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A linear weighted estimator for estimating population total or mean of the study variable

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ABSTRACT

In this paper we have extended the expressions of Bias and Mean square error of the estimators of Agarwal and Al-Mannai (2008) from first order of approximation to the second order of approximation. This helps further to the problem of reducing sampling error in sample surveys, by using the information on auxiliary variables. The usefulness of the estimators is demonstrated with the help of examples taken from the literature.

1. INTRODUCTION

To reduce the cost of the sample surveys, the statisticians prefer to estimate the parameters of several characteristics simultaneously. The sampling methodology depends upon the nature of the population and availability of the information and resources. If the information on auxiliary variable highly positively correlated with prime characteristics of interest is available, then the primary units are selected with probability proportional to size (pps) with replacement or without replacement. Selection of primaries with pps of an auxiliary variable, may improve the efficiency of the estimate of population mean/total [Agarwal et al (1978), Chaudhuri and Vos (1988)]. For those characteristics of interest having low or very low correlation with size measure, Rao (1966) introduced certain biased alternative estimators.

In the last four decades several linear weighted estimators are suggested for estimating population mean/total [see Singh (1967), Murthy (1967), Agarwal and Kumar (1980), Amahia et al (1989), Agarwal *et al* (2003)]. Recently, Agarwal and Al Mannai (2008) has made an attempt to show the importance of linear weighted estimators under ppswr sampling over the alternative estimator and also over the conventional estimators through an empirical study for a wide variety of populations available in the literature. We have extended the expressions of Bias and Mean square error of the estimators of Agarwal and Al-Mannai (2008) from first order of approximation to the second order of approximation. This helps further to the problem of reducing sampling error of the estimators, by using the information on auxiliary variables. The usefulness of the estimators is demonstrated with the help of examples. For the sake of clarity and the empirical studies, we have restricted our study when the information is available on two auxiliary variables

2. STATEMENT OF THE PROBLEM

Consider a finite population U of size N identifiable, distinct units $u_{1,}\, u_{2,}\, u_{i,.......}u_{N,}$ It is assumed that study variables y are defined on U. The information is available on two auxiliary variables x_1 for each unit of the population while the only information required on x_2 is the population mean $\overline{X}_2$. The selection probabilities based on x_1, are p_{1i} (= x_{1i}/X_1 ; $X_1=\sum_{i=1}^{N} x_{1i}$); i =1,2,….N.

The problem is to estimate the population mean/total of study variables y's with the reduced sampling error under the situations:

(i) When y has high linear relationship with x_2.

(ii) When y has linear negative relationship with x_2.

3. LINEAR WEIGHTED ESTIMATOR WHEN STUDY VARIABLE y AND x_2 ARE LINEARLY RELATED.

Let,

$$u_i = \frac{y_i}{Np_{1i}};\ v_i = \frac{x_{2i}}{Np_{1i}};$$

$$\overline{u} = n^{-1}\sum_{i=1}^{n} u_i = \overline{y}_{pps};\ \overline{v} = n^{-1}\sum_{i=1}^{n} v_i = \overline{x}_{2\,pps}$$

$$C_{lm}=\frac{\sum_i p_{1i}(v_i-\overline{X}_2)^l(u_i-\overline{Y})^m}{\overline{Y}^m\overline{X}_2^l};\ l,\ m = 0,\ 1,\ 2,\ldots$$

$$V(\overline{y}_{pps}) = \sigma_u^2 = \frac{1}{n}\sum_{i=1}^{N} p_{1i}(u_i-\overline{Y})^2 = \frac{C_{02}\overline{Y}^2}{n}$$

$$\sigma_v^2 = \sum_{i=1}^{N} p_{1i}(v_i-\overline{X}_2)^2 = \frac{C_{20}\overline{X}_2^{\,2}}{n}$$

$$\overline{y}_R = \frac{\overline{u}}{\overline{v}}\overline{X}_2;\ \overline{y}_p = \frac{\overline{u}\,\overline{v}}{\overline{X}_2};$$

$$\rho_{uv} = \sum_{i=1}^{N}\frac{p_{1i}(u_i-\overline{Y})(v_i-\overline{X}_2)}{\sigma_u\sigma_v} = \frac{C_{11}}{\sqrt{C_{02}C_{20}}}$$

3.1 WHEN THE STUDY VARIABLE Y AND X_2 ARE LINEARLY BUT NEGATIVELY CORRELATED

The proposed estimator $\overline{y}_1$ of the population mean $\overline{Y}$ is a linear weighted estimator:

$$\overline{y}_1 = w\overline{y}_p + (1-w)\overline{y}_{pps} \qquad (1)$$

where w is the weight to be determined.

The bias and mean squared error of $\overline{y}_1$ are respectively

$$B(\overline{y}_1) = wB(\overline{y}_p) \qquad (2)$$

and

$$M(\overline{y}_1) = w^2M(\overline{y}_p) + (1-w)^2V(\overline{y}_{pps}) + 2w(1-w)Cov(\overline{y}_p,\overline{y}_{pps}) \qquad (3)$$

The value of w which minimizes eqn (3) is

$$w_{\text{opt}} = \frac{V(\overline{y}_{pps}) - Cov(\overline{y}_p,\overline{y}_{pps})}{M(\overline{y}_p) + V(\overline{y}_{pps}) - 2Cov(\overline{y}_p,\overline{y}_{pps})} \qquad (4)$$

and

$$\mathrm{M}(\bar{y}_1)_{\min} = \frac{V(\bar{y}_{pps})M(\bar{y}_p) - Cov^2(\bar{y}_p, \bar{y}_{pps})}{M(\bar{y}_p) + V(\bar{y}_{pps}) - 2Cov(\bar{y}_p, \bar{y}_{pps})} \tag{5}$$

In order to find the bias and mean square error to the second degree of approximation, we define $\delta_{\bar{u}} = \frac{\bar{u} - \bar{Y}}{\bar{Y}}$; $\delta_{\bar{v}} = \frac{\bar{v} - \bar{X}_2}{\bar{X}_2}$, so that E ($\delta_{\bar{u}}$) = E ($\delta_{\bar{v}}$) =0.

$$B_2(\bar{y}_p) = (\frac{N-n}{N-1})(C_{11}\bar{Y}/n) \tag{6}$$

$$M_2(\bar{y}_p) = (\frac{N-n}{N-1})[(C_{02} + C_{20} + 2C_{11}) + \frac{2(N-2n)}{n(N-2)}(C_{12} + C_{21}) + \frac{3(N^2 + N - 6nN + 6n^2)}{n^2(N-2)(N-3)}C_{22}]\,(\bar{Y}^2/n) \tag{7}$$

$$Cov_2(\bar{y}_p, \bar{y}_{pps}) = (\frac{N-n}{N-1})[(C_{02} + C_{11}) + \frac{2(N-2n)}{n(N-2)}(C_{12})](\bar{Y}^2/n) \tag{8}$$

On substituting eqns (7) and (8) in eqn (4), and then in eqns (2) and (3) we get bias $B_2(\bar{y}_1)$ and MSE $M_2(\bar{y}_1)$ up to second degree of approximation. When the population size is sufficiently large, then the expression derived under second degree of approximation doesn't contribute much and can be ignored. Then we consider only the terms of first degree of approximation and the above eqn (7) and eqn (8) reduce to

$$M_1(\bar{y}_p) \cong n^{-1}\bar{Y}^2(C_{02} + C_{20} + 2C_{11}) \tag{9}$$

$$Cov_1(\bar{y}_p, \bar{y}_{pps}) \cong [(C_{02} + C_{11})](\bar{Y}^2/n) \tag{10}$$

$$w_{1\,\mathrm{opt}} \cong -(\frac{C_{11}}{C_{20}}) \tag{11}$$

To the first degree of approximations, the least bias and mean squared error of $\bar{y}_1$ are respectively

$$B_1(\bar{y}_1) \cong -(C_{11}^2/C_{20})(\bar{Y}/n) \tag{12}$$

and

$$M_1(\bar{y}_1) \cong [C_{02} - (C_{11}^2/C_{20})](\bar{Y}^2/n) \tag{13}$$

1.2 WHEN THE STUDY VARIABLES Y AND X_2 ARE LINEARLY AND POSITIVELY CORRELATED.

Agarwal and Kumar (1980) defined the linear weighted estimator as follows:

$$\bar{y}_0 = k\bar{y}_R + (1-k)\bar{y}_{pps} \tag{14}$$

The bias and MSE of $\bar{y}_0$ are

$$B(\bar{y}_0) = kB(\bar{y}_R) \tag{15}$$

$$M(\bar{y}_0) = k^2M(\bar{y}_R) + (1-k)^2V(\bar{y}_{pps}) + 2k(1-k)Cov(\bar{y}_R, \bar{y}_{pps}) \tag{16}$$

The value of k which minimizes $M(\bar{y}_0)$ is

$$k_{\text{opt}} = \frac{V(\bar{y}_{pps}) - Cov(\bar{y}_R, \bar{y}_{pps})}{M(\bar{y}_R) + V(\bar{y}_{pps}) - 2Cov(\bar{y}_R, \bar{y}_{pps})} \qquad (17)$$

and

$$\text{M}(\bar{y}_0)_{\min} = \frac{V(\bar{y}_{pps})M(\bar{y}_R) - Cov^2(\bar{y}_R, \bar{y}_{pps})}{M(\bar{y}_R) + V(\bar{y}_{pps}) - 2Cov(\bar{y}_R, \bar{y}_{pps})} \qquad (18)$$

The bias and mean square error to the second degree of approximation are:

$$B_2(\bar{y}_R) \cong \frac{N-n}{N-1}[(C_{20} - C_{11}) + \frac{2(N-2n)}{n(N-2)}(C_{21} - C_{30}) + \frac{3(N^2 + N - 6nN + 6n^2)}{n^2(N-2)(N-3)}$$
$$(C_{40} - C_{31}) + \frac{3N(N-n-1)(n-1)}{n^2(N-2)(N-3)}(C_{20}^2 - C_{20}C_{11})](\bar{Y}/n) \qquad (19)$$

$$M_2(\bar{y}_R) \cong \frac{N-n}{N-1}[(C_{20} + C_{02} - 2C_{11}) + \frac{2(N-2n)}{n(N-2)}(2C_{21} - C_{12} - C_{30}) +$$
$$\frac{3(N^2 + N - 6nN + 6n^2)}{n^2(N-2)(N-3)}(C_{40} - 2C_{31} - C_{22}) +$$
$$\frac{3N(N-n-1)(n-1)}{n^2(N-2)(N-3)}(3C_{20}^2 - 6C_{20}C_{11} + C_{20}C_{02} + 2C_{11}^2)](\bar{Y}^2/n) \qquad (20)$$

and

$$\text{Cov}_2(\bar{y}_R, \bar{y}_{pps}) \cong \frac{N-n}{N-1}[(C_{02} - C_{11}) + \frac{2(N-2n)}{n(N-2)}(C_{21} - C_{12}) + \frac{3(N^2 + N - 6nN + 6n^2)}{n^2(N-2)(N-3)}$$
$$(C_{22} - C_{31})](\bar{Y}^2/n) \qquad (21)$$

On substituting eqns (20) and (21) in eqn (17), and then in eqns (15) and (16) we get bias $B_2(\bar{y}_0)$ and MSE $M_2(\bar{y}_0)$ up to second degree of approximation.

The bias and MSE to the first degree of approximations are:

$$B_1(\bar{y}_0) \cong \frac{N-n}{N-1}(C_{20} - C_{11})(\bar{Y}/n) \qquad (18)$$

$$M_1(\bar{y}_0) \cong \frac{N-n}{N-1}(C_{20} + C_{02} - 2C_{11})(\bar{Y}^2/n) \qquad (19)$$

and

$$\text{Cov}_1(\bar{y}_R, \bar{y}_{pps}) \cong \frac{N-n}{N-1}(C_{02} - C_{11})(\bar{Y}^2/n) \qquad (20)$$

The value of k which minimizes the $M(\bar{y}_0)$ to the first degree of approximation is

$$k_{1\text{opt}} = \frac{C_{11}}{C_{20}} \qquad (21)$$

The bias and mean square error of $\bar{y}_0$ for k_{opt}, to the first degree of approximations are:

$$B_1(\bar{y}_0) = n^{-1}[C_{11} - C_{11}^2/C_{20}](\bar{Y}/n) \qquad (22)$$ and

$$M_1(\bar{y}_0) = [C_{02} - C_{11}^2/C_{20}](\bar{Y}^2/n) \qquad (23)$$

2. EMPIRICAL STUDY

To study the relative efficiency and the relative bias of linear weighted estimators over conventional estimator/s under probability proportional to size with replacement (ppswr) sampling we have considered a wide variety of populations that cover most of the practical situations we come across in real life surveys. These populations are taken from the available literature [Freund and Perles (1999), Hines and Montgomery (1990), Mendenhall *et. al* (2003), Milton and Arnold (2003), Neter *et al* (1985) and Ott (1984)].

Description of populations

Table-1 and 2 give the characteristics of the populations such as population size N, coefficients of variation of the study variable (y), the auxiliary variables x_1 and x_2, the correlation coefficients between (y, x_1) and (y, x_2). In table -1, the population size varies from 16 to 65, the coefficient of variation of y from 10.25 % to 86.46%, the coefficient of variation of x_1 from 5.66% to 73.37%, the coefficient of variation of x_2 from 3.02% to 91.02%. The correlation coefficient between (y, x_1) varies from 0.51 to 0.97, while the correlation coefficient between (y, x_2) varies from 0.34 to 0.94. The above described populations thus represent a variety of situations and we further divide these into three categories. The category (i) ρ_{yx_2} between 0.3 but < 0.5; category (ii) $0.5 < \rho_{yx_2} <$ 0.8, and category (iii) $\rho_{yx_2} > 0.8$.

In the table -2 the population size varies from 19 to 50, the coefficient of variation of y from 13.23 % to 64.88%, the coefficient of variation of x_1 from 8.74% to 29.16 %, the coefficient of variation of x_2 from 8.38 % to 42.01 %. The correlation coefficient between (y, x_1) varies from 0.44 to 0.98, while the correlation coefficient between (y, x_2) varies from - 0.83 to - 0.40.

Table -3 gives the relative gain in efficiency of the estimator $\bar{y}_0$ using second order of approximation over first order of approximation, and also over $\bar{y}_R$ and the reduction in bias of $\bar{y}_0$ [$B_2(\bar{y}_0)$] over $B_1(\bar{y}_0)$ and $B_2(\bar{y}_R)$. It can be noted that the gain of using second order of approximation over first order of approximation varies from10% to 102%. For more than 75% of the populations, this gain is more than 30%. Similarly, the relative gain in efficiency of the estimator $\bar{y}_0$ using second order ofapprox. over $\bar{y}_R$ (using second degree of approx.) varies from 15% to 580%. The reduction in $B_2(\bar{y}_0)$ is also very significant.

Table -4 gives the relative gain in efficiency of the estimator $\bar{y}_1$ using second order of approximation over $\bar{y}_1$ first order of approximation, and also over $\bar{y}_P$ [using second order of approximation] and the gain in relative biases of $\bar{y}_1$. It can be noted that the gain of using second order of approximation over first order of approximation varies from 47% to 155%. Similarly, the relative gain in efficiency of the estimator $\bar{y}_1$ using second order of approximation over $\bar{y}_P$ (using second degree of approx.) is enormous. There is considerable reduction in biases also.

Table-1 Characteristics of the populations when ρ_{yx_2} >0

	N	ρ_{yx_1}	ρ_{yx_2}	C_y	C_{x_1}	C_{x_2}	Category
	24	0.667	0.859	13.859	24.101	44.967	iii
	20	0.924	0.458	19.851	10.230	13.205	i
	65	0.554	0.742	44.271	36.558	91.023	ii
	25	0.514	0.497	21.066	19.638	16.204	i
	25	0.897	0.869	21.066	8.785	11.276	iii
	25	0.537	0.345	20.574	19.931	18.543	i
	20	0.790	0.755	40.049	58.433	33.791	ii
	32	0.730	0.395	29.622	18.902	29.793	i
	30	0.550	0.756	66.258	10.383	35.795	ii
	30	0.704	0.656	66.258	21.046	35.795	ii
	19	0.930	0.584	10.246	8.591	3.017	ii
	47	0.888	0.688	15.750	13.319	41.677	ii
	35	0.920	0.805	86.462	73.373	89.194	iii
	31	0.967	0.598	54.482	23.687	8.384	ii
	50	0.859	0.739	24.605	25.109	23.641	ii
	25	0.892	0.824	79.454	69.357	78.557	iii
	23	0.716	0.687	22.096	21.664	8.118	ii
	24	0.819	0.895	30.438	26.953	8.937	iii
	34	0.636	0.814	26.734	21.186	6.565	iii
	50	0.669	0.831	24.103	15.135	7.026	iii
	21	0.781	0.945	30.024	5.660	19.896	iii
	16	0.892	0.395	14.008	32.991	34.427	i
	32	0.730	0.355	29.622	18.902	29.793	i
min	**16**	**0.514**	**0.345**	**10.246**	**5.660**	**3.017**	
max	**65**	**0.967**	**0.945**	**86.462**	**73.373**	**91.023**	
q1	**23**	**0.668**	**0.541**	**20.820**	**14.227**	**10.106**	
q2	**25**	**0.781**	**0.742**	**26.734**	**21.046**	**23.641**	
q3	**33**	**0.892**	**0.819**	**42.160**	**26.031**	**35.795**	

Table-2 Characteristics of the populations when $\rho_{yx_2} < 0$

	N	ρ_{yx_1}	ρ_{yx_2}	C_y	C_{x_1}	C_{x_2}	
	23	0.498	-0.602	13.231	21.379	27.243	
	23	0.795	-0.602	13.231	8.745	27.243	
	35	0.893	-0.675	15.753	12.983	42.008	
	23	0.438	-0.641	64.883	29.160	20.832	
	50	0.839	-0.705	24.605	24.041	23.641	
	40	0.843	-0.698	25.788	24.288	25.251	
	22	0.893	-0.473	24.385	22.961	16.483	
	19	0.476	-0.645	51.010	27.773	21.407	
	27	0.907	-0.718	17.094	14.044	31.530	
	26	0.926	-0.651	16.627	12.790	36.545	
	21	0.890	-0.828	15.594	14.127	33.799	
	26	0.977	-0.421	59.014	26.002	8.933	
	31	0.967	-0.399	54.482	23.687	8.384	
min	**19**	**0.438**	**-0.828**	**13.231**	**8.745**	**8.384**	
max	**50**	**0.977**	**-0.399**	**64.883**	**29.160**	**42.008**	
q1	**23**	**0.795**	**-0.698**	**15.753**	**14.044**	**20.832**	
q2	**26**	**0.890**	**-0.645**	**24.385**	**22.961**	**25.251**	
q3	**31**	**0.907**	**-0.602**	**51.010**	**24.288**	**31.530**	

Table -3 Relative gain in efficiency of the estimator $\bar{y}_0$ **using** second order of approximation over first order of approximation, and over $\bar{y}_R$ **and gain in relative biases.**

	N	$\frac{M_1(\bar{y}_0)}{M_2(\bar{y}_0)}$	$\frac{M_2(\bar{y}_R)}{M_2(\bar{y}_0)}$	$\frac{B_1(\bar{y}_0)}{B_2(\bar{y}_0)}$	$\frac{B_2(\bar{y}_R)}{B_2(\bar{y}_0)}$	Category
	24	1.89	6.42	1.07	7.16	iii
	20	1.10	11.62	1.17	8.08	i
	65	1.68	57.67	1.83	5.82	ii
	25	2.00	42.55	-1.46	-2.52	i
	25	1.70	59.17	-1.21	-2.36	iii
	25	1.60	44.05	-1.65	-2.05	i
	20	1.30	1.15	1.30	-2.05	ii
	32	1.20	1.90	1.09	1.32	i
	30	1.94	17.59	-12.29	-13.86	ii
	30	1.98	22.02	-1.26	-2.19	ii
	19	1.40	17.03	1.09	-2.65	ii
	47	1.96	32.19	1.05	1.92	ii
	35	2.02	30.74	-1.02	-3.25	iii
	50	2.00	4.50	-1.09	-1.39	ii
	50	1.80	4.60	-1.09	-1.39	ii
	25	1.14	1.98	1.14	2.69	iii
	23	1.70	32.34	-6.29	-8.64	ii
	24	1.40	24.03	-6.74	-9.68	iii
	34	1.99	7.72	2.48	6.27	iii
	50	1.69	1.92	1.15	1.70	iii
	21	1.30	39.94	-1.21	-1.62	iii
	16	1.14	5.55	1.05	3.30	i
	32	1.40	1.90	1.18	1.32	i
min	16	1.10	1.15	-12.29	-13.86	
max	65	2.02	59.17	2.48	8.08	
q1	23.5	1.30	4.50	-1.26	-2.52	
q2	25	1.69	17.03	1.05	-1.39	
q3	34.5	1.96	32.34	1.15	2.69	

Table -4 Relative gain in efficiency of the estimator $\bar{y}_1$ using second order of approximation over first order of approximation, and over $\bar{y}_P$ **and gain in relative biases.**

	N	$\frac{M_1(\bar{y}_1)}{M_2(\bar{y}_1)}$	$\frac{M_2(\bar{y}_P)}{M_2(\bar{y}_1)}$	$\frac{B_1(\bar{y}_1)}{B_2(\bar{y}_1)}$	$\frac{B_1(\bar{y}_P)}{B_2(\bar{y}_1)}$	
	23	1.47	13.04	-1.73	30.81	
	23	1.52	58.71	1.49	54.94	
	35	1.58	71.68	1.51	101.30	
	23	2.55	5.28	1.03	3.47	
	50	1.59	35.76	1.10	24.37	
	40	1.59	35.76	1.10	24.37	
	22	1.55	63.56	1.02	44.29	
	19	2.14	6.78	1.03	4.67	
	27	1.58	27.12	1.50	63.99	
	26	1.57	31.21	1.51	72.09	
	21	1.54	14.14	1.50	63.45	
	26	2.06	6.40	4.67	1.33	
	31	2.08	6.01	1.37	4.44	
min	**19**	**1.47**	**5.28**	**-1.73**	**1.33**	
max	**50**	**2.55**	**71.68**	**4.67**	**101.30**	
q1	**23**	**1.55**	**6.78**	**1.03**	**4.67**	
q2	**26**	**1.58**	**35.76**	**1.37**	**30.81**	
q3	**31**	**2.06**	**49.14**	**1.50**	**63.45**	

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قسم الرياضيات في كلية العلوم ، جامعة البحرين ،مملكة البحرين

الملخص
قمنا فى هذا البحث بتوسيع مفاهيم (تعابير) التحيز ومتوسط تربيعات الاخطاء للمقدرات التى نشرت من قبل أجاروال و آل المناعي عام 2008 من تقريبات الرتبة الاولى الى تقريبات الرتبة الثانية ،تساعد هذه التقديرات على تقليل نسبة اخطاء موجات العينة وذلك باستخدام متغيرات مساعدة ، وتم توضيح جدوى المقدرات من امثلة معمولا" بها فى الادبيات.