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Studying Different Pricing Schemes Using Different Game Models in CRN: Market-Equilibrium and Non-Cooperative Model

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Abstract: Cognitive Radio Network (CRN) is an intelligent wireless communication system that aims to achieve efficient spectrum utilization through monitoring spectrum status, continuously. Spectrum utilization can be accomplished by sharing of spectrum holes from primary users (PUs) or primary service providers (PSPs) to secondary users (SUs).

In spectrum sharing process, (PSPs) or spectrum-sellers sell unused spectrum to secondary service providers (SSPs) or spectrum buyers according to specified price. In this paper, pricing issue will be investigated with two different market models, namely, market-equilibrium and Bertrand competitive market. In market-equilibrium, pricing depends on the balance between spectrum demand and spectrum supply. It is solved in a simplified manner that reduces computational process. In Bertrand competitive market, PSPs try to accomplish the highest payoff regarding others by manipulating in price. Genetic algorithm (GA) will be proposed with Bertrand competitive to reduce and optimize average pricing charged to SSP for more spectrum selling comparing with round-robin algorithm.

Keywords: Cognitive radio, Supermodular game theory, Bertrand competition, Genetic algorithm, Nash equilibrium.

1. Introduction

In Cognitive Radio Network (CRN), spectrum shared between licensed and unlicensed users to improve spectrum utilization [1]. Sharing process gives a chance to PSPs to gain some revenue by selling unused spectrum holes to SUs [2-7]. Selling and buying spectrum creates a spectrum-trading market. In any type of spectrum trading market, pricing issue based on the incentive of PUs to gain profit and requisitioning of SUs to use shared spectrum.

Actually, the spectrum-trading market has three different types, namely, market equilibrium, cooperative and competitive market. Market equilibrium, pricing depends on required spectrum by SSPs, which is satisfied by spectrum supply from PSPs. In cooperative pricing scheme, PSPs cooperate for maximum total profit. In competitive pricing scheme, each PSPs plays strategy which maximizes its individual profit [5].

In fact, shared spectrum is modeled using game theory [8, 9] or by other approaches like price theory or joint strategy (price theory and game theory).

Game theory was described with three main elements, namely, players or agents, strategy played and the payoff or utility gained from playing certain strategy. The player may play more than one strategy according to which strategy can achieve better payoff. The fundamentals of game theory were described in details in [10, 11]

Actually, game theory has four categories, namely, non-cooperative or competitive games, cooperative games, auction games and stochastic games [12]. The non-cooperative game, players make decisions independently. It has many types of Bertrand games [13] in which PSPs decide prices simultaneously. Any

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SU will buy spectrum from PSP who sells spectrum with the lowest price, then highest profit will be gained. Cooperative games [14], coalitions of players have joint strategies to gain mutual benefits likes Bargaining game [15] in which cooperation is enforceable by an outside party (e.g. a judge and police). In auction game, likes Stackelberg game, the strategy chosen by the leader can be observed by followers and then they adapt their decisions accordingly. Stochastic games are dynamic, competitive game with probabilistic strategy played by one or more players. The game is played in a sequence of stages.

A special type of games will be introduced in this paper called Supermodular games [15] which have the property of convergence to Nash equilibrium. The basic idea of this game depends on the selection of one player that takes an action with higher value; the others want to do the same. Consequently, with multiple PSPs, the Supermodular game will be competitive. In literature [16], authors demonstrate solving competitive market under penalty constraints of payoff function, and demand from SSPs. In this paper, market-equilibrium pricing scheme will be considered as an extension of our work in the literature [16].

In detail, the main contributions of this paper will be described as follows:

- An analytic solution for Market-equilibrium pricing scheme, and then evaluating Nash point through numerical results.
- Solving Competitive pricing model considered for spectrum trading and substitutability using GA to optimize pricing strategy selection with respect to all PSPs in the network.
- Extensive analysis of system results for Bertrand competitive and equilibrium market.

The rest of paper is organized as follows: Sect. 2 discusses market-equilibrium scheme and understanding Nash equilibrium. Sect. 3 investigates the basics and the properties of Supermodular game and Bertrand competition theory. System model will be presented in Sect. 4 The solution of spectrum price problem will be investigated in Sect. 5. Simulation results will be introduced in Sect. 6. Finally, conclusions will be made in Sect. 7.

2. Analytic Solution For Market-Equilibrium Pricing Scheme

Actually, in spectrum trading market, Each PSP is not aware of the situation of others in the network. Pricing process depends on providing the supply to market according to the demand from SUs. In this market, spectral efficiency (bits/sec/Hz) will be considered due to its impact. The spectrum demand of SUs is effected by modulation type, signal to Noise ratio (SNR) in allocated spectrum and price that is offered by PSPs. Spectral efficiency depends on SNR or channel quality. Spectral efficiency is evaluated by [17]:

$$SE = log_2(1 + k_c\gamma) \tag{1}$$

Where,

$k_c = \frac{1.5}{ln(0.2/BER^{tar})}$, and γ is SNR and BER^{tar} is target bit error rate (BER).

Considering utility gained by SUs from spectrum sharing process, which is given by [18]:

$$u(\boldsymbol{sB}) = \sum_{i=1}^{N} sB_i\, SE_i^{(s)} - \frac{1}{2}\left(\sum_{i=1}^{N} sB_i^2 + 2e\sum_{i\neq j} sB_i\, sB_j\right) - \sum_{i=1}^{N} pr_i\, sB_i \tag{2}$$

Where $\boldsymbol{sB} = [sB_1, sB_2, \dots \dots . sB_N]$ is a set of shared spectrum from PSPs, $SE_i^{(s)}$ is spectral efficiency of wireless transmission by SUs using frequency f_i , pr_i is price offered by PSPs for spectrum frequency f_i and e is spectrum substitutability ($0 \leq e \leq 1$) which is also used with Supermodular game model.

The profit that is gained by PSPs due to sharing process consists of revenue and cost where (Profit = revenue - cost). Revenue that is gained from PUs connected to PSP i and revenue from sharing process with SUs that equals to $(pr_i\, sB_i + n_1K_i)$. Cost due to QoS degradation of PUs is $(n_2K_i(B_i^{req} - SE_i^{(p)}\frac{B_i - sB_i}{K_i})^2$ therefore, the total profit is donated by [18]:

$$\rho(\boldsymbol{Pr}) = pr_i sB_i + n_1K_i - n_2K_i(B_i^{req} - SE_i^{(p)}\frac{B_i - sB_i}{K_i})^2 \tag{3}$$

Where n_1, n_2 are constant parameters, B_i^{req} is required bandwidth for each PU, B_i is available spectrum for each PSP i, K_i is the number of PUs which are attached to PSP i and $SE_i^{(p)}$ is spectral efficiency for PSP i.

To solve this market and evaluate equilibrium status, the demand, and the supply functions are evaluated by differentiating the utility and the profit functions, respectively with respect to b_i as follows:

$$\frac{\partial u(sB)}{\partial sB_i} = \frac{\partial \rho(Pr)}{\partial sB_i} = 0 \tag{4}$$

This leads to:

$$D(\boldsymbol{Pr}) = \frac{\left(SE_i^{(s)} - pr_i\right)\left(e(N-2)+1\right) - e\sum_{i\neq j}\left(SE_i^{(s)} - pr_j\right)}{(1-e)\left(e(N-1)+1\right)} \tag{5}$$

$$S(pr_i) = B_i - \frac{K_i}{SE_i^{(p)}}\left(B_i^{req} - \frac{pr_i}{2n_2\, SE_i^{(p)}}\right) \tag{6}$$

Where $D(\boldsymbol{Pr})$ and $S(p_i)$ are the demand and the supply functions, respectively. $\boldsymbol{Pr} = [pr_1.pr_2.\ldots\ldots pr_N]^{\mathrm{T}}$ is price vector offered by PSP in spectrum trading process.

Then, from Eq.5 with Eq.6 and solving them simply for pr_i then,

$$pr_i = \frac{\frac{K_i B_i^{req}}{SE_i^{(p)}} - \frac{e\sum_{i\neq j}\llbracket(SE\rrbracket_i^{(s)} - pr_j^t)}{(1-e)(e(N-1)+1)} + \frac{SE_i^{(s)}(e\,(N-2)+1)}{(1-e)(e(N-1)+1)} - B_i}{\frac{e(N-2)+1}{(1-e)(e(N-1)+1)} + \frac{K_i}{2\left(SE_i^{(p)}\right)^2 n_2}} \tag{7}$$

3. Supermodular Game Basics and Bertrand Competition

Supermodular games are an interesting type of games that are exhibited as strategic complementarity; this means that when one player takes higher action, the others wants to do the same. It is based on a rich mathematical foundation of lattice theory and comparative statics [19].

The strategy space of every player is partially ordered set and the utility of playing higher strategy increases when the opponents also play higher strategy. In the following, the fundamentals of Supermodular games will be introduced. Then Bertrand competition will be investigated.

A. Supermodular Game

Suppose $f(x)$ is real valued function in lattice X, and X' is subset of X. If $x \prec x'$ then x' is upper the bound of X'. If $x \succ x'$ then x' is lower bound of X'. Then, x' is greatest or (least) element in X', respectively. If set of upper or (lower) bound of X' has least or (greatest) element, then we have least upper or (greatest lower) bound of X', respectively. Noting that, least upper called supremum or joint $(x' \vee x)$:=max (x', x), and greatest lower called infimum or meet $(x' \wedge x)$:=min (x', x). If two elements x and x' of partially ordered set X have joint and meet, then X is lattice and X' is sublattice of X.

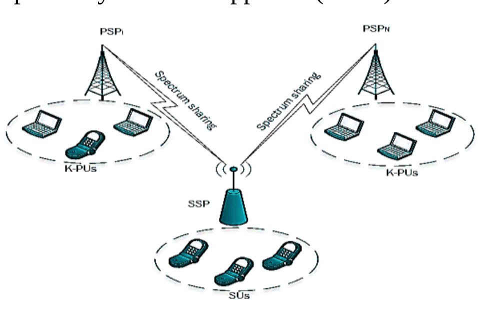

Figure 1. Spectrum sharing model.

If $f(x) + f(x') \leq f(x \vee x') + f(x \wedge x')\ \forall x, x' \in X$, Therefore, $f(x)$ is supermodular function, hence if we have game (N, S, f_i : i ∈{1,…,N}), it will have the following properties [11]:

- S_i is compact subset of $\mathbb{R}$;
- f_i is upper semi continuous in (S_i, S_{-i}) ;and
- f_i has increasing difference in (S_i, S_{-i}) .

Suppose that X and T are partially ordered sets and $f(x.t)$ is real valued function on subset S (strategy set) of X × T, if $f(x',t') - f(x,t') \geq f(x',t) - f(x,t)$ means that the gain to choose higher action a x' is increasing when t is higher $(i.e.\ t')$ that is called increasing difference property or supermodularity.

The player tries to take higher action. The others try to do the same. By other words, increasing one component of player n's strategy set does not decrease the marginal profit of the other component in the same strategy.

If the function $f: R^n \rightarrow R$ is twice continuously differentiable in open interval $[\bar{X}, X]$ in R^n lattice, then necessary and sufficient condition of function f having supermodularity is $\frac{\partial^2 f(x,y)}{\partial x_i \partial y_j} \geq 0\ \forall i \neq j$ where $\{1.\ldots i.\ldots.j\ldots.N\}$ are players of the game.

By applying Topkis' theorem [17], it shows that each player's best response function is increasing with the actions of other players.

Suppose (S, u) is Supermodular game, and let $BR_i(s_{-i}) = \arg\max_{s_i \in S_i} U_i(s_i.s_{-i})$ is the best response of player i given other player's strategies (s_{-i}) and $U_i(s_i.s_{-i})$ is utility of player. Then the the the $BR_i(s_{-i})$ has greatest and least element $\overline{BR_i}\ (s_{-i})$, $\underline{BR_i}\ (s_{-i})$ which represent largest and smallest Nash in pure strategies, respectively. If $s'_{-i} \geq s_{-i}$, then $\overline{BR_i}\ (s'_{-i}) \geq \overline{BR_i}\ (s_{-i})$ and $\underline{BR_i}\ (s'_{-i}) \geq \underline{BR_i}\ (s_{-i})$.

B. Bertrand Competition

Bertrand game is a competitive game, which describes interaction among spectrum sellers that set price and spectrum buyers. The game depends on substitutability among spectrum available from PSPs and freedom degree of SSP to freely switch through this spectrum [20]. Substitutability depends on the homogeneity of shared spectrum from different PSPs. SSPs buy available spectrum that was provided with a lower price.

4. SYSTEM MODEL

Let us assume that CR system have N of PSPs such that each PSPi ($i \in \{1. \dots . N\}$) has spectrum size B_i and service K_i of PUs. Each PSP satisfies its own PUs spectrum's requirements and sells unused chunks of spectrum to SSPs or SUs (spectrum buyer) as shown in Fig.1. Selling and buying process creates spectrum-trading market. In competition market, PSPs do not cooperate and each is aware about strategies of the others. In Bertrand model, best response of PSP i can be computed in terms of the price. Bertrand game model is applied on price competition among PSPs to obtain the Nash equilibrium pricing.

Distributed [5, 21] algorithms and round-robin optimization [22] are used before to optimize different problems. In Distributed pricing algorithm, PSP uses its own information (the demand from SSP and price of the previous iteration) to adjust the price for next iteration. Round robin depends on the least upper elements in strategy which represent optimizing limits then, search for the optimal value of price that gives the best response to the utility function. A genetic algorithm which is described in [23, 24] will be presented to obtain optimum strategy selection, in which PSP wants to maximize its individual payoff depending on the spectrum demand of SSP.

5. SOLUTIONS OF SPECTRUM PRICE PROBLEM

A. Spectrum demand

Assuming single SSP has spectrum demand, which is denoted by [18]:

$$D_i(Pr.e) = a_i - b_i pr_i + e\sum_{i \neq j} pr_j \tag{9}$$

Where a_i is market capacity of spectrum size B_i, P is price vector offered by PSP, $\boldsymbol{Pr} = [pr_1. \dots . p_i r. \dots . pr_N]^T$ is equivalent to PSP strategy, p_i is price per spectrum unit B_i or strategy and e is substitutability coefficient that shows how SSP can freely switch among available spectrum ($0 \leq e \leq b_i$). Therefore, if $d = 0$, SSP can't switch through free spectrums otherwise spectrum from more than one PSP can be completely substituted.

B. Payoff function or utility function

Payoff function describes gain from chosen certain strategy. In competitive Bertrand model, profit depends on single decision variable (price). The demand function is given as follows [18]:

$$u_i(\boldsymbol{Pr}.e) = (pr_i - c_i)D_i(Pr,e) - \nu_i\left(B_i^{req} - \frac{B_i - D_i(Pr.e)}{K_i}\right)^2 \tag{10}$$

Where ν_i is penalty coefficient, c_i is cost per spectrum unit and B_i^{req} is required bandwidth for each PU. Therefore, second term in Eq.10 represents required bandwidth minus allocated spectrum for each PU attached to PSP that is yield to spare spectrum for sharing.

Proposition: The game with utility function in Eq.10 is Supermodular game.

Proof: assume the price is restricted for each PSP, the $u_i(\boldsymbol{Pr}.e)$ is twice differentiable and the strategy (price) is single decision variable in Bertrand game and has supermodularity property.

The first-order derivative of Eq.10 is denoted by,

$$b_i pr_i^2 - \left(a_i + e\sum_{i \neq j} pr_j\right) pr_i + \frac{2\nu_i\left(B_i^{req} - \frac{B_i}{K_i}\right) + c_i}{\left(K_i + 2\frac{\nu_i}{K_i}\right)} = 0 \tag{11}$$

Moreover, the second derivative is described by,

$$\frac{\partial^2 u_i(\boldsymbol{Pr}.e)}{\partial pr_i \partial pr_j} = e + \frac{2b_i\nu_i e}{K_i^2} \geq 0 \tag{12}$$

This means that increasing on a component of strategy does not decrease profit of the other components. Applying Supermodular game on Bertrand competition leads to largest and smallest Nash equilibrium for iterated strategies, which are non-decreasing functions.

Mathematically, Nash equilibrium is obtained by equalizing first derivative with zero (i.e. $\frac{\partial u_i(\boldsymbol{Pr}.d)}{\partial p_i} = 0$) and this leads to Eq. 11.

If Eq.11 solved for pr_i, therefore, the best response that satisfies PSP profit is obtained using,

$$pr_i^* = max_{i \in N}(pr_i, pr_{-i}) \tag{13}$$

By substituting pr_i^* into Eq.10 to give maximum profit. The best response is evaluated by,

$$U_i(pr_{-i}) = \max_{pr_i \in P_i(pr_{-i})} u_i(pr_i . pr_{-i}; e) \tag{14}$$

For PSP_i, it has strategy set P_i and p_{-i} is strategy of player i's opponents, (pr_i, pr_{-i}) is joint strategy vector that refers to feasible strategy set of player i given other strategies of the opponents is expressed as $p_i: (pr_i . pr_{-i}) = Pr_i | pr_{-i}$.

C. Genetic algorithm

Genetic algorithm [23, 24] is used to optimize profit function as fitness function according to the upper limit of the price that is obtained from demand function. The algorithm produces multiple generations of price vectors for each PSP, and it is terminated when the price is less than price limit that is evaluated. This creates some equilibrium in Bertrand market and prevents one of PSP to be dominant if its price is less than chosen limit of demand. Moreover, the algorithm reduces the abrupt change in price that is offered by PSPs. In the following, proposed GA will be introduced to show how does it work.

a. *Initializing objective function with random sets (chromosomes) of strategies (genes) such that overall sets represent the population.*

b. *Note: the population of strategy set is ranked by price bounds that previously defined the demand function.*

c. *Substitute with a value of price strategy (chromosome) in the profit function (objective function).*

d. *Use roulette wheel selection to choose the fittest strategy that gives a higher value for profit function.*

e. *Apply crossover process mate strategy set according to crossover probability that defines the location of crossover.*

f. *Random position of strategy set (gene) is randomly changed using the probability of mutation.*

g. *Crossover and mutation (reproduction) produce a new generation of price strategy (offspring).*

h. *New strategy sets are undergoing steps 3, 4, 5.*

i. *At each iteration t, produced strategies are compared with the previous in iteration $t-1$ and check if they are in predefined range of price bounds. If there is no more change in produced price strategies, algorithm is terminated. Otherwise, algorithm is continued.*

Note: a solution that is obtained is considered best strategy profile according to the role of "survival for strongest". Moreover, it is constrained with price bound (upper bound) that is obtained by solving demand equation that defines marginal profit of PSP.

6. SIMULATION RESULTS

Considering equilibrium-market system with two PSPs, each has 16 PUs (i.e. K=$K_1 = K_2 = 16$), total frequency spectrum W= [20, 20] MHZ, $BER^{tar} = 10^{-4}$, $n_1 = n_2 = 5$,channel quality for primary user is 10 dB, channel quality of SUs varies between 9 to 22 dB and substitutability $e = 0.7$.

Note that parameters will be changed for next section of Bertrand competitive scenario.

Fig.2 shows the relation between two prices, where P1 is price of PSP_1 and P2 is price of PSP_2. When P1 increases P2 also increases. The same thing occurs for different values of d as shown in Fig.3.

In Fig.4, the evaluation of Nash value points to the demand of spectrum from SSP equals to the spectrum supply from PSP. The intersection between demand function and supply represents market equilibrium point (Nash point). It may be no market equilibrium if there is no intersection between the demand and the supply functions for another values of B_i^{req}.Note that supply is increasing function in price while demand is decreasing function in pricing as shown in Fig.5.

Considering competitive market system with two PSPs, each has 10 PUs (i.e. $K_i = 10$) and, total spectrum of each PSP is B= [200, 300] MHZ. Let spectrum requirements of PUs from their PSPs are

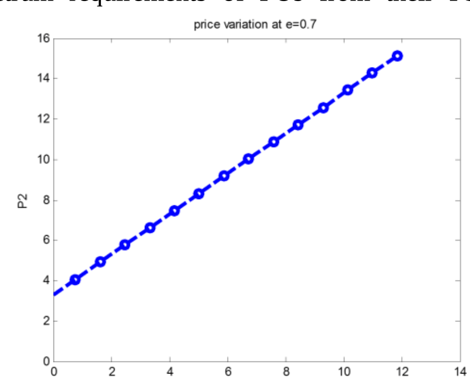

Figure 2. price variation.

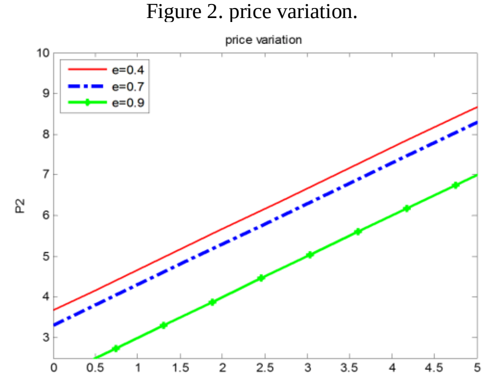

Figure 3. Price variation at different values of e.

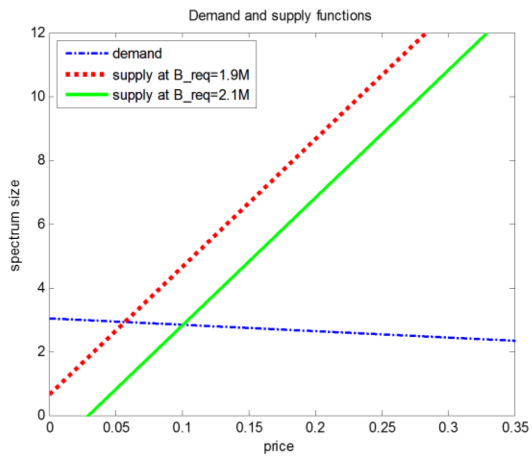

Figure 4. Demand and supply functions.

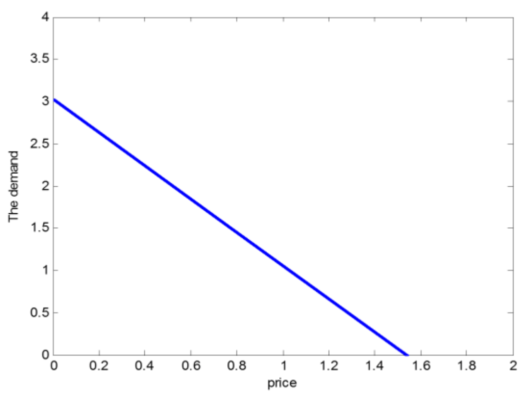

Figure 5. demand function.

B^{req}= [20, 30]MHZ, spectrum capacity market for each PSP is a=[200, 300]MHZ, the penalty term is v=[1, 3], d= 0.1, 1, and the cost per spectrum unit is c=[2, 4] .

Genetic algorithm optimizes profit function as a fitness function in price such that the average in fitness function (profit) is not less price limit to prevent more price reducing among PSPs. It reduces average pricing by 27% comparing with [13] as shown Fig.6. In addition, substitutability among spectrum, d from 0.3 to 0.8 for PSP1 and PSP2 has impact on price convergence such that average lower price for small value of e.

With increasing substitutability with convergent values for each PSP as shown in Fig.7, PSP1 and PSP2 produce more spectrum availability for SSP to switch among free spectrum. Prices converge better than introduced in Fig.6 for the same iterations. PSP has no chance to reduce its price to gain more profit. Price is not constant for all iterations but it changes with fixed value of interval for each iteration due to the demand of changing and profit competition. Then, from two

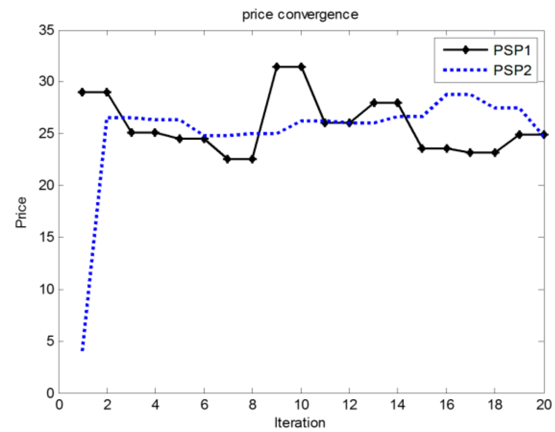

Figure 6. Price versus iteration for two PSP.

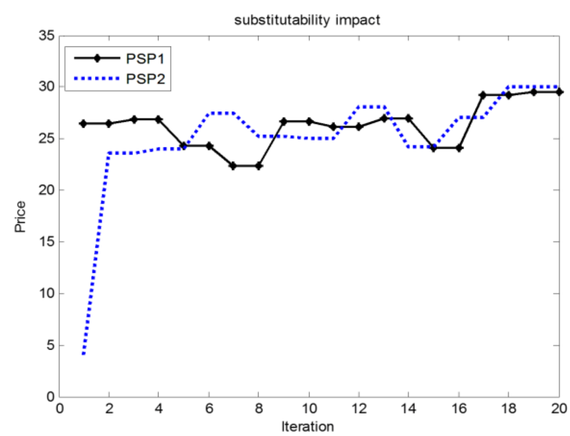

Figure 7. Substitutability effect.

pricing system, market-equilibrium reach stability faster than non-cooperative but the non-cooperative market is more practical than market-equilibrium.

7. Conclusions

In this paper, the cognitive radio of market-equilibrium analytically resolved for obtaining Nash equilibrium. Moreover, Bertrand competition spectrum sharing market has been considered in which multiple PSPs shares unused spectra for SSPs. The effects of spectrum demand of SSP and profit gained in competition market have been considered using a genetic algorithm to provide optimized profit gained by choosing best individuals for each generation of price evaluated then natural selection for the best individual is returned to profit function which gives optimum profit with respect to the market. Taking upper limit for pricing, spectrum substitutability influences which lead to price variations.

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