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New Median Ranked Set Sampling for Skew Distributions

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Abstract: A new median ranked set sampling procedure for positively skew distributions (NMRSSS) is proposed and used to estimate population mean. The estimators based on the proposed scheme are compared with the estimators based on ranked set sampling (RSS), median ranked set sampling (MRSS) and new median ranked set sampling (NMRSS) procedures. It is shown that the relative precisions of the estimators based on NMRSSS are higher than the estimators based on RSS, MRSS and NMRSS procedures.

Keywords: Exponential distribution; Gamma distribution; Heavy right tail distribution; Lognormal distribution; Mean square error; New median ranked set sampling; Pareto distribution; Relative precision; Skew distributions; unbiased estimator; Weibull distribution.

1. INTRODUCTION

Ranked set sampling (RSS) for estimating a population mean was proposed by McIntyre (1952) as a cost-efficient alternative to simple random sampling (SRS) if the observations can be ranked according to the characteristics under investigation by means of visual inspection or other methods not requiring actual measurements. The RSS procedure has been extensively used in agriculture, environmental, ecological and recently in human studies where the exact measurement of unit is either difficult or expensive. McIntyre indicated that the use of RSS is more powerful and superior to SRS procedure to estimate the population mean. However, the mathematical foundation for RSS was provided by Dell and Clutter (1972) and Takahasi and Wakimoto (1968). Dell and Clutter (1972) also showed that the estimator for population mean based on RSS is at least as efficient as the estimator based of SRS with the same number of observations even when there are ranking errors. The selection of ranked set sample of n involves drawing n random samples with n units in each sample. The n units in each sample are ranked with respect to a variable of interest by using judgement or other methods not requiring actual measurements. The unit with the lowest rank is measured from the first sample, the unit with the second lowest rank is measured from the second sample, and this procedure is continued until the unit with the highest rank is measured from the last sample. The n^2 ordered observations in n samples can be displayed as

$$\begin{array}{cccc} x_{(11)} & x_{(12)} & \cdots & x_{(1n)} \\ x_{(21)} & x_{(22)} & \cdots & x_{(2n)} \\ \cdots & \cdots\cdots & \cdots & \cdots \\ \cdots & \cdots\cdots & \cdots & \cdots \\ x_{(n1)} & x_{(n2)} & \cdots & x_{(nn)} \end{array}$$

We measure only n ($x_{(ii)}$, i=1,2,.., n) observations and they constitute the RSS. The n observations are independently but not identically distributed. In RSS, n is usually small and therefore, to increase the sample size, the above procedure is repeated r times. For convenience, we assume that r=1.

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Many investigators used RSS in the parametric setting (see Bhoj and Ahsanullah, 1996; Bhoj 1997a and 1997b; Lam, Sinah and Zong, 1994, 1995; Stokes, 1995). Most of the distributions considered by these investigators belong to the family of random variables with cumulative distribution function of the form $F((x-\mu)/\sigma)$, where μ and σ are the location and scale parameters, respectively. Bhoj (2001) proposed ranked set sampling procedure with unequal samples (RSSU) to estimate the population mean and showed that it is superior to RSS for all distributions considered in his study. RSSU was shown to be superior for MRSS procedure when the distributions are symmetrical or moderately skew. Bhoj and Kushary (2014) proposed RSSU with unequal replications and showed that its relative precision for estimating population mean is higher than that based on MRSS procedure. More recently, Bhoj and Kushary (2016) proposed the estimator for population mean based on RSSU for heavy right tail distributions which is uniformly better than the estimators based on RSS and MRSS procedures. There are many variations of ranked set sampling to find the better estimator for the population mean. However, in this paper, we will concentrate on the median ranked set sampling (MRSS) and new median ranked set sampling (NMRSS) procedures.

2. NEW MEDIAN RANKED SET SAMPLING

Bhoj (1997a) and Muttlak (1997) modified the RSS procedure which further reduced the variance of the estimator for population mean. Bhoj (1997a) proposed a general modified ranked set sampling in the parametric setting where he proposed to select only two order statistics when the sample size n=2m is even. He suggested to select the j^{th} order statistics from the first m samples and k^{th} order statistic from last m samples. The choices of the two order statistics depend on the distribution under consideration and the parameter(s) to be estimated. This modified ranked set sampling becomes the median ranked set sampling (MRSS) if one is interested in estimating the mean of a symmetrical distribution. Muttlak (1997) proposed to use MRSS in a nonparametric setting.

In MRSS, we draw n random samples of size n from the population and rank n observation in each sample. If n is even, we measure m^{th} order statistic from the first m samples and $(m+1)^{th}$ order statistics from the last m samples where n=2m. If n is odd, we measure the observation with rank (n + 1)/2 from each sample. When n=2, RSS and MRSS procedures are identical. The relative precision of MRSS over RSS is superior when n is odd. Bhoj (2002) noted that the values of relative percentage increases in relative precisions are higher when we move from even to odd values of n, and they are lower when we switch from odd to even values of n. Therefore, Bhoj (2002) proposed a new median ranked set sampling (NMRSS) for even n=2m. In NMRSS, we draw samples of two sizes. We draw first m samples of size (n-1) and last m samples of size (n+1). Then we rank the observations in each sample as in RSS and MRSS. We only measure the median from each sample. Bhoj (2002) proposed the estimator for the population mean, μ, based on NMRSS. He showed that for unimodal symmetric distributions around μ, the NMRSS is better than MRSS procedure for n=2,4 and 6. For moderate skew distributions, NMRSS is better than MRSS for small n. However, the proposed estimator is not better than the one based on MRSS for the distributions with higher skewness and sample sizes higher than 2. In this paper, our main objective is to propose the estimator for the population mean based on NMRSS observations which will be better than the one based on MRSS for highly skew distributions and sample sizes greater than two.

3. ESTIMATORS FOR THE POPULATION MEAN

McIntyre (1952) proposed the estimator for population mean, μ, based on RSS as

$$\bar{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_{(ii)}.$$

$\bar{\mu}$ is an unbiased estimator of μ with the property that $Var(\bar{\mu}) < Var(\bar{x})$, where $\bar{x}$ is the sample mean based on simple random sample of size n .

The estimator, $\hat{\mu}$, based on MRSS, as defined in Section 2, is

$$\hat{\mu} = \begin{cases} \frac{1}{n}\left[\sum_{i=1}^{m} x_{(im)} + \sum_{i=m+1}^{n} x_{(im+1)}\right] & \text{for even n,} \\ \frac{1}{n}\sum_{i=1}^{n} x_{(ik)}, & \text{where } k = (n+1)/2. \end{cases}$$

$\hat{\mu}$ is an unbiased estimator for μ when the distribution is symmetric around μ with the property that $Var(\hat{\mu}) \le Var(\bar{\mu})$, see Bhoj(1997a) and Mutlack(1997).

In this paper, we are planning to use the skew distributions. For these distributions, $\hat{\mu}$, is a biased estimator for μ except when n=2. In this case, for comparison purposes with other estimators, we use the mean square error (MSE) of $\hat{\mu}$ where $MSE = variance + (bias)^2$. Muttlak (1997) demonstrated that for most distributions, MSE of $\hat{\mu}$, is less than Var($\bar{x}$) for small sample sizes.

The estimator, μ^*, based on NMRSS, as defined in section 2, is

$$\mu^* = \frac{w\sum_{i=1}^{m} x_{(im)}}{m} + \frac{(1-w)\sum_{i=m+1}^{n} x_{(im+1)}}{m}, \text{ where } w = (n-1)/2n.$$

This estimator is weighted average of NMRSS observations and it works well when the distributions under consideration are symmetric around μ or have low skewness. However, μ^* does not perform well when the distributions under consideration are highly skew or extremely skew with heavy right tail for $n > 2$.

In this paper we propose the estimators for μ which are weighted linear combinations of NMRSS observations for extremely skew distributions. In this paper, we have chosen Gamma (3), Gamma (5), Weibull (2) and Exponential distributions which have moderately large or large skewness. In addition, we have chosen Lognormal, Pareto (2.5) and Pareto (5) distributions which are extremely skew with heavy right tails.

We propose the following set of nonparametric estimators for μ based on NMRSS observations:

$$\tilde{\mu}_k = \frac{w_1\sum_{i=1}^{m} x_{(im)}}{m} + \frac{w_2\sum_{i=m+1}^{n} x_{(im+1)}}{m}, \quad \text{where } k = 1,2,\ldots,5.$$

The variance of $\tilde{\mu}_k$ is given by

$$Var(\tilde{\mu}_k) = \sigma^2[w_1^2\, V_{m(n-1)}/m \;+\; w_2^2\, V_{(m+1)(n+1)}/m],$$

where $V_{m(n_i)}$ denotes the variance of the m^{th} order statistics from a sample of size n_i.

We considered various values of w_1 and w_2 that were based on the ratio w_2/w_1, and are given by

$$R = \frac{w_2}{w_1} = \frac{1}{n} + \frac{n}{n-1} + \frac{h}{n-1} d_n, \quad \text{where}$$

$$d_n = \frac{n-3.5827}{1.8345n-5.2518}, \quad n = 2{,}4 \text{ and } 6, \text{ and } h \geq 0.$$

The values of w_1 for three values of n are determined so that the new estimators for the population mean based on NMRSSS procedure perform better than the estimators based on RSS, MRSS and NMRSS schemes for the chosen seven distributions. The weight w_1 is determined by

$$w_1 = \frac{1+(n/(n^2-6)).c_n}{1+R}, \quad \text{where}$$

$$c_n = \begin{cases} \frac{n}{D} & \text{for } n = 2 \\ \frac{(n-3)(9.25n-23)}{D} & \text{for } n = 4, 6, \text{ and} \end{cases}$$

$$D = 50n.$$

In order to keep the number of weights within reasonable limits, in this paper, we chose only five values of h= 0.55, 0.65, 0.75, 0.85 and 0.90. These values of h give the values of the ratios, w_2/w_1, which are in the middle of the range of ratios for seven distributions. We get five estimators for population mean from these values of h.

4. COMPARISON OF ESTIMATORS

We compare the various estimators for μ based on RSS, MRSS, NMRSS and NMRSSS procedures. For this purpose, we define the following nonparametric relative precisions (RPNs):

$$RPN_k = \frac{Var(\bar{\mu})}{MSE(\tilde{\mu}_k)} \quad \text{for} \quad k = 1,2, \dots 5.$$

$$RPN_6 = \begin{cases} \dfrac{Var(\bar{\mu})}{MSE(\mu^*)} & \text{if } \mu^* \text{ is a biased estimator} \\ \dfrac{Var(\bar{\mu})}{Var(\mu^*)} & \text{if } \mu^* \text{ is an unbiased estimator} \end{cases}$$

$$RPN_7 = \begin{cases} \dfrac{Var(\bar{\mu})}{MSE(\hat{\mu})} & \text{if } \hat{\mu} \text{ is a biased estimator} \\ \dfrac{Var(\bar{\mu})}{Var(\hat{\mu})} & \text{if } \hat{\mu} \text{ is an unbiased estimator} \end{cases}$$

We note that $\bar{\mu}$ is always an unbiased estimator for μ . However, $\tilde{\mu}_k$ is a biased estimator for μ for skew distributions. It is clear that RPN_k/RPN_7 can be used for comparison of the estimators based on NMRSSS proposed in this paper and MRSS procedure. Similarly the values of RPN_k/RPN_6 can be used for comparison of the estimators based on the estimators proposed in this paper and the one proposed by Bhoj (2002) based on NMRSS procedure. The values of RPN_k, k=1,2,...,7. are presented in Table A1 for various distributions and n=2,4 and 6. The variances and biases of the various estimators based on NMRSSS, NMRSS and MRSS are given in Tables 2 and 3, respectively. In tables 1, 2 and 3, the first five columns, give the relative precision, variances and biases of the estimators based on NMRSSS procedure, while column 6 and 7 give the relative precisions, variances and biases based on the estimators based on NMRSS and MRSS procedures, respectively.

We observed that $\tilde{\mu}_k$, k=1,2, ...,5. based on NMRSSS are all superior to the estimators for μ based on RSS and MRSS for all distributions and three sample sizes. The gains in the precisions of $\tilde{\mu}_k$, k=1,2,...,5. over the estimator based on RSS are substantial. The gains in precisions of the estimators based on NMRSSS over the MRSS estimators are quite good. $\tilde{\mu}_k$ is also superior to the estimator based on NMRSS proposed by Bhoj (2002) for n=4 and n=6 for all distributions and the gains in precisions are substantial. Bhoj (2002) recommended the NMRSS scheme with n=2 for all distributions and it works well. However, even for n=2, $\tilde{\mu}_k$, k=1,2,...5. is superior to μ^* for six out of seven diverse distributions considered in this paper. The only exception is Pareto(5) which is extremely heavy right tail distribution.

We observe from Table 1 that the relative precision of the estimator based on NMRSSS increases with n for moderately skew Gamma distributions. For extremely skew distributions, such as , Weibull (2), Lognormal, Pareto (2.5) and Pareto (5), the relative precision increases when n increases from two to four, and it decreases when n increases from four to six. We note from Table 2 that the variances based on NMRSSS, MRSS and NMRSS procedures decrease as n increases. Table 3 shows that for two Gamma and Pareto (5) distributions bias in $\tilde{\mu}_k$ decreases as n increases. Also bias increases as n increases from two to four and then decreases when n increases from four to six for Weibull (2) and Pareto (2.5). In the case of exponential and lognormal distributions, bias in $\tilde{\mu}_k$ increases as n increases. The bias of estimators based on NMRSS and MRSS procedures increases with n for all distributions. Therefore MSE of the estimator increases for large n. In the case of Weibull (2) distribution for n=6, the estimators based on NMRSS and MRSS are even less efficient than the estimator based on RSS.

5. CONCLUSIONS and DISCUSSION

In this paper, we proposed new median ranked set sampling procedure (NMRSSS) for highly skew distributions. The set of estimators for the population mean based on NMRSSS are proposed under nonparametric settings. The proposed estimators are weighted linear combinations of NMRSS observations where the weights are functions of sample size. These estimators are compared with the estimators based on the ranked set sampling (RSS), median ranked set sampling (MRSS) and new median ranked set sampling (NMRSS). It is shown that the estimators for μ based on NMRSSS procedure are superior to the estimators based on RSS and MRSS procedures for all seven distributions under consideration and the three samples sizes. It is observed that the gains in relative precisions of the estimators based on NMRSSS over RSS estimators are quite high. The gains in precision of NMRSSS over MRSS estimator are good. These gains depend on a particular distribution under consideration and sample size.

The estimator based on NMRSSS is superior to the estimator based on NMRSS procedure for $n > 2$ for all distributions. When n=2, the estimator based on the NMRSSS procedure is superior to the one based on NMRSS procedure for six out of seven distributions considered in this paper.

Based on the computations of relative precisions, we recommend the NMRSSS scheme to estimate the population mean based on even values of n when the samples are drawn from moderate skew or highly skew distributions.

In this paper we have proposed a simple method of estimating population mean of positively heavy skewed distributions whose means and variances of order statistics are readily available. In our estimators we have used medians from the samples and finding the variance of the median in nonparametric setting would be much harder. Our method may be used for skew distributions whose means and variances are not readily available. In such situations bootstrap is a viable approach. The researchers suggested several methods and the algorithms of their methods of bootstrapping a RSS, see Modaress, Hui and Zheng (2006), Amiri and Modaress(2017), and Frey(2014).

Skewed symmetric distributions have received more attention with applications in economics, finance , insurance, and telecommunications. One such distribution is the skewed Cauchy distribution which suffers from limited applicability because of the lack of finite moments. In order to overcome this problem Nadarajah and Kotz (2007) proposed a skewed truncated Cauchy distribution. Recently Teimouri and Nadarajah (2017) proposed an algorithm for simulating truncated Cauchy random variables in an efficient way. One can use extensive simulation to estimate the means and variances of order statistics which can be used to calculate relative precisions of our proposed estimators for skewed truncated Cauchy distribution.

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TABLE 1. NONPARAMETRIC RELATIVE PRECISIONS FOR THE ESTIMATORS FOR μ

Distribution	n	RPN_1	RPN_2	RPN_3	RPN_4	RPN_5	RPN_6	RPN_7
Gamma(3)	2	1.1850	1.1813	1.1773	1.1733	1.1712	1.1651	1.0000
	4	1.2691	1.2691	1.2689	1.2688	1.2688	1.2093	1.2090
	6	1.3507	1.3506	1.3504	1.3502	1.3501	1.0228	1.0491
Gamma(5)	2	1.1466	1.1427	1.1387	1.1346	1.1325	1.1309	1.0000
	4	1.2535	1.2534	1.2533	1.2532	1.2532	1.2082	1.1980
	6	1.2705	1.2704	1.2703	1.2702	1.2701	1.1056	1.1200
Weib(2)	2	1.0775	1.0731	1.0687	1.0643	1.0621	1.0536	1.0000
	4	1.1104	1.1102	1.1101	1.1100	1.1099	1.1099	1.1043
	6	1.0757	1.0756	1.0755	1.0753	1.0753	1.0529	1.0612
Exponential	2	1.3602	1.3572	1.3539	1.3504	1.3486	1.3333	1.0000
	4	1.2814	1.2812	1.2811	1.2809	1.2808	1.2251	1.2712
	6	1.0308	1.0306	1.0304	1.0302	1.0300	0.8238	0.8734
Lognormal	2	2.2655	2.2753	2.2838	2.2911	2.2944	2.2224	1.0000
	4	2.3440	2.3439	2.3438	2.3437	2.3436	2.2070	2.2766
	6	1.7258	1.7255	1.7252	1.7249	1.7248	1.3954	1.4807
Pareto(2.5)	2	3.7310	3.7863	3.8383	3.8871	3.9104	3.7181	1.0000
	4	5.8928	5.8927	5.8926	5.8925	5.8924	4.8527	5.0387
	6	5.6682	5.6672	5.6663	5.6653	5.6648	3.1227	3.3120
Pareto(5)	2	1.7301	1.7319	1.7331	1.7336	1.7337	1.8753	1.0000
	4	2.7615	2.7616	2.7617	2.7617	2.7617	1.7873	1.8586
	6	4.5108	4.5113	4.5117	4.5120	4.5122	1.1415	1.2131

TABLE 2. VARIANCE FOR THE ESTIMATORS FOR μ

Distribution	n	Var1	Var2	Var3	Var4	Var5	Var6	Var7
Gamma(3)	2	0.8581	0.8606	0.8632	0.8659	0.8673	0.8923	1.0605
	4	0.2546	0.2546	0.2546	0.2546	0.2546	0.2410	0.2540
	6	0.1245	0.1245	0.1245	0.1245	0.1245	0.1097	0.1125
Gamma(5)	2	1.4635	1.4681	1.4730	1.4781	1.4807	1.5216	1.7430
	4	0.4394	0.4394	0.4394	0.4394	0.4395	0.4160	0.4341
	6	0.2162	0.2162	0.2162	0.2163	0.2163	0.1906	0.1944
Weib(2)	2	0.0668	0.0670	0.0673	0.0676	0.0677	0.0694	0.0736
	4	0.0207	0.0207	0.0207	0.0207	0.0207	0.0196	0.0200
	6	0.0103	0.0103	0.0103	0.0103	0.0103	0.0091	0.0092
Exponential	2	0.2552	0.2556	0.2560	0.2564	0.2567	0.2656	0.3750
	4	0.0709	0.0709	0.0709	0.0709	0.0709	0.0671	0.0747

	6	0.0334	0.0334	0.0334	0.0334	0.0334	0.0295	0.0310
Lognormal	2	0.7621	0.7569	0.7522	0.7481	0.7462	0.7965	1.9672
	4	0.1590	0.1590	0.1589	0.1588	0.1588	0.1501	0.1912
	6	0.0665	0.0665	0.0665	0.0665	0.0665	0.0587	0.0654
Pareto(2.5)	2	0.2350	0.2304	0.2263	0.2224	0.2206	0.2472	1.0243
	4	0.0325	0.0324	0.0324	0.0324	0.0324	0.0306	0.0417
	6	0.0128	0.0128	0.0128	0.0128	0.0128	0.0113	0.0129
Pareto(5)	2	0.0198	0.0197	0.0196	0.0196	0.0196	0.0206	0.0424
	4	0.0046	0.0046	0.0046	0.0046	0.0046	0.0044	0.0053
	6	0.0020	0.0020	0.0020	0.0020	0.0020	0.0018	0.0020

TABLE 3. BIAS FOR THE ESTIMATORS FOR μ

Distribution	n	Bias1	Bias2	Bias3	Bias4	Bias5	Bias6	Bias7(MRSS)
Gamma(3)	2	-.1919	-.1930	-.1940	-.1949	-.1954	-.1341	0.0000
	4	-.1334	-.1334	-.1335	-.1335	-.1336	-.2118	-0.1788
	6	-.0673	-.0674	-.0674	-.0674	-.0675	-.2463	-0.2317
Gamma(5)	2	-.2381	-.2392	-.2402	-.2412	-.2417	-.1403	0.0000
	4	-.0823	-.0823	-.0824	-.0824	-.0824	-.2165	-0.1809
	6	0.0598	0.0598	0.0597	0.0597	0.0597	-.2489	-0.2342
Weib(2)	2	-.0396	-.0397	-.0399	-.0401	-.0401	-.0222	0.0000
	4	-.0111	-.0111	-.0112	-.0112	-.0112	-.0350	-0.0296
	6	0.0143	0.0142	0.0142	0.0142	0.0142	-.0407	-0.0383
Exponential	2	-.1430	-.1440	-.1449	-.1458	-.1462	-.1250	0.0000
	4	-.1751	-.1752	-.1752	-.1753	-.1753	-.1979	-0.1667
	6	-.1806	-.1806	-.1807	-.1807	-.1807	-.2306	-0.2167
Lognormal	2	-.3259	-.3282	-.3304	-.3325	-.3335	-.2977	0.0000
	4	-.4238	-.4239	-.4240	-.4241	-.4241	-.4578	-0.3969
	6	-.4464	-.4464	-.4465	-.4466	-.4466	-.5197	-0.4944
Pareto(2.5)	2	-.1989	-.2003	-.2015	-0.2027	-0.2033	-0.1683	0.0000
	4	-.2150	-.2151	-.2151	-0.2152	-0.2152	-0.2549	-0.2244
	6	-.1958	-.1958	-.1958	-0.1959	-0.1959	-0.2855	-0.2732
Pareto(5)	2	-.0689	-.0693	-.0696	-.0699	-.0701	-.0446	0.0000
	4	-.0361	-.0362	-.0362	-.0362	-.0362	-.0693	-0.0596
	6	-.0033	-.0033	-.0033	-.0033	-.0033	-.0793	-0.0752