The Average Run Length Performance of a Multivariate Exponentially Weighted Moving Average Control Chart Procedure with Application

A. M. Kandil[1], M. S. Hamed[1*], S. M. Mohamed[2] and H. M. Shehata[3]

[1]*Department of Applied Statistics, Faculty of Commerce, Benha University, Egypt*
[2]*Department of Applied Statistics, Institute of statistical studies and research, Cairo University, Egypt*
[3]*Department of Commercial Affairs, Workers University, Egypt*

Abstract: One of the most powerful tools in quality control is the statistical control chart. First developed in the 1920's by Walter Shewhart, the control chart found widespread use during World War II and has been employed, with various modifications ever since. The drawbacks to multivariate charting schemes is their inability to identify which variable was the source of the signal. The multivariate exponentially weighted moving average (MEWMA) developed by Lowry, et al (1992) is an example of a multivariate charting scheme whose monitoring statistic is unable to determine which variable caused the signal. In this paper, the run length performance of multivariate exponentially weighted moving average (MEWMA) chart with application is studied. Industry fertilizers is important one of the chemical industries in Egypt, so that this work concerns the fertilizers industries quality control, especially urea fertilizer with application on Delta fertilizer and chemical industries which is considered on of the leading companies the field of fertilizer production in Middle east with application of multivariate quality control procedures to achieve best one procedure for multivariate quality control.

Keywords: Quality Control, Multivariate Analysis, MEWMA, (Multivariate Exponentially Weighted Moving Average).

1. INTRODUCTION

With today's use of computers, it is common to monitor several correlated quality characteristics simultaneously. Various types of multivariate control charts have been proposed to take advantage of the relationships among the variables being monitored. Alt (1985), Jackson (1985), Lowry and Montgomery (1995), and Mason et al. (1997) discuss much of the literature on this topic.. The formatter will need to create these components, incorporating the applicable criteria that follow.

2. THE EWMA CONTROL CHART PROCEDURE:

Timm (1996) developed a control chart using the EWMA to control a process mean. The EWMA techniques give the most recent observation the greatest weight with all previous observations weights decreasing in a geometric (exponential) progression from the recent back to the first. To demonstrate the EWMA technique, suppose that we observe sample means X_1, X_2, X_3, … in the univariate case, where,

$$X \sim N (\mu_0, \sigma_x^2) \tag{1}$$

MacGregor (1995) as introduced the univariate Exponentially Weighed Moving Average (EWMA) control chart:

$$Z_i = rX_i + (1 - r) Z_{i-1}, \quad i = 1, 2, 3, … \tag{2}$$

Where r is a smoothing constant and Z_i is the value of the EWMA after observation i, where i represents the observation number as well as an index of a point in time, where we required $Z_0 = \mu_0 = 0$ without loss of generality and $0 < r \leq 1$.

And he supposed that if X_1, X_2, X_3, … are independently and identically distributed $N (0, \sigma^2)$ random variables, the mean of Z_i is 0 and the variance is

E-mail address: abdelfattah.kandil@gmail.com, moswilem@gmail.com, dr_mahdym62@yahoo.com,hanaaaaa2007@yahoo.com

$$\text{var} (Z_i) = \sigma_Z^2 =$$
$$\left\{ \frac{r \left[1 - (1-r)^{2i} \right]}{2-r} \right\} \sigma_X^2, \quad i = 1, 2, 3, ... \quad (3)$$

Thus, he suggested that when the in-control value of the mean is 0, the control limits of the EWMA chart are at $\pm L \sigma_{Zi}$, where L and r are the parameters of the chart.

Lucas and Saccucci (1990) have discussed that the choice of L and r for the interval $(0.05 \leq 0.25)$ work well in practice, with $r = 0.05$, $r = 0.10$ and $r = 0.20$ being popular choices. A good rule of thumb is to use smaller values of r to detect smaller shifts. They have also found that $L = 3$ (the usual three $-$ sigma limits) works although when r is small $-$ say, $r \leq 0.1$ - there is an advantage in reducing the width of the limits by using a value of L between about 2.6 and 2.8 although their control limits, were based on the asymptotic form of

σ_{Zi} which is given by $\quad \sigma_{Zi} \cong \sigma_X \sqrt{\dfrac{r}{2-r}} \quad .$

3. MULTIVARIATE EXTENSION OF THE EWMA CONTROL CHART:

Lowry, Woodall Champ, Rigdon (1992) has generalized the concept of the univariate EWMA control chart to the multivariate case. They defined the MEWMA vectors as:
$$Z_i = R X_i +$$
$$(1 - R) Z_{i-1}, \quad i = 1, 2, 3, ... \quad (4)$$
Where $Z_0 = 0$ and $R = \text{diag} \left(r_1, r_2, ..., r_p, \ 0 < r_j \leq 1 \right), j = 1, 2, ..., p$ and $p > 1$

The MEWMA control chart gives an out of control signal as soon as:
$$T_i^2 = Z_i^` \Sigma_i^{-1} Z_i > L \quad (5)$$
Where $L > 0$ is chosen to achieve a specified in-control ARL, and Σ_{Zi} is the covariance matrix of Z_i.

Mason et al. (1997) provide a technique to interpret the out-of-control signal and identify an assignable cause from a multivariate control chart. Their method can be applied to MEWMA control chart.

Mason et al's method has assumed that Phase I of process control is when the first subgroups are drawn from the process and little is known about the joint distribution of the quality characteristics.

Phase II is when sufficient information is known about the process and subgroups are drawn to test if the process is in control.

Often there is no reason to apply different exponential weights to past observations of the p deferent quality characteristics. In this situation, Lowry et. (1992) assumed the equal weights across characteristics

where $r = r_j, \quad j = 1, ..., p,$ the MEWMA vectors can then be written as
$$Z_i = r X_i + (1 - r) Z_{i-1}, \quad i = 1, 2, 3, ... \quad (6)$$

Under the assumption of equal weights, Lowry et al. (1992) have shown that the covariance matrix of Z_i can be written in terms of the exponential weight r and the covariance matrix of the process data $\sum_X$ as :

$$\Sigma_{Zi} = \left\{ \frac{r \left[1 - (1-r)^{2i} \right]}{2-r} \right\} \Sigma_X \quad (7)$$

Note that if $r = 1$, the MEWMA chart is equivalent to Hotellin's T^2 chart.

Lowry et al. have suggested that when the process is likely to stay in-control for some time period, the asymptotic from of the covariance matrix $\sum_X$ used to calculate the MEWMA test statistic:
$$\Sigma_{Zi} = \left\{ \frac{r}{2-r} \right\} \Sigma_X \quad (8)$$

Lowry et al. (1992) have also, added the use of exact variance of the EWMA statistic leads to a natural fast initial response for the EWMA chart. Thus, the initial that, out-of-control conditions can also, applied to the MEWMA chart. Because, however, it may be more likely that the process will stay in control for a while and then shift out of control, they assumed for chart design that the asymptotic (as $i \to \infty$) covariance matrix, is give by Equation (8).

Linderman and Love (2000) have pointed out that the used of exact covariance matrix and the asymptotic covariance matrix lead to two different procedures; actually, they have concentrated their efforts concerned solely on the MEWMA chart using the asymptotic covariance matrix and the exact covariance matrix.

They have depended on the use of MEWMA chart in order to study the p quality characteristics associated with a process. The process begins in the in-control state with mean vector $\mu_0 = 0$ and covariance matrix $\sum_X$. They supposed also that the process is subject to a single assignable cause, which shifts the process mean from μ_0 to a point on the constant probability density contour D; defined by
$$D = \lfloor \mu_1 | \mu_1^` \Sigma_X^{-1} \mu_1 = \delta^2 \rfloor \quad (9)$$
Where δ, the parameter describing the size of shift, is known. Note that Equation (9) is the constant probability density contour for a p- dimensional multivariate normal distribution. This contour forms an ellipsoid which is centered at μ_0 and has axes at $\pm \delta \sqrt{\xi_j e_j}$, where $\sum_X e_j = \xi_j e_j$, for $j = 1, ..., p$.

Khoo (2003) has suggested that the main choice for the value of r in Equations (7) and (8) is, founded on the magnitude of shift where a quick detection is required. In general, small values of r used for quick detection of small shifts, where as larger values of r used for quick detection of large shifts. He has, also added that in case of (i → ∞) the exact covariance matrix in Equation (7) is approximately equal to the asymptotic covariance matrix in Equation (8); pointing out that this, the two covariance matrices in Equation (7) and (8) differ only in initial periods when i values are small. However, the advantages of the exact MEWMA chart over the asymptotic MEWMA chart are that the Exact MEWMA chart enables quicker detection of initial out-of-control conditions and is more sensitive for detecting shifts involving smaller values of r.

4. THE AVERAGE RUN LENGTH (ARL) PERFORMANCE:

The average run length is metric used to determine the control chart's ability in order to determine if the process is in control or out of control. Lowry et al. (1992) established that ARL performance for a MEWMA control chart is directionally invariant and determined solely by the non-centrality parameter. This means that the average run length depends only on the distance between the in control and out of control mean, and not on the direction. The non-centrality parameter given by Lowry et al. (1992), as:

$$\delta^2(\mu_y) = (\mu_y - \mu_0)' \sum_X^{-1}(\mu_y - \mu_0) \qquad (10)$$

Where μ_y is the mean when the process is out of control. They supposed that if $\mu_i = \mu$, $i = 1, 2, ...,$ the non-centrality parameter is given as:

$$\delta^2 = \left(\mu' \sum_X^{-1} \mu\right)$$

Recent researches have been developed to approximate the ARL for a MEWMA control chart. These methodologies have assumed that the in control distribution is given as $N_p(\mu_0, \sum_X)$. In addition, all these methodologies have made use of the ARL following fact that the performance depends only on the non-centrality parameter. As a result, to determine the ARL for a MEWMA control chart with an in-control distribution $N_p(\mu_0, \sum)$ and out-of-control distribution $N_p(\mu_1, \sum)$, we can determine the ARL performance of a MEWMA control chart with an in control distribution $N_p(0, 1)$ and out of control distribution $N_p(\mu_\delta, I)$, where $\mu_\delta = (\delta, 0, ..., 0)$. That is, the non-centrality parameter between both problems is identical.

Actually, the ARL performance, which only depends on the non-centrality parameter, is central to methodologies that have suggested to approximate ARL0 and ARL1 for a MEWMA chart. The approximation

methodologies include simulation Lowry et al. (1992), a bivariate Markov chain method Runger & Prabhu (1996), and integral equation Rigdon (1996, (a), (b)).

5. The Application:

Delta Fertilizers and Chemical Industries is considered on of the leading companies in the field of fertilizers production in Egypt. About 4500 employees are working for it, on the various managerial levels. Urea production is one of the major products of the company. The production of urea occurs through three stages, summarized as follows:

A) HIGH PRESSURE STAGE:

In this stage, urea is produced through two reactions; the first reaction occurs by condensation of Ammonia Gas and Carbon dioxide under high pressure and temperature for the sake of the production of intermediate material, known as Carbamate .The second reaction happens by separating the water from the Carbamate in order to a chive urea. In this, stage the condensation of urea approximately 56%.

It contains 16 variables, these are:

X1	E-201Outlet Temperature
X2	Outlet Cold NH3 from E- 201
X3	Co2 to Train
X4	Co2 pressure to Synthesis
X5	Co2 after E-22
X6	R-201
X7	Temperature in Reactor R-201
X8	Temperature in Reactor R-201
X9	Temperature in Reactor R-201
X10	Temperature in Reactor R-201
X11	Stripper level
X12	Liquid Leaving the Stripper
X13	Stream from E-204 to j-201
X14	Conditioned water to Scrubber E-204
X15	Conditioned water from Scrubber E-204
X16	Stream from j-203

Table analysis of laboratory in this stage:

t1.1	NH_3	Rector outlet
t1.2	CO_2	Rector outlet
t1.3	UR	Rector outlet
t1.4	B_1	Rector outlet
t1.5	H_2O	Rector outlet
t2.1	NH3	Stripper outlet
t2.2	CO_2	Stripper outlet
t2.3	UR	Stripper outlet
t2.4	B_1	Stripper outlet
t2.5	H_2O	Stripper outlet

B) LOW PRESSURE STAGE:

In this stage, the condensation of urea liquid rises from 56% to 71%. This happens through the decomposition of the remaining Carbamate and the elimination of water under low pressure.

It contains seven variables, these are:

y1	Urea Solution from stripper E-202
y2	Steam to E-205
y3	Urea Carbonate Solution from stripper T-201 to E-205
y4	Gas leaving T-201
y5	Level in TK-201
y6	P-203
y7	Urea Solution in TK-201

Table analysis of laboratory in this stage:

t3.1	NH_3	D 202 Outlet
t3.2	CO_2	D 202 Outlet
t3.3	UR	D 202 Outlet
t3.4	B_1	D 202 Outlet
t3.5	H2O	D 202 Outlet
t4.1	NH_3	In TK 201
t4.2	CO_2	In TK 201
t4.3	UR	In TK 201
t4.4	B_1	In TK 201
t4.5	H_2O	In TK 201
t5.1	NH_3	In PI 302
t5.2	CO_2	In PI 302
t5.3	UR	In PI 302

C) EVAPORATION AND PRILLING STAGE:
This stage occurs by two stage:

Firstly, Evaporation stage:

In this stage, the condensation of urea rises from 71% to 98.7% approximately and the urea liquid trams forms to urea melt. This happens under high pressure and temperature.

Secondly, Prilling Stage:

In this stage, the urea melt is through formed into prilling in the prilling tower.

It contains four variables, these are:

Z1	Urea Solution from D-204 to E-209
Z2	D- 205 Vacuum
Z3	Urea to periling tower X-202
Z4	E- 211 Vacuum

Table analysis of laboratory in this stage:

t6.1	B1
t6.2	H2O
t6.3	Pills > 3.35
t6.4	Pills 3.35: 2.4
t6.5	Pills 2.4 : 1.4
t6.6	Pills 1.4 :1.0
t6.7	Pills < 1.0
t6.8	UR

5.1 DATA DESCRIPTION:

For the application of multivariate quality control, chart data originate from urea production process, which consists of the three stages and the analysis of laboratory, which discussed above.

The number of the sample is 732 observations taken per hour.

The advantages of this sample that, it has several variables and several stage of the production. This advantage of the production is the basic reason for choosing this production to allow us to study the multivariate quality control charts.

In this application, we shall introduce the most common using techniques of multivariate quality control chart; MEWMA.

A multivariate EWMA chart consists of:

- Plotted points, each of which represents the multivariate statistic for each observation.
- Upper control limits (red), which provide a visual means for assessing whether the process is in-control.

  MINITAB marks points outside of the control limits with a red symbol.

  We select combination of r and ARL for plotting several MEWMA charts for each stage of the production study.
  ARL and r are the parameters of the MEWMA chart, the minimum allowable ARL is 1 and the value of r must be a number between zero and one.
  Note that the default value of r = 0.1 and the default value of ARL= 200.
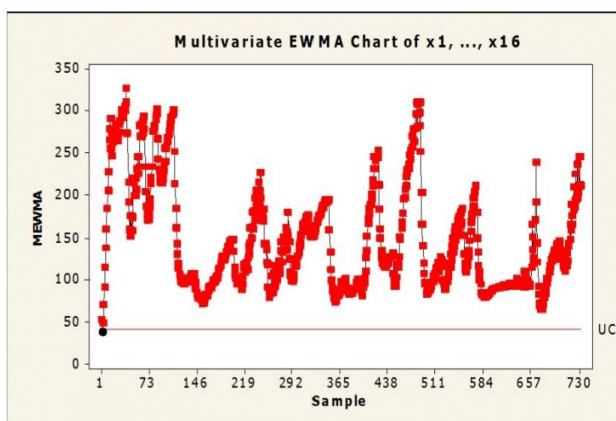
Fig. (1) MEWMA chart of x_1, ..., x_{16} with ARL = 200, r = 0.1

We can summarize the MEWMA chart of x1,…,x16 and t1.1,…,t2.5 (where ARL = 200 and r = 0.1) as follows:

- The upper control limit is 40.0. Therefore, we expect the MEWMA statistics to fall below 40.0.
- Test results indicate that all points beyond the control limits except point 2 and 3 fall below the upper control limit.
- Test results indicate that the process is in- control for two points and out –of control for 730 point. Then the out-of-control rate 99.73% and the in-control rate 0.27%.
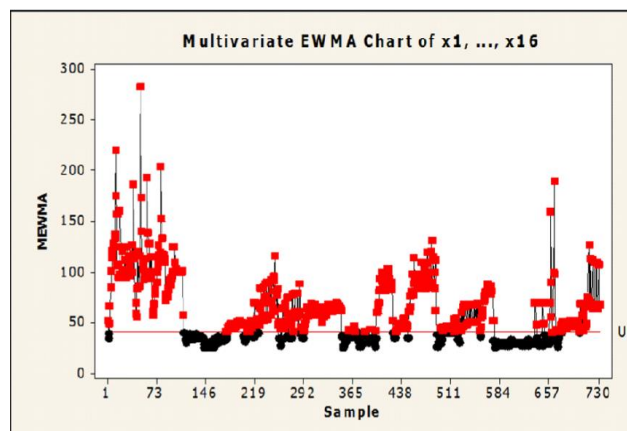
Fig. (2) MEWMA chart of x_1, ..., x_{16} with ARL = 200, r = 0.3

We can summarize the MEWMA chart of x1,…,x16 and t1.1,…,t2.5 (where ARL = 200 and r = 0.3) as follows:

- The upper control limit is 40.5. Therefore, we expect the MEWMA statistics to fall below 40.5.
- Test results indicate that 491 points beyond the control limits.
- Test results indicate that the process is in-control for 241 points and out –of control for 491 point. Then the out-of-control rate 67.08% and the in-control rate 32.92%.
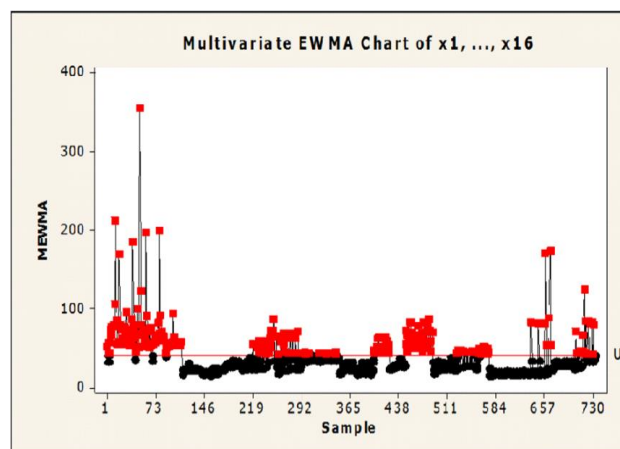

Fig. (3) MEWMA chart of x_1, ..., x_{16} with ARL = 200, r = 0.5

We can summarize the MEWMA chart of x1,…,x16 and t1.1,…,t2.5 (where ARE = 200 and r = 0.5) as follow:

- The upper control limit is 41.0. Therefore, we expect the MEWMA statistics to fall below 41.0.
- Test results indicate that 274 points beyond the control limits.
- Test results indicate that the process is in- control for 458 points and out –of

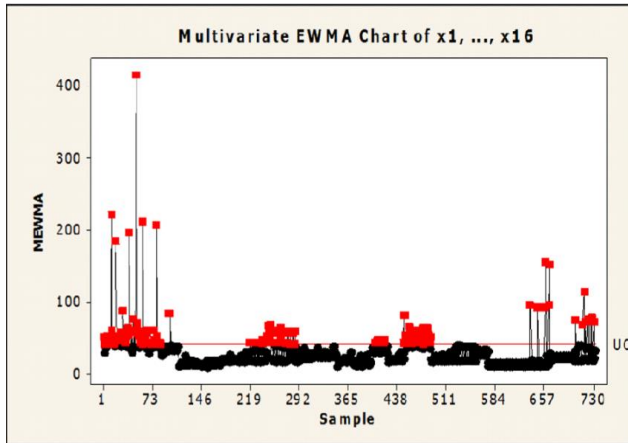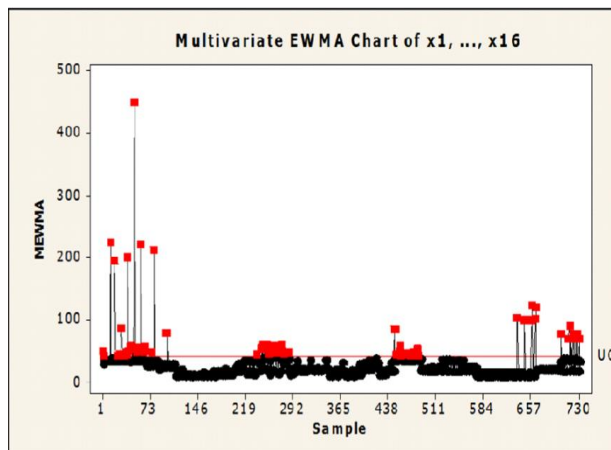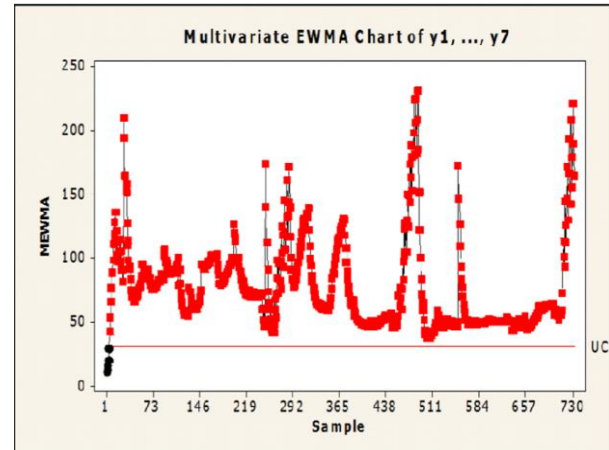control for 274 point. Then the out-of-control rate 37.43% and the in-control rate 62.57%.

Fig. (4) MEWMA chart of x_1, ..., x_{16} with ARL = 200, r = 0.7

We can summarize the MEWMA chart of x1,…,x16 t1.1,…,t2.5 (where ARL = 200 and r = 0.7) as follows:

- The upper control limit is 41.5. Therefore, we expect the MEWMA statistics to fall below 41.5.
- Test results indicate that 140 points beyond the control limits.
- Test results indicate that the process is in- control for 592 points and out –of control for 140 point. Then the out-of-control rate 19.13% and the in-control rate 80.87%.

Fig. (5) MEWMA chart of x_1, ..., x_{16} with ARL = 200, r = 0.9

We can summarize the MEWMA chart of x1,…,x16 and t1.1,…,t2.5 (where ARL = 200 and r = 0.9) as follows:

- The upper control limit is 42.0. Therefore, we expect the MEWMA

statistics to fall below 42.0.
- Test results indicate that 83 points beyond the control limits.
- Test results indicate that the process is in- control for 649 points and out –of control for 83 point. Then the out-of-control rate 11.34% and the in-control rate 88.66%.

Fig. (6) MEWMA chart of y_1, ..., y_7 with ARL = 200, r = 0.1

We can summarize the MEWMA chart of y1,…,y7 and t3.1,…,t4.5 (where ARL = 200 and r = 0.1) as follows:

- The upper control limit is 30.4. Therefore, we expect the MEWMA statistics to fall below 30.4.
- Test results indicate that all points beyond the control limits except point 1,2,3,4 and 5 fall below the upper control limit.
- Test results indicate that the process is in- control for five points and out –of control for 727 point. Then the out-of-control rate 99.32% and the in-control rate 0.68%.
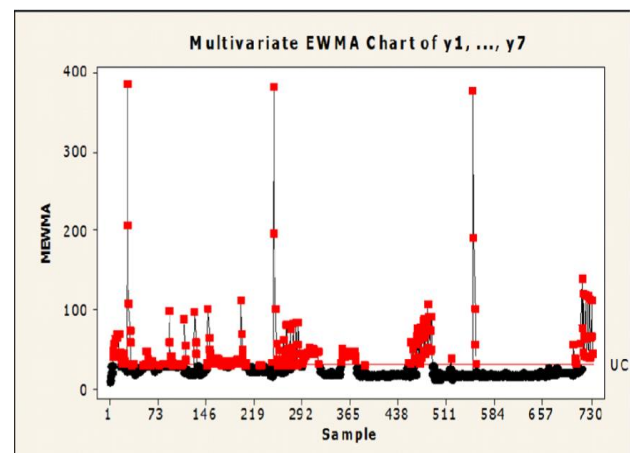

Fig. (7) MEWMA chart of y_1, ..., y_7 with ARL = 200, r = 0.3

We can summarize the MEWMA chart of y1,…,y7and t3.1,…,t4.5 (where ARL = 200 and r = 0.3) as follows:

- The upper control limit is 30.9. therefore ,we expect the MEWMA statistics to fall below 30.9.
- Test results indicate that 256 points beyond the control limits .
- Test results indicate that the process is in-control for 476 point and out-of control for256 point . then the out-of-control rat 34.97% and the in-control rat 65.03%.
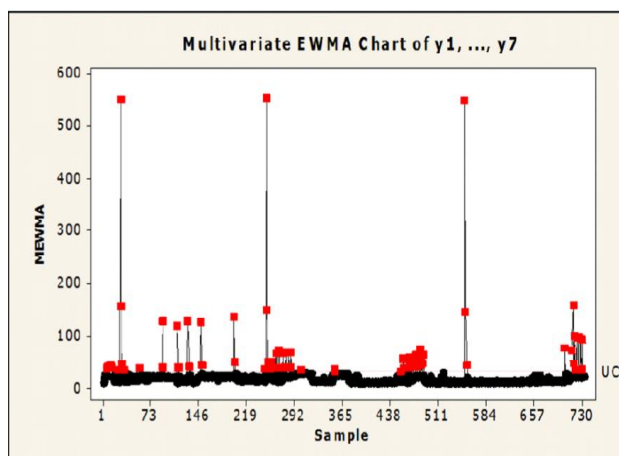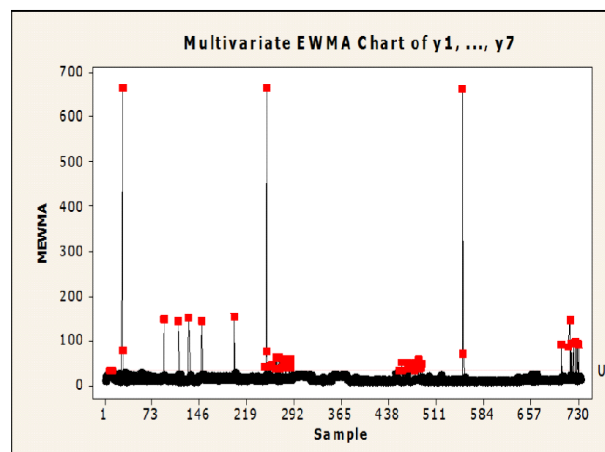

Fig. (8) MEWMA chart of y_1, ..., y_7 with ARL = 200, r = 0.5

We can summarize the MEWMA chart of y1,…,y7and t3.1,…,t4.5(where ARL = 200 and r = 0.5) as follows:

- The upper control limit is 31.4. therefore ,we expect the MEWMA statistics to fall below 31.4.
- Test results indicate that 83 points beyond the control limits .
- Test results indicate that the process is in-control for 649 point and out-of control for83 point . then the out-of-control rat 11.34% and the in-control rat 88.66 %.

Fig. (9) MEWMA chart of y_1, ..., y_7 with ARL = 200, r = 0.7

We can summarize the MEWMA chart of y1,…,y7and t3.1,…,t4.5 (where ARL = 200 and r = 0.7) as follows:

- The upper control limit is 31.9. Therefore, we expect the MEWMA statistics to fall below 31.9.
- Test results indicate that 58 points beyond the control limits.
- Test results indicate that the process is in- control for 674 points and out –of control for 58 points. Then the out-of-control rate 7.92% and the in-control rate 92.08%.
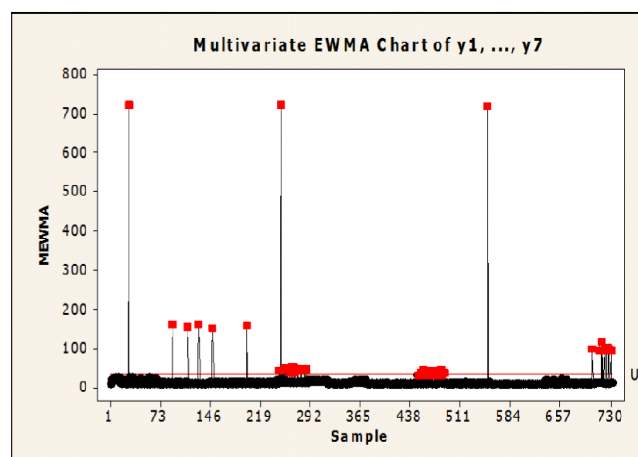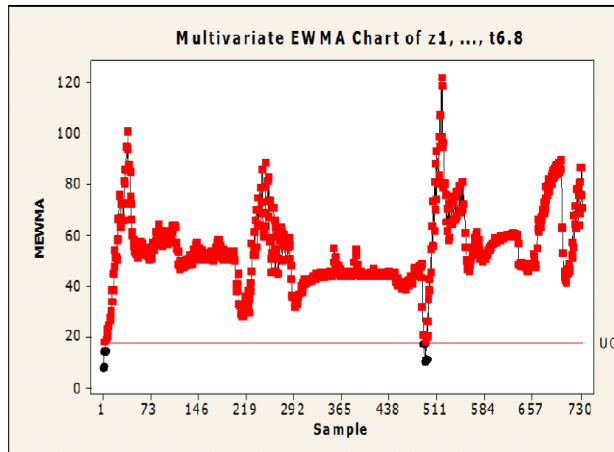

Fig. (10) MEWMA chart of y_1, ..., y_7 with ARL = 200, r = 0.9

We can summarize the MEWMA chart of y1,…y7 and t3.1,…,t4.5 (where ARL = 200 and r = 0.9) as follows:

- The upper control limit is 32.3. Therefore, we expect the MEWMA statistics to fall below 32.3.
- Test results indicate that 54 points

beyond the control limits.

- Test results indicate that the process is in- control for 678points and out –of control for 54 point. Then the out-of-control rate 7.38% and the in-control rate 92.62%.

Fig. (11) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 200, r = 0.1

We can summarize the MEWMA chart of z1,…,z4 and t6.1,…,t6..8 (where ARL = 200 and r = 0.1) as follows:

- The upper control limit is 17.6. Therefore, we expect the MEWMA statistics to fall below 17.6.
- Test results indicate that all points beyond the control limits except point 1,2,3 and 4 fall below the upper control limit.
- Test results indicate that the process is in- control for four points and out –of control for 728 point. Then the out-of-control rate 99.45% and the in-control rate 0.55%.
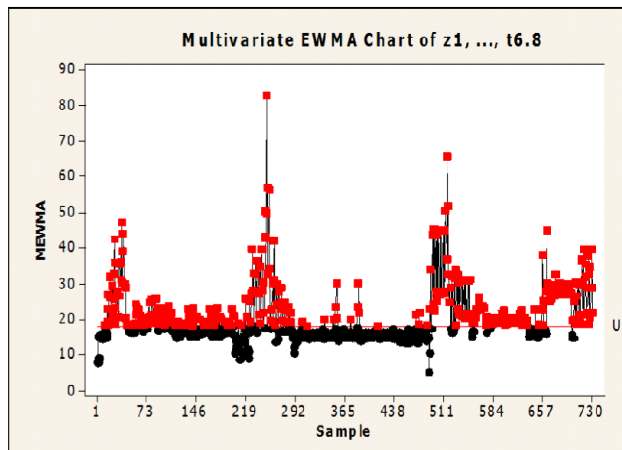

Fig. (12) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 200, r = 0.3

We can summarize the MEWMA chart of z1,…,z4 and t6.1,…,t6.8 (where ARL = 200 and r = 0.3) as follows:

- The upper control limit is 18.05. Therefore, we expect the MEWMA statistics to fall below 18.05.
- Test results indicate401 point through beyond the control limits.
- Test results indicate that the process is in- control for 331 point and out –of control for 401 point. Then the out-of-control rate 54.78% and the in-control rate 45.22%.
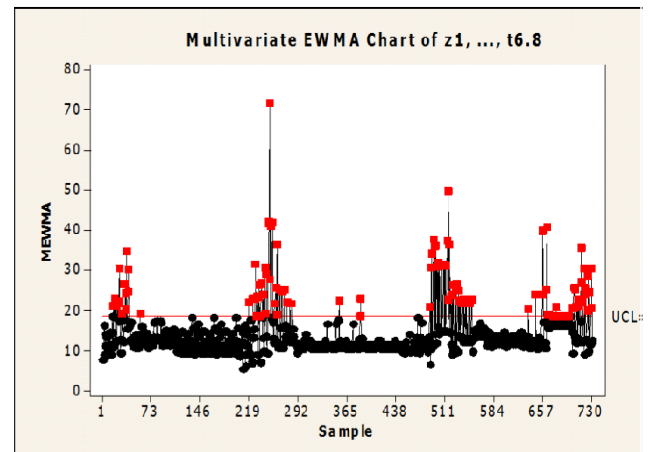

Fig. (13) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 200, r = 0.5

We can summarize the MEWMA chart of z1,…,z4 and t6.1,…,t6.8 (where ARL = 200 and r = 0.5) as follows:

- The upper control limit is 18.54. Therefore, we expect the MEWMA statistics to fall below 18.54.
- Test results indicate 115 point through beyond the control limits.
- Test results indicate that the process is in-control for 617 point and out –of control for 115 point. Then the out-of-control rate 15.7% and the in-control rate 84.3%.
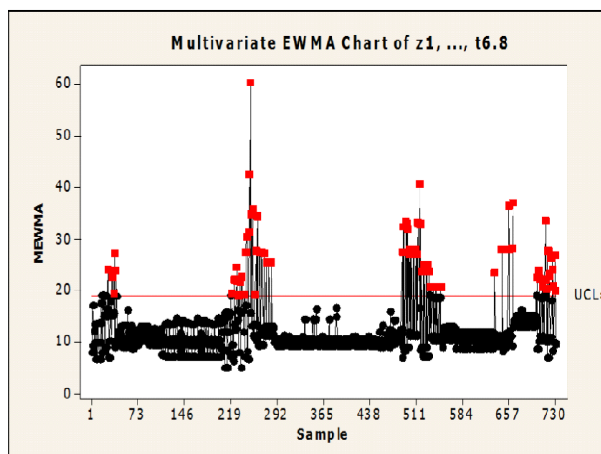
Fig. (14) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 200, r = 0.7

We can summarize the MEWMA chart of $z1,…,t6.8$ (where ARL = 200 and r = 0.7) as follows:

- The upper control limit is 19.03 Therefore, we expect the MEWMA statistics to fall below 19.03.
- Test results indicate 78 point through beyond the control limits
- Test results indicate that the process is in- control for 654 point and out –of control for 78 point. Then the out-of-control rate 10.66% and the in-control rate 89.34%.
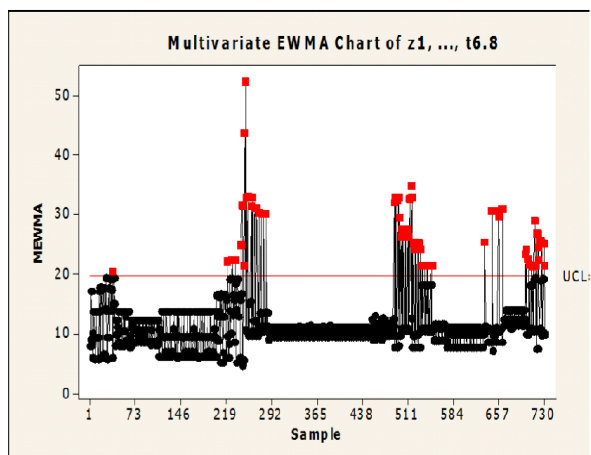

Fig. (15) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 200, r = 0.9

We can summarize the MEWMA chart of $z1,…,z4$ and $t6.1,…,t6.8$ (where ARL = 200 and r = 0.9) as follows:

- The upper control limit is 19.52. Therefore, we expect the MEWMA statistics to fall below 19.52.
- Test results indicate 67 point through beyond the control limits.
- Test result indicate that the process is

in- control for 665 point and out –of control for 67 point. Then the out-of-control rate 9.15% and the in-control rate 90.85%.
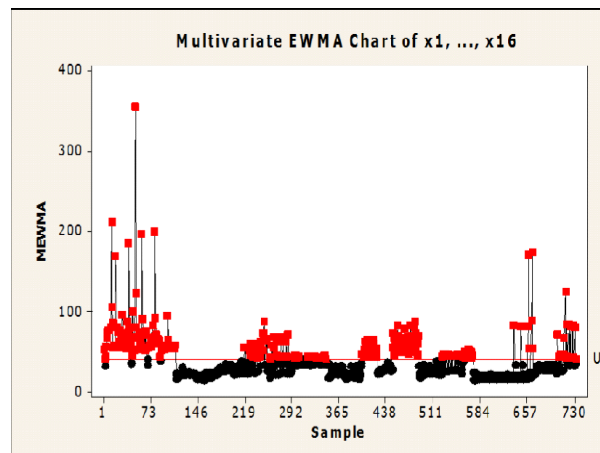

Fig. (16) MEWMA chart of x_1, ..., x_{16} with ARL = 100, r = 0.5

We can summarize the MEWMA chart of $x1,…,x16$ and $t1.1,...,t2.5$ (where ARL = 100 and r = 05) as follows:

- The upper control limit is 40.2. Therefore, we expect the MEWMA statistics to fall below 40.2.
- Test results indicate that 278 points beyond the control limits.
- Test results indicate that the process is in- control for 454 points and out –of control for 278 point. Then the out-of-control rate 37.98% and the in-control rate 62.02%.
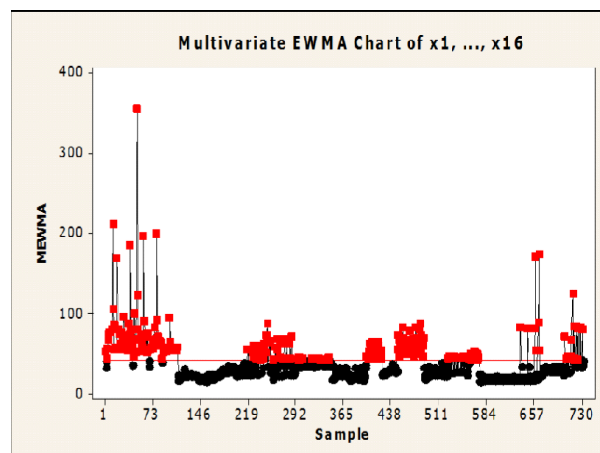

Fig. (17) MEWMA chart of x_1, ..., x_{16} with ARL = 300, r = 0.5

We can summarize the MEWMA chart of $x1,…,x16$ and $t1.1, ...,t2.5$ (where ARL = 300 and r = 05) as follows:

- The upper control limit is 41.4. Therefore, we expect the MEWMA statistics to fall below 41.4.
- Test results indicate that 271 points beyond the control limits.
- Test results indicate that the process is in- control for 461points and out –of control for 271 point. Then the out-of-control rate 38.02%   and the in-control rate 62.98%.
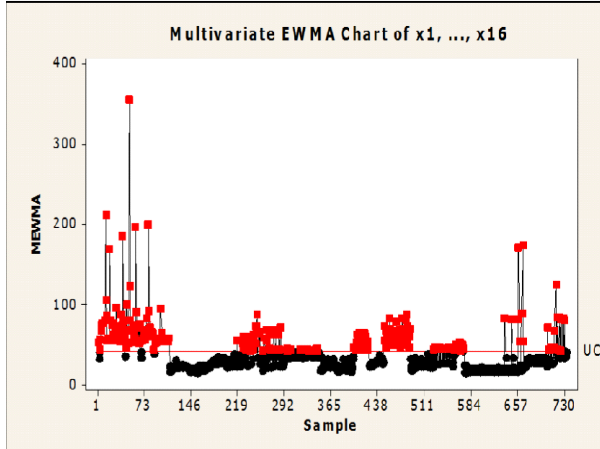

Fig. (18) MEWMA chart of x_1, ..., x_{16} with ARL = 400, r = 0.5

We can summarize the MEWMA chart of x1,…,x16 and t1.1,…,t2.5 (where ARL = 400 and r = 05) as follows:

- The upper control limit is 41.7. Therefore, we expect the MEWMA statistics to fall below 41.7.
- Test results indicate that 268 points beyond the control limits.
- Test results indicate that the process is in- control for 464points and out –of control for 268 point. Then the out-of-control rate 36.61%   and the in-control rate 63.39%.
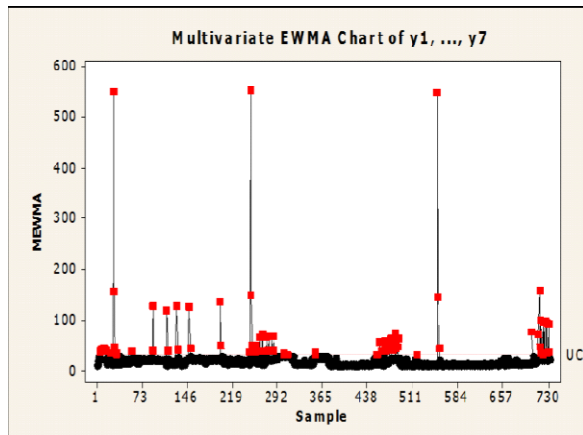

Fig. (19) MEWMA chart of y_1, ..., y_7 with ARL = 100, r = 0.5

We can summarize the MEWMA chart of y1,…,y7 and t3.1,…,t4.5 (where ARL = 100 and r = 0.5) as follows:

- The upper control limit is 30.6. Therefore, we expect the MEWMA statistics to fall below 30.6.
- Test results indicate that 89 points beyond the control limits.
- Test results indicate that the process is in- control for 634 points and out –of control for 89 point. Then the out-of-control rate 12.16% and the in-control rate 87.84%.
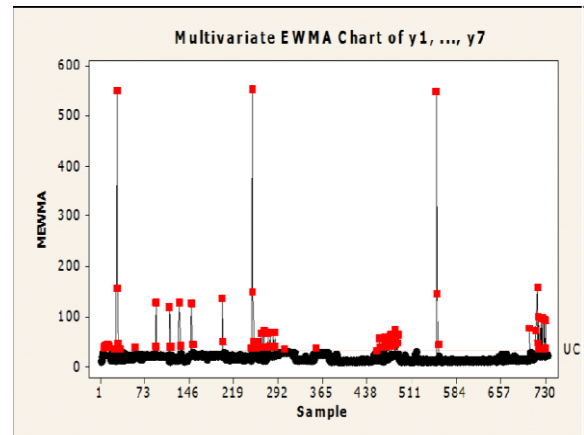

Fig. (20) MEWMA chart of y_1, ..., y_7 with ARL = 300, r = 0.5

We can summarize the MEWMA chart of y1,…,y7 and t3.1,…,t4.5 (where ARL = 300 and r = 0.5) as follows:

- The upper control limit is 31.8. Therefore, we expect the MEWMA statistics to fall below 31.8.
- Test results indicate that 82 points beyond the control limits.
- Test results indicate that the process is in- control for 650 points and out –of control for 82 point. Then the out-of-control rate 11.20% and the in-control rate 88.80%.

Fig. (21) MEWMA chart of y_1, ..., y_7 with ARL = 400, r = 0.5

We can summarize the MEWMA chart of y1,…,y7 and t3.1,…,t4.5 (where ARL = 400 and r = 0.5) as follows:

- The upper control limit is 32.1. Therefore, we expect the MEWMA statistics to fall below 32.1.
- Test results indicate that 81 points beyond the control limits.
- Test results indicate that the process is in- control for 651 points and out –of control for 81 point. Then the out-of-control rate 11.07% and the in-control rate 88.93%.
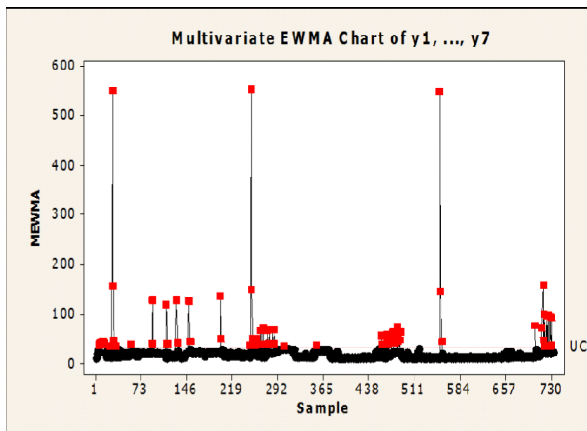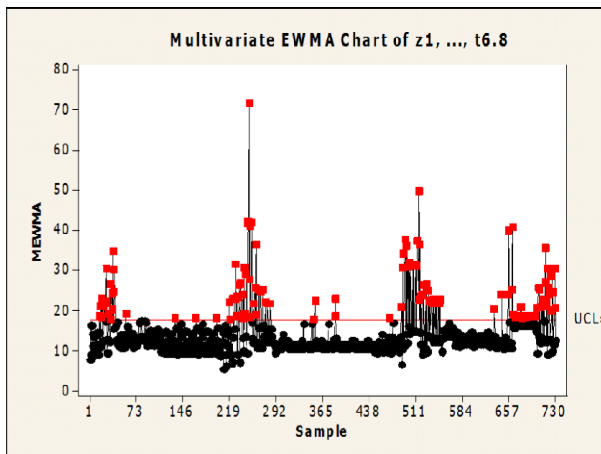

Fig. (22) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 100, r = 0.5

We can summarize the MEWMA chart of z1,…,z4 and t6.1,…,t6.8 (where ARL = 100 and r = 0.5) as follows:

- The upper control limit is 17.78 Therefore, we expect the MEWMA statistics to fall below 17.78.
- Test results indicate 135 point through beyond the control limits.
- Test results indicate that the process is

in- control for 597 point and out –of control for 135 point. Then the out-of-control rate 18.44% and the in-control rate 81.56%.
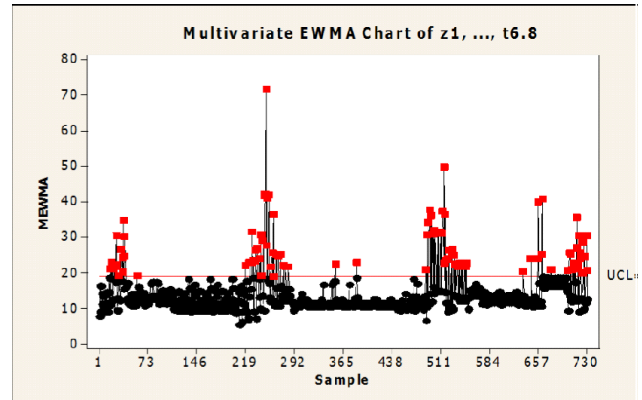
Fig. (23) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 300, r = 0.5

We can summarize the MEWMA chart of z1,…,z4 and t6.1,…,t6.8 (where ARL=300 and r = 0.5) as follows:

- The upper control limit is 18.99. Therefore, we expect the MEWMA statistics to fall below 18.99.
- Test results indicate 102 point through beyond the control limits.
- Test results indicate that the process is in- control for 630 point and out –of control for 102 point. Then the out-of-control rate 13.93% and the in-control rate 86.07%.

Fig. (24) MEWMA chart of z_1, ..., $t_{6.8}$ with ARL = 400, r = 0.5

We can summarize the MEWMA chart of z1,…,z4 and t6.1,…,t6.8 (where ARL = 400 and r = 0.5) as follows:

- The upper control limit is 19.31. Therefore, we expect the MEWMA

statistics to fall below 19.31.

- Test results indicate 98 point through beyond the control limits.
- Test results indicate that the process is in- control for 634 point and out –of control for 98 point. Then the out-of-control rate 13.39% and the in-control rate 86.61%.
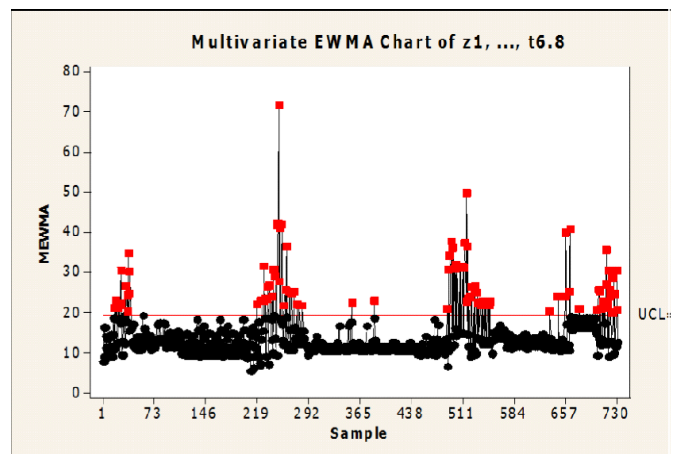- Test results indicate that for the same ARL=200 the out of control rate decreased by increasing the value of r
- Test results indicate that UCL increased by increasing the value of r.
- We note that for the same value of r the out-of control decreased by increasing the value of ARL but with small rate.
- Finally, we note that we use small value of r to determine small shift in the process mean and use large value of r to determine large shift in the process mean.

5.2 TEST RESULTS OF APPLICATION STUDY:

It is shown that in the High pressure stage, test result of MEWMA (when r = 0.1 and ARL=200) the out-of-control percentage 99.73% and the in-control percentage 0.27%. The difference occurred in the percentage is due to the sensitivity of MEWMA of the small shifts, and the out-of-control percentage (when r = 0.3, r = 0.5, r = 0.7, r = 0.9 and ARL = 200) are respectively (67.080%, 37.43%, 19.13%, 11.34%) and the in-control percentage (32.92%, 62.57%, 80.87%%, 88.66%).

It is shown that in the Low-pressure stage, test results of MEWMA (when r = 0.1 and ARL = 200) indicate the out-of-control percentage 99.32% and the in-control 0.68%, while, in the MEWMA the out-of-control percentage(when r = 0.3, r = 0.5, r = 0.7, r = 0.9, and ARL = 200) respectively (34.97%, 11.34%, 7.92%, 7.38%) and the in-control percentage (65.03%, 88.66%, 92.08%, 92.62%,).

It is shown that in the Evaporation and Prilling stage, test result in MEWMA indicates (when r = 0.1, r = 0.3, r = 0.5, r = 0.7, r = 0.9 and ARL = 200) the out-of-control percentage respectively (99.45%, 54.78, 15.70%, 10.66%, 9.15%) and the in-control percentage (0.55%, 45.22%, 84.3%, 89.34% 90.85%).

These results allow us to determine whether the joint process variability is in-control or out-of-control.

It is shown that the out-of-control and the in-control percentage changes by using difference values of and ARL.

It was shown that there is a relationship between the value of r and the out-of-control percentage, the out-of-control percentage increased by increasing the value of r.

Test results indicate that in the High pressure stage, for example the points (114, 118, 123, 128, 133, 134, 138, 140, 143) out-of-control and the variables which causes the out-of-control signals are (x6, t2.4, x16) . Also points (151, 156, 161, 166, 167, 171, 173) out-of-control and the variables which causes the out-of-control signal are (x2, x3, t2.1, t2.4) and so on.

It is shown that in the design of MEWMA control charts, small values of r are more efficient in detecting small process mean shifts and large values of r are more efficient in detecting large process mean shifts.

6. Finally:

The company should use multivariate quality control chart; MEWMA, to monitor the quality of the urea production.

On using the MEWMA chart to determine whether or not the process in control, the company should choose small values of r to detect small process mean shifts and choose large values of r to detect large process mean shifts.

Table (1): Summarize MEWMA charts

	ARL = 200, r = 0.1			ARL = 200, r = 0.3			ARL = 200, r = 0.5			ARL = 200, r = 0.7			ARL = 200, r = 0.9		
	UCL	Out-of-control Rate	In-control rate	UCL	Out-of-control rate	In-control rate	UCL	Out-of-control rate	In-control rate	UCL	Out-of-control rate	In-control rate	UCL	Out-of-control rate	In-control rate
MEWMA for x_1, ..., x_{16}	40.00	99.73 %	0.27 %	40.50	67.08%	32.92%	41.00	37.43%	62.57%	41.50	19.13 %	80.87 %	42.00	11.34%	88.66%
MEWMA for y_1, ..., y_6	30.40	99.32%	0.68%	30.9	34.97%	65.03%	31.4	11.34%	88.66%	31.90	7.92%	92.08%	32.30	7.38%	92.62%
MEWMA for z_1, ..., $t_{6.8}$	17.60	99.45 %	0.55 %	18.05	54.78%	45.22%	18.54	15.70	84.3%	19.03	10.66 %	89.34 %	19.52	9.15%	90.85%

Table (2): Summarize MEWMA charts

	ARL = 100, r = 0.5			ARL = 200, r = 0.5			ARL = 300, r = 0.5			ARL =400, r = 0.5		
	UCL	Out-of-control rate	In-control rate	UCL	Out-of-control rate	In-control rate	UCL	Out-of-control rate	In-control rate	UCL	Out-of-control rate	In-control rate
MEWMA for x_1, ..., x_{16}	40.20	37.98 %	62.02 %	41.00	37.43%	62.57%	41.40	38.02%	62.98%	41.70	36.61 %	63.39 %
MEWMA for y_1, ..., y_6	30.60	12.16 %	87.84%	31.4	11.34%	88.66%	31.8	11.20%	88.80%	32.10	11.07%	88.93%
MEWMA for z_1, ..., $t_{6.8}$	17.78	18.44 %	81.56 %	18.54	15.70%	84.3%	18.99	13 .93%	86.07%	19.31	13.39 %	86.61 %

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