Varying Order Cost. *American Journal of Applied Sciences*. 2(8): 1213-1217.
- [5] Inderfurth, K. and Vogelgesang, S. (2011). Concepts for Safety Stock Determination under Stochastic Demand and Different Types of Random Production Yield, Otto Von Guericke University Magdeburg. pp. 1 – 23. [Http://www.fww.ovgu.de/femm](http://www.fww.ovgu.de/femm).
- [6] Jung, J. Y., Blau, G., Pekny, J. F., Reklaitis, G. V., and Eversdyk, D. (2008). Intergrated Safety Stock Management for Multi-stage Supply Chains under Production capacity constraints. *Computers and Chemical Engineering*. 32: 2570-2581.
- [7] Luthra, N. and Roshan, R. (2011). A New Framework for Safety Stock Manager. Cognizant 20-20 insights. Pp 1-8 . [Http://www.academia.edu](http://www.academia.edu).
- [8] Mark, S. (2009). Access to Essential Medicines as a Component of the Right to Health: Realizing the Right to Health. Swiss Human Rights Book.
- [9] Minner, S. (2000). Strategic Safety Stocks in Supply Chains. Lecture Notes in Economics and Mathematical Systems, 490pp.
- [10] NACA. (2015). National Agency for the Control of Aids: Nigeria Global Aids Response Country Progress Report. https://www.unaids.org/sites/default/files/country/.../NGA_narrative_report_2015.pdf
- [11] Radasanu, A. C. (2016). Inventory Management, Service Level and Safety Stock. *Journal of Public Administration, Finance and Law*. 145-153.
- [12] Taha, H., (2007). Operations Research; an Introduction, 8th ed. Prentice Hall, Upper Saddle River, New Jersey. pp. 838.
- [13] Kembe, M.M., Agada, P.O., and Owuna, D. (2014). A queuing model for hospital bed occupancy management: A case study. *International J. of Compu. and Theoretical Statistics*. Vol 1, No. 1, 13-28.
- [14] UNAIDS (2011). UNAIDS Word aids day report. Switzerland:UNAIDS. [Http://www.unaids.org/en/resources/documents/2011/20111121_JC2216_WorldAIDSday_report_2011](http://www.unaids.org/en/resources/documents/2011/20111121_JC2216_WorldAIDSday_report_2011)
- [15] Walkowiak, H. and Keene, D. (2009). Managing Medicines and Supplies for HIV/AIDS Program Scale-Up from the Ground up: Building Comprehensive HIV/AIDS Care Programs in Resource-Limited Settings.