International Journal of Computing and Digital Systems
ISSN (2210-142X)
Int. J. Com. Dig. Sys. 12, No.1 (Dec-2022)

https://dx.doi.org/10.12785/ijcds/1201118

Complexity Space Modelling for Industrial Manufacturing Systems

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Received 16 Mar. 2021, Revised 31 May. 2021 Accepted 14 Jul. 2022, Published 31 Dec. 2022

Abstract: The static and dynamic complexity of an industrial engineered system are integrated in a complexity space modelling approach, where information complexity boundaries expand over time and serve as an indicator for system instability in a static complexity space. In a first step, model-based static and dynamic conceptions of complexity are introduced and described. The necessary capabilities are theoretically demonstrated, alongside a set of assumptions concerning the behavior of industrial system complexity and its functions as a core foundation for the proposed complexity space model. In a second step, the successful application of the proposed modelling approach on a real-world industrial system is presented. Case study results are briefly presented and discussed as a first proof of concept for the general applicability of the proposed modelling approach for current and future industrial systems. In a final step a short research outlook is provided.

Keywords: complexity, modelling, industrial manufacturing system, system analysis

1. Introduction

The internet of things (IoT) is a vast interdisciplinary field with many areas of contributing technologies. The application of IoT involves different factors like hardware, software, and humans acting under resource limitations (cost, complexity, energy sources and memory space available). IoT can be defined as a network of devices that interacts with the physical surroundings and which communicates over wireless networks in the contexts of value creation.[1] This means that the internet is expanded beyond traditional devices like smartphones, computers, or tablets. This leads to a situation where millions of devices are interconnected and generate and interchange data in unprecedented scale under the conception of Big Data.[2,3] The resulting IoT ecosystems are therefore highly information based and complex, with thousands or even millions of interconnected devices that generate, transfer, and transform high dimensional data sets between various system elements, ranging like sensors, advanced server architectures or cyber-physical production systems. This leads to systems being more interconnected, non-linear and as a result more complex for the linearly thinking human mind to comprehend and to predict and therefore at the same time cause and effect for its central characteristic of increasing system complexity.[4,5,6] The idea of the central importance of managing increasingly technologized and complex systems can be regarded as essential for handling the design and transformation process of modern complex engineered systems of organizational value creation. The integration of IoT to manufacturing systems, through the combination of operational technology with information technology, fits this definition, where IoT is expected to improve human life quality and economic productivity. IoT is giving rise to the industrial IoT (IIoT) concepts, advanced and complex machine-machine communication in the form of cyber-physical system architectures and Industry 4.0, with impactful application fields, like smart cities and smart factories or the agricultural, medical and logistics sector. [1,2,7,8,9]

A. Motivation & Novelty

This paper contributes to the topic of complexity modelling for industrial engineered systems by exploring the possibility to conceptually model the complexity of a system in a three-dimensional Euclidean space with the

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means of a set of theoretical axiomatic assumptions concerning the definitions of static and dynamic complexity. The core motivation of the paper is to answer to the key-challenge of developing conceptual complexity models which can fulfill uncertainty-reducing, communicative or strategic purposes in the decision-making process between different stakeholders (for example system engineer and manager) and which can serve as a first baseline to be developed to more determinate and executable simulation models in the future, for example through in-depth and specialized mathematical formalisms or coded computational methods like computer algorithms. [10,11] To achieve this, the paper theoretically and practically explores and proposes a novel conceptual approach to model and quantify the complexity of modern and future industrial systems in a way that supports the visualization and potentially simulation of the complexity of both the physical and the virtual system layers and their respective information flow in a generally applicable three-dimensional model. The proposed model is to be regarded as an early-stage artifact that integrates two different complexity dimensions, provides axiomatic requirements for more specialized, formal and mathematically operable models and which allows exploratory analysis (EA) of complex engineered systems. EA is focused on describing ranges of possible system development trajectories and extreme behavior patterns or drastic changes in the system while focusing on endogenous and system internal complexity dynamics. [10,12] To adhere to the concept of EA, the aim of this paper to introduce a perspective on complexity modelling that represents industrial system complexity through conceptions of static and dynamic complexity dimensions via an integrated, compounded state in a conceptual model, the concept of complexity space. The proposed model now aims to have the following functions:

- Characterize the basic constituents and/or governing dynamics of industrial system complexity in a coherent framework via the introduction of complexity space.
- Provide a coherent understanding of the dimensions and factors that unify the complexity of engineered systems.
- Serve as an early-stage artifact component or starting point for more advanced modelling and simulation approaches for complex engineered systems.
- Enable early-stage exploratory analysis (EA) for industrial system analysis.
- Supporting the decision-making process between different system stakeholders through reducing uncertainty about the systems properties, for example in the strategic system management or design process of the system.

The proposed model and its assigned values and functions must also be understood as the intention to further improve and expand on the work on complexity space modelling, strategic complexity management and complexity theory for industrial engineered systems by Freund, Al-Majeed & Millard. [6,13,14] The paper now follows the following core steps: (i) introduction of the applied conception of complexity, (ii) description of the three-dimensional modelling approach, (iii) presentation and short discussion of a real-world case application, (iv) discussion and outlook on future research.

2. Applied Conception of Complexity

Complexity has many metrics, dimensions and definitions and has been defined as the measure of uncertainty or difficulty in achieving the functional requirements of a system within the ranges of its design. [11,13,14,15,16] Two conceptions of complexity shall be applied for the proposed modelling approach:

- Static / structural complexity
- Dynamic complexity

Static or structural complexity shall be defined as how the industrial system is structured (e.g. number of processors/machines, machine connections and interconnections). Dynamic complexity is defined as a measure of the unpredictability in the behavior of the system over a time-period based on information entropy. A common example of dynamic complexity is any type of unwanted system behavior, like a machine breakdown. [14,15,16,17] Dynamic complexity is thus the core obstacle to achieving the systems target function. [13,14,15,16,17] Both types of complexity shall serve as the two foundational dimensions of the applied complexity Modelling approach. Both dimensions represent reliable measurement dimensions for complexity, for example Defense Advanced Research Projects Agency (DARPA) of the US Government expects complexity of next generation products to reach 1.0E+08, measured in parts and lines of code. [15] The notions of static and dynamic complexity make also visible that the presented model focuses on system intra-dependency, the internal complexity of the layout of the manufacturing system. For simplicity, the model does not regard stand-alone equipment complexity, environmental system complexity or any external factors that may impact system complexity. In the next section the applied definition of static industrial system complexity is now described.

A. Static industrial system complexity

The concept of static industrial system complexity (S_C) shall be defined by the static, time-independent architectural layout of a manufacturing process represented by machines /operations (m), their connections via links (l), and their interconnectedness via gates (g) as shown by equation (1).

$$S_C=\{m,l,g\} \quad (1)$$

This definition offers a more nuanced definition than just the often-used number of system parts as a starting point for system complexity Modelling. It must be mentioned at this point that the number of parts, connection and gates do represent a multi-dimensional quantity, as for example a machine may contain several subsystems. It is thus necessary to apply pre-defined levels of abstraction to allow system representation in the form of pre-set system boundaries and pre-defined system entities. [14,15,16,17,18,19,20]

These pre-set and pre-defined boundaries and entities shall be utilized as abstractions, to allow a formalized modelling of industrial manufacturing systems to narrow down on the issue on system complexity via a clearly defined set of parameters.[18,19,20] Any manufacturing system itself shall be regarded as a flux of material (input) going through a transformation process (adding information), consisting out of machines, links and gates, which then results in a flux of output materials (products) with a higher complexity.[18,19,20] This is illustrated by Figure 1.

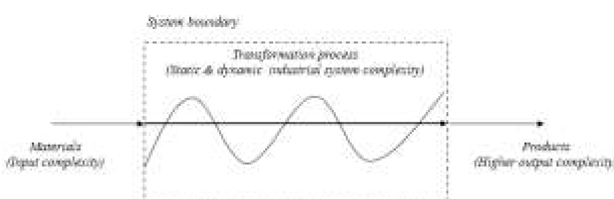

Figure 1: Manufacturing system

In the context of industrial system complexity, the term machine (m) shall be defined as a physical processor of information in the transformation process of a manufacturing system, an active element or artifact, that performs actions in the form of processing information via transformation of energy, material and information. An action is defined as a change in the state of the model, e.g. any action contained in the transformation process. Different processors can execute in parallel, and they proceed with the performance of actions independently or dependent of each other. This means that different processors can be active at the same time or can function in a sequential manner. A machine shall also be capable to function as an expanded processor. This means that a machine encompasses a given set of sub-processors in the form of operations. For example, a manufacturing machine could contain two sub-operations in the form of a packaging machine and a manual operator. [18,19] The term links (l) shall refer to interaction pathways between machines in the transformation process where information is passed from one machine to the other, for example in the form of materials over conveyor belts, intermediate products, or wireless data flows. It is thus modelled that material or immaterial objects can flow from one processor only if processors are connected via links. The term gate (g) shall refer to connection points where links connect machines within the system. Gates specify interaction and decision-points between processors and thus define the modus operandi of how different processors interact with each other in a system, for example through digital interfaces, machine interfaces, manual quality tests, sensors, or others. [19,20] Consequently a gate transforms the information that is send by one processor to another via links, so that the receiving processor unit can process and transform the received information in a correct fashion.

The conception of static complexity leads to the conclusion that the static and time-independent complexity (S_C) of an industrial production system shall be reduced to, captured, and quantified by three dimensions:

- Structural complexity (Cs): machine layout
- Connectivity complexity (Cc): link layout
- Interconnectivity complexity (Ci): gate layout

Figure 2 illustrates the three dimensions by showcasing the block chart of a hypothetical production system (S1) based on the complexity dimensions machines, links and gates.

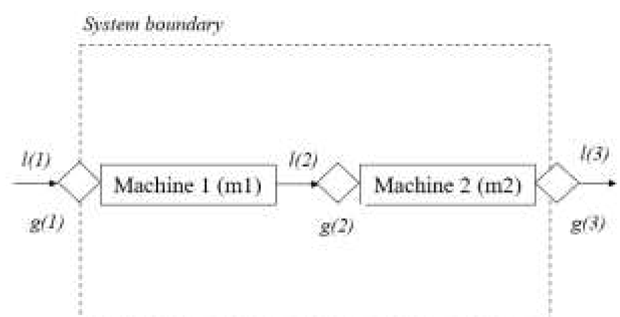

Figure 2: Example manufacturing system

Figure 2 shows, that S1 consists out of a machine layout with two machines, with m(1) and m(2), which are connected by a link layout with three links, with l(1)-l(3), and which interconnect with a gate layout of three gates, with g(1)-g(3).

B. Complexity Space

The modelling of the static complexity of a manufacturing system shall now be expressed by the theoretical three-dimensional compound state volume that results from the three dimensions combined, which shall be named complexity space of a system and is illustrated by Figure 3 and with the volume (V_{Cspace}) as illustrated in equation (2).

$$V_{Cspace} = Cs \times Cc \times Ci \quad (2)$$

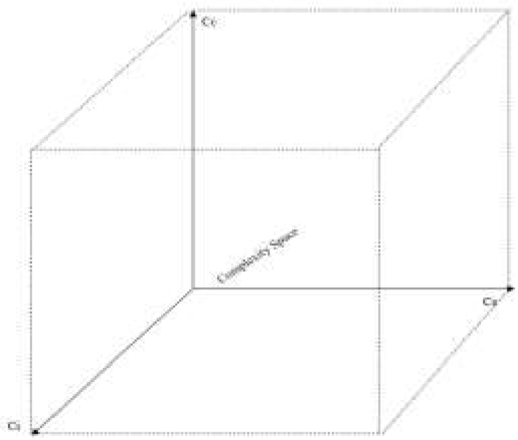

Figure 3: Complexity space

The theory of complexity space can now be applied as a foundational basis for system complexity Modelling and visualization of static industrial system complexity (S_C) of a manufacturing system. Equation (3) reflects this.

$$V_{Cspace} = S_C \tag{3}$$

Equation (3) shows, that (V_{Cspace}) can now be utilized to represent the compound state of (S_C) for this paper. In the next chapter, the three dimensions of (V_{Cspace}) are now described in detail.

3. Complexity Space

The three dimensions (Cs, Cc, Ci) of complexity space (V_{Cspace}) are assumed to comprise the variables of the compound state of the complexity of the static structure of a modeled manufacturing system. The basic arrangements and relations between the individual system parts in the form of machines, gates and links and are now further described. The logarithm to the base of 2 is utilized to decrease the impact of higher numbers in the different dimensions and to allow a quantification in units of bits. [14,19,21]

A. Structural Complexity

(Cs) shall be defined by a systems structural, static layout of machineries (m). Consequently, (Cs) of an industrial production system is expected to be maximized if (m) is maximized, as shown in equation (4).

$$Log_2(m)=Cs_{max} \tag{4}$$

Where m=number of machines and Cs=structural complexity of the system.

B. Connectivity Complexity

(Cc) shall be defined by a systems structural, static layout containing transfer links (l) between the system machinery layouts. Consequently, (Cc) of an industrial production system is expected to be maximized if (l) is maximized a shown in equation (5).

$$Log_2(l)=Cc_{max} \tag{5}$$

Where l=number of links and Cc=connectivity complexity of the system.

C. Interconnectivity Complexity

(Ci) shall be defined by a systems structural, static layout of number of gates (g) connecting different transfer links to the system static structural machinery layout and types of gates, for example data or material gates. Consequently, (Ci) of a manufacturing system is expected to be maximized if (g) is maximized as shown in equation (6).

$$Log_2(g)=Ci_{max} \tag{6}$$

Where g=number of gates and Ci=interconnectivity complexity of the system.

The definition of the complexity dimensions shows that the total volume of the complexity space (V_{Cspace}) of an industrial system can be maximized by maximizing each complexity dimension and is calculated in units of bits via the use of a base-2 logarithm to encode all static system states in information and to reduce the overall impact of larger dimension sizes on the overall complexity space volume.

(V_{Cspace}) can now be calculated based on Equation (2), (4), (5), (6) as a space of information in units of bits as shown in equation (7).

$$V_{Cspace} = Log_2(m) \times Log_2(l) \times Log_2(g) \tag{7}$$

Where V_{Cspace} = complexity space volume of the system and $Log_2(m)$ = Cs, $Log_2(l)$=Cc and $Log_2(g)$=Ci.

After introducing the concept of complexity space and complexity space volume as the metric for static system complexity in detail, the next section now describes the method of complexity space profiles.

D. Complexity Space Profiles

To illustrate and compare the extend of each complexity space dimension, the method of complexity space profile (CSP) shall be utilized. A CSP shall serve as a two-dimensional overview, a profile, of complexity space that allows to display and compare all relevant complexity space properties of a system. Figure 4 now provides an example CSP for a hypothetical system (S2) with the properties described in Equation (8).

$$S_C= \{m=10, l=10, g=10\}, \text{ with } V_{Cspace} = 36{,}6 \tag{8}$$

Figure 5 provides an example CSP for a hypothetical system (S3) with the system properties shown in Equation (9).

$$S_C= \{m=10, l=10, g=100\}, \text{ with } V_{Cspace} =73{,}3 \tag{9}$$

Figure 4 now shows the CSP of S2.

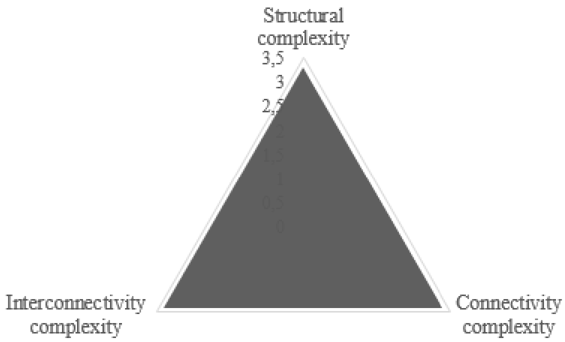

Figure 4: CSP of S2

Figure 5 now shows the CSP of S3.

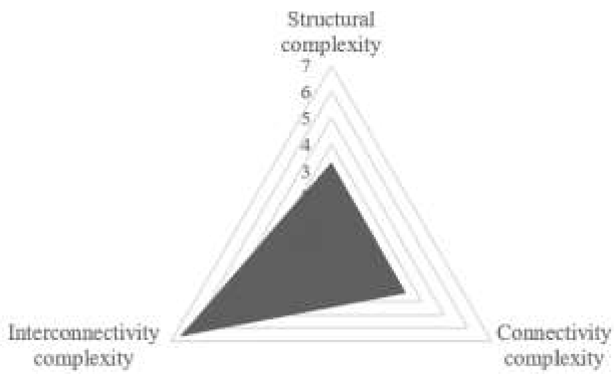

Figure 5: CSP of S3

Figure 8 and Figure 9 underline the functionality of complexity space and the resulting complexity space profiles, since the provided CSP of S2 and S3 allow the analysis of the distribution of S_C and overall extent of complexity space volume in a manufacturing system and the comparison of S_C between different systems. In the provided examples, the CSP make clearly visible that S2 has an equal distribution of complexity space, while in S3 the complexity dimension (Ci) is significantly larger than (Cs) and (Cc) leading to up the concept of complexity space bias (CSB).

E. Complexity Space Bias

Complexity space profiles (CSP) allow to determine complexity space bias (CSB). In general, three types of CSB are theoretically possible.

- Unbiased (O-CSB): All dimensions of a CSP are of equal size.
- One-dimensional CSB (1D-CSB): One complexity dimension in a CSP is significantly enlarged.
- Two-dimensional CBS (2D-CSB): Two complexity dimensions in a CSP are significantly enlarged.

For example, the CSP displayed in Figure 4 can now be classified as 0-CSB, while Figure 5 displays CSP with a 1D-CSB.

After introducing and describing the method of CSP and the concept of CSB the next section now shows how different sub-systems of a system can be modelled via complexity space.

F. Multiple System Levels in Complexity Space

Figure 6 illustrate a multi-system-layer complexity space model by introducing a hypothetical layer of complexity spaces of a hypothetical automotive factory system (S1 contains S2, S2 contains S3) positioned in complexity space.)

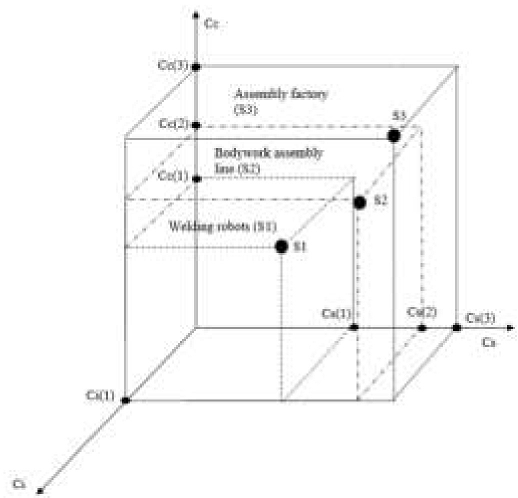

Figure 6: Example levels of a manufacturing system in complexity space

Based on the provided example it can be shown that multiple system levels of a manufacturing system can be captured and visualized in complexity space model at once, allowing the representation of different system layers in the model through the utilization of complexity space and CSP. Based on this notion it is now possible to introduce and integrate a dynamic complexity component to the model in the form of a definition of dynamic complexity.

4. DYNAMIC COMPLEXITY

A common dominator of dynamic complexity shall be introduced through the notion of system information entropy. The entropy of a system is in this context regarded as a measure of disorder in the system and fits the applied conception of Deshmukh, Talavage and Barash.[16] Based on this notion, a dynamic element of industrial system complexity in the form of information complexity and serves as an indicator for system instability when integrated in the complexity space model of a manufacturing system.

A. Properties of Dynamic Complexity

Dynamic system complexity is associated with three main properties:

- large quantity of information (Qi)
- variety of information (Vi)
- information content (Hi)

These properties correspond dynamically and time-dependent to the transformation efforts in a manufacturing system to achieve the high output complexity in correspondence to a given production goal within a given industrial system. [17,18,19] Consequently the dynamic system complexity in the form of information complexity (C_N) is proposed to represent the quantity, variety and information content of information contained in a system at a given point in time. Equation (9) illustrates this.

$$C_N = \{Qi, Vi, Hi\} \tag{9}$$

Where C_N = Information complexity, Oi= information quantity, Vi= information variety and Hi= information content.

B. Information Complexity as Machine Memory Space

To allow practical application of the model, a given machine (m) in a manufacturing system shall be expected to utilize a given amount of information (N) to contribute to the transformation process of the system. To allow a more nuanced and practical definition of the term information it is possible to introduce the concept of machine memory space (mms).[21,22,23] As Figure 1 already indicates, a machine (m) in an manufacturing system and the system itself shall be regarded as algorithms, a sequence of well-ordered instructions (input), that serve to solve a well-formulated problem (output) to obtain the overall goal of the system.[23] (mms_{system}) now describes the total amount of memory space units and therefore the extent of the encoded information content (Hi), quantity (Qi) and variety (Vi) needed by the static layout of a system to produce the expected solution as an output in relation to its input instructions. Equation (10) illustrates this.

$$mms_{system} = \{Qi, Vi, Hi\} \tag{10}$$

Where mms_{system}= amount of system memory space, Oi= information quantity, Vi= information variety and Hi= information content.

For example, in the case of a linear programming problem this process shall be defined as the problem of either minimizing or maximizing a linear function subject to a finite set of linear constraints, for example with a simplex algorithm. [23,24,25] The total information complexity contained in a system ($C_N(T)$) can now be calculated as shown in equation (10).

$$Log_2(mms_{system}) = C_N(T) \tag{10}$$

Where mms_{system}= amount of system memory space and $C_N(T)$= total information complexity contained in a system.

It can now be stated that a system must be regarded as non-complex if no or only minimal information is flowing, irrespective of the size of complexity space volume. It can now be stated that a system must be regarded as non-complex if no or only minimal information is flowing, irrespective of the size of complexity space volume.

C. Information Complexity within Complexity Space

The introduced definition of information complexity can now be integrated in the concept of three-dimensional complexity space and shall be assumed to take the form of an information complexity sphere with a volume (V_{Sphere}) situated in (V_{Cspace}), with (V_{Cspace})>(V_{Sphere}), ((V_{Sphere}), (V_{Cspace}))>0. To achieve this a hypothetical information complexity inception point (I(S)) is assumed to exist at the center of complexity space. From I(S) the total information complexity ($C_N(T)$) is expected to expand in all directions into complexity space over time as the static layout of the system circulates, stores and generates information via machines, gates and links. For simplification, the volume of the information complexity shall be defined by the conception of information complexity as a spherical body that occupies complexity space, where ($C_N(T)$) is regarded as the radius (r) of the information complexity sphere situated in the complexity space, as shown in equations (11) and (12) and illustrated in Figure 7.

$$V_{Sphere} = 4/3 \times \pi \times C_N(T)^3 \tag{11}$$

$$V_{Sphere} = 4/3 \times \pi \times Log_2(mms_{system})^3 \tag{12}$$

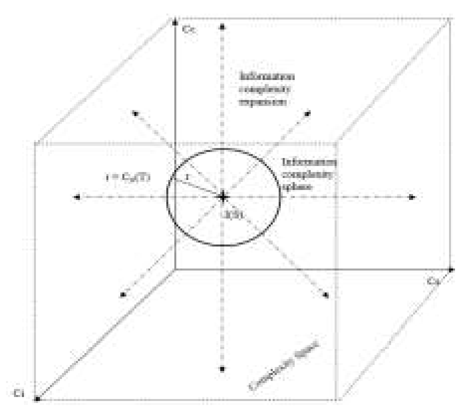

Figure 7: Information complexity in complexity space

D. Time-dependent Information Complexity Expansion

Based on the conception of industrial systems as algorithms it shall furthermore be assumed that the informational complexity of an industrial system increases over a timespan t0-tn when the input / output instructions of the system change over time and no mitigating or inhibiting regulations of the system are in place.

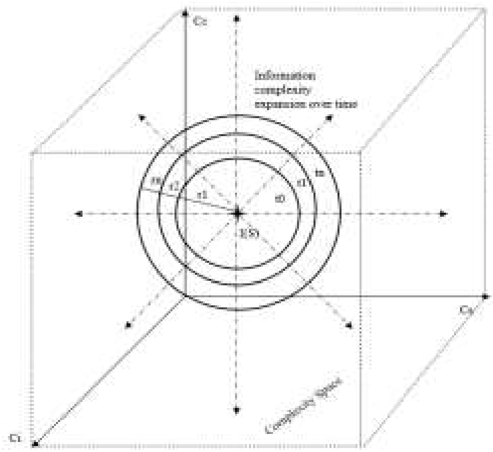

Figure 8: Expansion of information complexity in complexity space over time

Consequently, a manufacturing system that is expected to function under varying input / output instructions as an algorithm to meet changing system transformations can be expected to maximize the volume of its information complexity sphere over time. This is shown by Lehman´s Law of increasing complexity, which states that a systems complexity must increase over time if the system is not artificially regulated and by the laws of entropy in a system, exemplified in the second law of thermodynamics. [15,21,26,27] Information complexity thus suggests the expenditure until the boundary of the systems complexity space is reached over time. The application of Ashby´s Law of requisite variety, which states that a decrease of disturbance and outcome variety must always be accompanied by a proportional increase in regulation variety, allows to draw conclusions concerning an increase of regulation effort or hidden cost of the system over time in proportion to informational complexity. [28,29]

E. System Distortion

The volume of the complexity space of a manufacturing system resulting from the dimensions (Cs), (Ci) and (Cc) predefines the theoretical limits for the expansion of (C_N) and the maximum volume of the information complexity sphere. If expansion of (C_N) is not inhibited, the radius of the information complexity sphere (rn) must reach the boundaries of one or more dimensions of the complexity space in a time (tn) and creates the distortion point D(S) in the given dimension, according to the CSB of the system. When D(S) is reached the system shall be in a distorted state, leading up maximum deviation of the system target function in the distorted dimension and information complexity is unable to expand further in this dimension. Distortion in a system shall thus be defined as the upper limit of useful system operational ability where the system behaviour becomes non-linear and potentially chaotic. Figure 9 illustrates this.

If complexity space bias is 0-CSB, as illustrated by the provided examples in Chapter 3, all dimensions of the complexity space are distorted at the same time and the same radius of (C_N), leading up to simultaneous full system distortion in every dimension. If the complexity space is differently shaped, for examples box shaped in a 1D or 2D-CSB case, thus when complexity space dimensions are of unequal size, a biased system distortion occurs. A system shall be defined as one-dimensionally distorted if D(S) is expected to be reached in one given dimension first. A system shall be defined as two-dimensionally distorted if D(S) is reached in two of three dimensions of the complexity space at the same time. A system shall be defined as unbiased if all dimensions are reached at the same time.

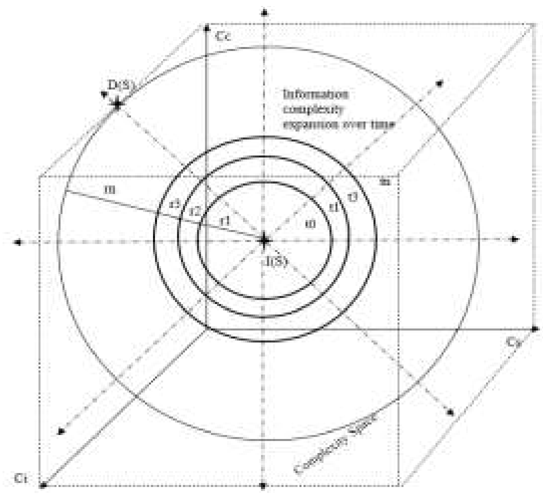

Figure 9: System distortion point

Figure 10 now illustrates the expansion of information complexity in complexity space via a logistic growth function based on Equation (13).

$$C_N(T)(t) = C_{N0} \text{ x } \exp(kt) / (1+d/k \text{ x } C_{N0} (\exp(kt)-1)) \quad (13)$$

Where $C_N(T)(t)$ = amount of information complexity at a given point in time t, C_{N0} = amount of information complexity at t0, k=growth factor, d= degression factor.

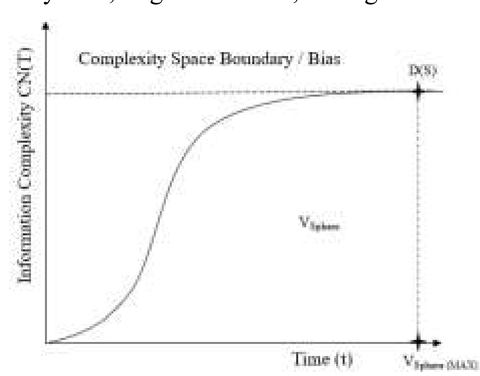

Figure 10: Logistic growth function of information complexity expansion in complexity space

Based on Equation (13), Figure 10 shows that $C_N(T)$ is now expected to show logistic growth behaviour over time (t0-tn) in the boundaries of complexity space. While doing this, $C_N(T)$ is not only limited by the inherent degression factor of the function (d) but also by the volume of complexity space dimensions and the resulting complexity space bias of the system in which information complexity expands until the D(S), with $V_{Sphere\,(MAX)}$, is reached

After introducing the most relevant functions of the proposed complexity space model, the next chapter now showcases and discusses the results of a first application of complexity space modelling on a real-world system in the European beauty & health industry to illustrate the applicability and limitations of the proposed modelling approach.

5. Case Study: Complexity Space Model of a Manual Assembly Line

The structure of this chapter starts with the description of case and the chosen and applied case study research design and describes the operationalization of complexity space modelling of a real-world system. In a final step the results of the case study and their applicative value are discussed.

A. Case Description

The proposed complexity space modelling approach is applied on a manufacturing system of the type of manual assembly line at an international health & beauty manufacturer with the goal to establish a complexity space model of the system and to showcase the general functionality and applicability of the proposed modelling approach on real-world manufacturing systems. The analysed system comprises a linear assembly line of 4 working stations manned with one worker each. It is necessary to acknowledge that the analysed system must be regarded as a stable, simple, non-problematic system at its core and therefore serves as demonstrative, illustrative purpose as a case study to establish a first proof of concept. Figure 11 illustrates the basic layout of the modeled system.

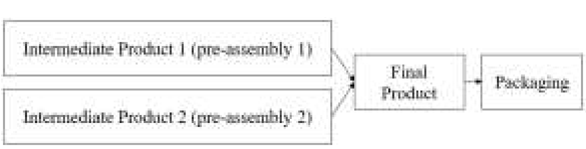

Figure 11: Basic layout manual assembly line

Figure 11 shows that the modeled manufacturing system contains the assembly of two intermediate products which get assembled in a final product and are then packaged in a final step before getting shipped to the customer.

B. Method: Document Analysis

To achieve system complexity modelling, the company provided documentation as the data basis for system modelling. 10 computer-based and internet-transmitted documents of various types collected in a database were individually reviewed by research team and used to generate a complexity space model of the analysed manufacturing system. The utilized documents for complexity space modelling encompass factory layouts, production schedules, cost calculations, maintenance reports as well as videos of the facility.

C. Complexity Analysis

The conducted analysis of the case via the complexity space modelling approach is achieved through conducting four different steps which are defined in Table 1.

Table 1: Complexity Analysis via Complexity Space Modelling

Step of analysis	Description
1.Structural complexity analysis	Establishment of complexity space model based on the three dimensions of structural complexity.
2.Complexity Space Profile (CSP) & Bias (CSB)	Interpretation of complexity space model through the means of the CSP and the resulting CSB.
3.Dynamic complexity analysis	Establishment of a conception of information complexity.
4.Aggregated analysis	Integration of structural and complexity analysis in one system model.

After showcasing the four different steps necessary to conduct complexity space modeling & analysis on the provided case, the next sections now show the application of the different steps.

D. Structural Complexity Analysis: Complexity Space

Based on the analysed documents, the resulting properties of the modeled system are now defined by the complexity dimensions shown in Equation (14).

$$S_C = \{m=4, l=5, g=8\}, \text{ with } V_{Cspace} = 13{,}9 \qquad (14)$$

Equation (14) shows that the modelled system is characterized by 4 machines (m), 5 links (l) and 8 gates (g). Based on the Equations (4) - (7) this results in an overall complexity space volume V_{Cspace} =13,9.

Figure 12 now illustrates the resulting complexity space of the analysed system.

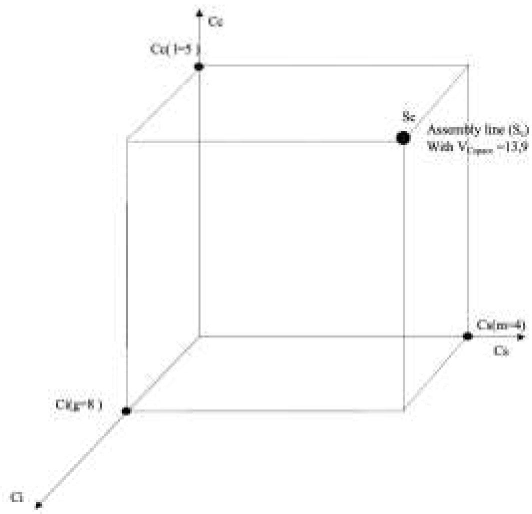

Figure 12: Complexity space model of assembly line

Figure 12 shows, that it is possible to construct a complexity space for the analysed system based on the calculations shown by Equations (4) – (7) and therefore to coherently allocate all dimensions of structural complexity of the analysed case in a model-coherent way via the compound state of complexity space.

In a next step the complexity space profile (CSP) and bias (CSB) are identified to allow interpretation of the established complexity space.

E. System Complexity Space Profile & Bias

Based on the propositions made in Chapter 3, the complexity space profile (CSP) and bias (CSB) of the modelled system is now illustrated in Figure 13.

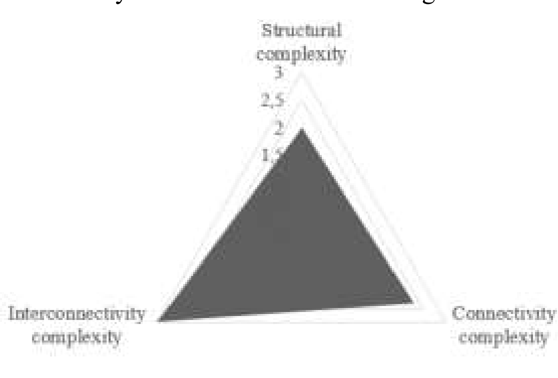

Figure 13: System Complexity Space Profile of manual assembly line

Figure 13 shows that the modelled system can be categorized as a two-dimensional CSB (2D-CSB), with the strongest bias being interconnectivity complexity, the second strongest bias connectivity complexity. This indicates that the structural complexity has a major source in the interconnectivity of system parts and connectivity, thus in the arrangement of the gates and the layout of machinery. Based on Chapter 4, it can now be concluded that the arrangement of gates is to be expected to be the hypothetical main source of system distortion.

F. Dynamic Complexity Analysis

Due to the analysed system being a manual assembly line the information complexity ($C_N(T)$) of the system is difficult to quantify in practice, since no dedicated machine-memory space is utilized to achieve the transformation process in the system even though the system itself can be described as a sequence of well-ordered instructions (input), that serve to solve a well-formulated problem to obtain the overall goal of the system. Consequently, information is flowing in the system in the physical form of materials and manual human labor. Based on Equation (9)

$$C_N = \{Qi, Vi, Hi\} \tag{9}$$

It can be concluded through a qualitative assessment that the overall information quantity, variability, and content is non-complex, and that the system is undistorted by the degree of existing information complexity. The provided documents also show that sufficient controls are in place to keep the system running in a stable and reliable fashion and which inhibit information complexity expansion effectively.

G. Aggregated Complexity Analysis

When modelling this kind of information complexity in the established complexity space model for the case shown in Figure 12, the information complexity sphere (V_{Sphere}) can be expected to be non-distortive, leading to undistorted complexity space. Additionally, the expansion of the information complexity sphere can be expected to be minimal or even non-existent. Based on this, Figure 14 illustrates a hypothetical approximation of information complexity in the modelled complexity space of the case.

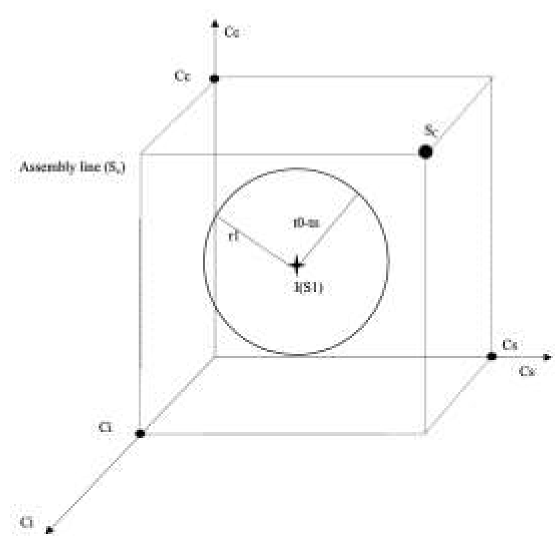

Figure 14: Complexity space model of assembly line with information complexity approximation

As stated in Chapter 4 it can now be determined that the analysed system must be regarded as non-complex since only minimal information is flowing, irrespective of the size of complexity space volume.

H. Summary

The results of individual steps of the conducted case study analysis are now displayed in Table 2.

Table 2: Complexity Analysis via Complexity Space Modelling

Step of analysis	Results
Structural complexity analysis	A complexity space with the properties: $S_C = \{m=4,\ l=5,\ g=8\}$, with $V_{Cspace} = 13,9$ is established.
Complexity Space Profile (CSP) & Bias (CSB)	A two-dimensional CSB (2D-CSB), with the strongest bias being interconnectivity complexity, the second strongest bias connectivity complexity is identified.
Dynamic complexity analysis	The overall information quantity, variability, and content show that the system is non-complex, and that the system is undistorted by the degree of existing information complexity. An approximation of information complexity is integrated into the established complexity space.
Aggregated analysis	The information complexity sphere (V_{Sphere}) is modeled to be inhibited and non-distortive, leading to undistorted complexity space. The expansion of the information complexity sphere can be expected to be minimal or even non-existent. The analysed system represents a stable and functional system.

The results of the conducted case study are now discussed in the next section.

I. Discussion

The provided case study shows that the proposed complexity space modelling approach is generally applicable to the conceptual modelling of the complexity space of a real-world manufacturing system based on existing and partly standardized company documentation. Due to the analogue and non-complex nature of the analysed manual assembly line the extent of contained information complexity was only determinable in a qualitative way and got integrated into the established complexity space as an approximated information complexity sphere. This indicates the necessity for further case studies with fully digitized production systems. Nevertheless, the generated complexity space model of the system can be regarded as an adequate representation of the real-world systems structure, volatility, and performance reliability in terms of its complexity. Overall, the results allow the goal of early-stage exploratory analysis.

J. Implications of Results: Applicative Value

The provided complexity space modelling case study application allows to draw the following first implications for the applicative value of the approach:

- The model allows to characterize the basic constituents and/or governing dynamics of industrial system complexity in a coherent framework.
- A coherent understanding of the dimensions and factors that unify the complexity of the analysed complex system is achieved.
- The results can serve as a baseline component for more advanced modelling and simulation approaches for complex engineered systems
- The model can support the reduction of uncertainty in the decision-making process between different system stakeholders.

The mentioned implications show that the proposed complexity space modelling approach achieves its primary goals of enabling early-stage exploratory system analysis while serving as a potential conceptual baseline for more advanced system models and simulation.

6. CONCLUSION

A complexity space modelling approach for industrial system complexity is introduced and aims to serve as a conceptual modelling approach with the primary function of early-stage exploratory system analysis and enabling more advanced modelling and simulation approaches. The model is based on the axiomatic conception of a three-dimensional static complexity space in which informational complexity is modelled as a sphere that expands dynamically over time until expansion is limited by the boundaries of complexity space. It can be concluded in the context of the model, that any engineered system maximizes information complexity over time and thus also maximizes entropy over time, making the system increasingly prone to error, hazardous and cost intensive over time, if the system information complexity expansion is not adequately artificially controlled via an external control system of proportionate size and ability. Based on the provided case study application in the form of a complexity space model of a manual assembly line, it is shown that complexity space modelling can be generally applied to create conceptual models of the complexity of real-world industrial systems which can be used for more advanced modelling and simulation approaches.

A. Research Outlook

There are now many open directions for future work. Firstly, it would be interesting to further analyze the behavior of information growth in current and future IoT-based industrial systems. Secondly, based on the case study applications, the model could be extended in a way that it would allow to derive analytic and strategic implications for decision-makers for system optimization, for example through the translation of the complexity space model into a complexity assessment matrix or risk assessment matrix. Finally, it would be interesting to study current real-world IIoT systems, like cyber-physical systems, through complexity space modelling in more detail.

ACKNOWLEDGMENT

We thank the board of reviewers for their helpful suggestions and feedback.

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