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Estimation of Finite Population Mean Under Measurement Error

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Abstract: In this paper, we have proposed two log- product -type estimators and a new estimator for estimation of finite population mean under measurement error by using auxiliary information. The expressions for Bias and mean squared error of proposed estimators are evaluated up to first order of approximation. Based on theoretical results obtained, a numerical study by generating Normal population using R programming language is also included to compare the efficiency of proposed estimators with other relevant estimators.

Keywords: Auxiliary variable, bias, Mean square error, Measurement error, Study variable.

1. INTRODUCTION

In sampling survey, characteristic of estimators (based on data) presume that observed values are indeed true values. Often, above condition is not met in practice accounting to errors in measurement. This measurement (or response) error during data collection stage is grossly contributed by respondent (or enumerator or both). These errors refer to the differences between individual's observed values and their corresponding true values. In a household survey either purposely or accidentally a respondent may report his/her income different (more or less) from actual income. In this case the difference between incomes reported by respondents and actual income constitutes measurement (or response) error. In field of sampling, significant attention has been devoted to the study and estimation of measurement errors by using auxiliary information in estimating finite population parameters. Many authors have made considerable contribution in estimating finite population mean in the presence of measurement error by incorporating auxiliary information. Shalabh [1] defined estimation in presence of measurement error using ratio method. Manisha and Singh [2],[3] suggested estimation of population mean and role of regression by incorporating measurement error. Singh and Karpe [4] proposed ratio-product estimator for finite population mean involving measurement errors. Recently, Kumar et al.[5] studied some ratio type estimators in the presence of measurement error by utilizing auxiliary information. Malik and Singh [6] defined a family of estimators for estimation of finite population mean using SRS scheme under measurement error. Singh et al. [7] proposed an estimator for estimation of finite population mean for difference estimators using measurement error. Several other authors including [8],[9],[10] etc. also considered problem of estimation of finite population mean under measurement error using auxiliary information. In this paper, we have proposed two log-product type estimators and a new estimator for estimation of finite population mean using auxiliary information in presence of measurement error under simple random sampling without replacement (SRSWOR) scheme.

2. NOTATIONS

Consider a finite population $\Omega=\{\Omega_1,\Omega_2,.........,\Omega_N\}$ of size N units and let we draw a sample of size n units from it using SRSWOR scheme. Let (x_i, y_i) (i=1, 2… n) be observed values on X and Y corresponding to true values (X_i, Y_i) (i=1, 2… N) respectively.

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Let $u_i = y_i - Y_i$ and $v_i = x_i - X_i$ be the measurement errors on study and auxiliary variable respectively. Where, u_i and v_i are stochastic in nature with mean zero and variance σ_u^2 and σ_v^2 and are independent. Population covariance and correlation coefficient between X and Y are σ_{xy} and ρ respectively. Coefficient of variation on X and Y is given by $C_x = \frac{\sigma_x}{\bar{X}}$ and $C_y = \frac{\sigma_y}{\bar{Y}}$

$$\text{Let,}\quad \omega_u = \frac{1}{\sqrt{n}}\sum_{i-1}^{n} u_i\,,\quad \omega_y = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(y_i - \bar{Y}) \quad \text{and}$$

$$\omega_v = \frac{1}{\sqrt{n}}\sum_{i-1}^{n} v_i\,,\quad \omega_x = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(x_i - \bar{X})$$

$$s_0 = \bar{y} - \bar{Y} \text{ and } s_1 = \bar{x} - \bar{X} \quad \text{such that } E(s_0) = E(s_1) = 0$$

$$\text{Also, } E(s_0^2) = \frac{\sigma_y^2}{n}\left(1 + \frac{\sigma_u^2}{\sigma_y^2}\right) = \delta_{ym}\ ,\ E(s_1^2) = \frac{\sigma_x^2}{n}\left(1 + \frac{\sigma_v^2}{\sigma_x^2}\right) = \delta_{xm} \text{ and } E(s_0 s_1) = \frac{\rho\sigma_y\sigma_x}{n} = \delta_{yxm}.$$

3. ESTIMATORS IN LITERATURE

In this section we consider several relevant estimators in literature of survey sampling. The expressions for Bias and MSE under measurement error are given up to the first order of approximation.

i.) The usual unbiased estimator is $\phi_1 = \bar{y}$ and its MSE under measurement error is given as:

$$MSE_{min}(\phi_1) = \delta_{ym} \tag{3.1}$$

ii.) Cochran [11] gave usual ratio estimator given as: $\phi_2 = \bar{y}\frac{\bar{X}}{\bar{x}}$

The expression for its minimum MSE under measurement error is given by,

$$MSE_{min}(\phi_2) = \delta_{ym} + \frac{\bar{Y}^2}{\bar{X}^2}\delta_{xm} - 2\frac{\bar{Y}}{\bar{X}}\delta_{yxm} \tag{3.2}$$

iii.) Murthy [12] defined usual product estimator in the following form: $\phi_3 = \bar{y}\frac{\bar{x}}{\bar{X}}$

Its minimum MSE under measurement error is given as:

$$MSE_{min}(\phi_3) = \delta_{ym} + \frac{\bar{Y}^2}{\bar{X}^2}\delta_{xm} + 2\frac{\bar{Y}}{\bar{X}}\delta_{yxm} \tag{3.3}$$

iv.) The Regression estimator is defined in the following form: $\phi_4 = \bar{y} + k(\bar{X} - \bar{x})$

where, k is a constant and its optimum value is given by $k_{opt} = \frac{\delta_{yxm}}{\delta_{xm}}$

and its minimum MSE under measurement error is given as,

$$MSE_{min}(\phi_4) = \delta_{ym} - \frac{\delta_{yxm}^2}{\delta_{xm}} \tag{3.4}$$

v.) Kumar et al. [4] proposed estimators for population mean as follow:

$$\phi_5 = \bar{y}\exp\left(\frac{\mu_x - \bar{x}}{\mu_x + \bar{x}}\right) \quad \text{and} \quad \phi_6 = w_1\bar{y} + w_2(\mu_x - \bar{x})$$

Expressions for their min. MSE of ϕ_5 and ϕ_6 under measurement error are given by Eq. (3.5) and Eq. (3.6) respectively,

$$MSE_{min}(\phi_5) = \frac{\sigma_y^2}{n}\left[1 - \frac{C_x}{C_y}\left(\rho - \frac{C_x}{4C_y}\right)\right] + \frac{1}{n}\left[\frac{\mu_y^2}{4\mu_x^2}\sigma_v^2 + \sigma_u^2\right] \quad (3.5)$$

$$MSE_{min}(\phi_6) = \mu_Y^2 - \frac{b_3 b_4^2}{b_1 b_3 - b_2^2} \quad (3.6)$$

where $b_1 = \mu_Y^2 + \delta_{ym}$, $b_2 = -\delta_{yxm}$, $b_3 = \delta_{xm}$, $b_4 = \mu_Y^2$

4. PROPOSED ESTIMATORS UNDER MEASUREMENT ERROR

Mishra et al.[13] proposed two log-product type estimators for finite population mean. Here, influenced by Mishra et al. [13] we propose two log product type estimators and a new estimator for estimation of finite population mean under measurement error by utilizing auxiliary information. Notations to be used in the forthcoming section are as defined in section 2. The expressions for the Bias and MSE's of proposed estimators are obtained for the terms up to first order of approximation.

$$1.)\ P_1^* = \bar{y} + \lambda \log\left(\frac{\bar{x}}{\bar{X}}\right) \quad (4.1)$$

Expanding Eq. (4.1) and retaining terms up to second power of s_i's (i=0, 1), we have

$$P_1^* = (\bar{Y} + \bar{y} - \bar{Y}) + \lambda \log\left(\frac{\bar{X} + \bar{x} - \bar{X}}{\bar{X}}\right) \quad \text{or}$$

$$P_1^* = (\bar{Y} + s_0) + \lambda \log\left(\frac{\bar{X} + s_1}{\bar{X}}\right)$$

$$P_1^* - \bar{Y} = s_0 + \lambda \frac{s_1}{\bar{X}} - \frac{\lambda}{2}\frac{s_1^2}{\bar{X}^2}$$

The expressions for Bias and MSE of P_1^* are given in Eq. (4.2) and Eq. (4.3),

$$Bias(P_1^*) = E(P_1^* - \bar{Y}) = -\frac{\lambda}{2\bar{X}^2}\delta_{xm} \quad (4.2)$$

$$\text{and } MSE(P_1^*) = E(P_1^* - \bar{Y})^2 = \delta_{ym} + \frac{\lambda^2}{\bar{X}^2}\delta_{xm} + \frac{2\lambda}{\bar{X}}\delta_{yxm} \quad (4.3)$$

Differentiating partially Eq. (4.3) w. r. to λ and Equating to zero, we get the optimum value of $\lambda = \lambda_{opt}$,

$$\lambda_{opt} = -\frac{\delta_{yxm}}{\delta_{xm}}\bar{X}$$

Substituting the optimum value λ_{opt} in Eq. (4.3) and simplifying, we get the expression for the $MSE_{min}(P_1^*)$ given by Eq. (4.4).

$$MSE_{min}(P_1^*) = \delta_{ym} - \frac{\delta_{yxm}^2}{\delta_{xm}} \quad (4.4)$$

$$2.)\ P_2^* = (t_1 + 1)\bar{y} + t_2 \log\left(\frac{\bar{x}}{\bar{X}}\right) \quad (4.5)$$

Expanding Eq. (4.5), taking terms up to the first order of approximation, we get following form

$$P_2^* - \overline{Y} = (t_1\overline{Y} + t_1 s_0 + s_0) + t_2\left(\frac{s_1}{\overline{X}} - \frac{1}{2}\frac{s_1^2}{\overline{X}^2}\right) \quad (4.6)$$

Further, taking expectation of Eq. (4.6), we get expression for Bias given by Eq. (4.7) and expression for MSE is obtained by squaring the expectation term is given by Eq. (4.8),

$$\text{Bias}(P_2^*) = E(P_2^* - \overline{Y}) = t_1\overline{Y} - \frac{t_2}{2\overline{X}^2}\delta_{xm} \quad (4.7)$$

$$\begin{aligned} MSE(P_2^*) &= E(P_2^* - \overline{Y})^2 \\ &= t_1^2(\overline{Y}^2 + \delta_{ym}) + \delta_{ym} + 2t_1\delta_{ym} + \frac{t_2}{\overline{X}^2}\left(t_2\delta_{xm} + 2\overline{X}t_1\delta_{yxm} + 2\overline{X}\delta_{yxm}\right) \end{aligned} \quad (4.8)$$

Minimizing Eq. (4.8) w. r. to t_1 and t_2 we get, optimum values of constants as t_{1opt} and t_{2opt} as given below,

$t_{1opt} = \frac{AD - CE}{E^2 - AB}$ and $t_{2opt} = \frac{BC - DE}{E^2 - AB}$ where,

Putting the optimum values t_{1opt} and t_{2opt} in Eq. (4.8) we get the expression for $MSE_{min}(P_2^*)$,

$$MSE_{min}(P_2^*) = \delta_{ym} + \frac{BC^2 + AD^2 - 2CDE}{E^2 - AB} \quad (4.9)$$

where, $A = \overline{Y}^2 + \delta_{ym}$, $B = \frac{\delta_{xm}}{\overline{X}^2}$, $C = \delta_{ym}$, $D = \frac{\delta_{yxm}}{\overline{X}}$, $E = \frac{\delta_{yxm}}{\overline{X}}$

$$3.)\ P_3^* = (a_1 + 1)\bar{y} + \frac{\overline{X}}{\bar{x}}a_2 \quad (4.10)$$

Expanding Eq. (4.10) and retaining the terms up to the first order of approximation, we get following form,

$$P_3^* = \overline{Y} + s_0 + a_1\overline{Y} + a_1 s_0 + \left(1 - \frac{s_1}{\overline{X}} + \frac{s_1^2}{\overline{X}^2}\right)a_2$$

$$P_3^* - \overline{Y} = s_0 + a_1\overline{Y} + a_1 s_0 + a_2 - \frac{s_1 a_2}{\overline{X}} + \frac{s_1^2}{\overline{X}^2}a_2 \quad (4.11)$$

Taking expectation of Eq. (4.11) we get the Bias expression and squaring Eq. (4.12) we get MSE as given in Eq. (4.13),

$$\text{Bias}(P_3^*) = E(P_3^* - \overline{Y}) = a_1\overline{Y} + a_2 + \frac{a_2}{\overline{X}^2}\delta_{xm} \quad (4.12)$$

$$\begin{aligned} MSE(P_3^*) &= E(P_3^* - \overline{Y})^2 \\ &= \delta_{ym} + a_1^2(\overline{Y}^2 + \delta_{ym}) + a_2^2\left(1 + \frac{\delta_{xm}}{\overline{X}^2}\right) + 2a_1\delta_{ym} - 2a_2\frac{\delta_{yxm}}{\overline{X}} + 2a_1 a_2\left(\overline{Y} - \frac{\delta_{yxm}}{\overline{X}}\right) \end{aligned} \quad (4.13)$$

Differentiating Eq. (4.13) partially w. r. to constants a_1 and a_2, and equating to zero, we get optimum values of $a_1 = a_{1opt}$ and $a_2 = a_{2opt}$ given as, $a_{1opt} = \frac{A_1 D_1 - C_1 E_1}{E_1^2 - A_1 B_1}$ and $a_{2opt} = \frac{B_1 C_1 - D_1 E_1}{E_1^2 - A_1 B_1}$

Substituting the values a_{1opt} and a_{2opt} in Eq. (4.13), we obtain $\min.MSE(P_3^*)$ as given in Eq. (4.14),

$$MSE_{min}(P_3^*) = \delta_{ym} + \frac{B_1C_1^2 + A_1D_1^2 - 2C_1D_1E_1}{E_1^2 - A_1B_1} \quad (4.14)$$

where, $A_1 = \overline{Y}^2 + \delta_{ym}$, $B_1 = 1 + \frac{\delta_{xm}}{\overline{X}^2}$, $C_1 = \delta_{ym}$, $D_1 = -\frac{\delta_{yxm}}{\overline{X}}$, $E_1 = \overline{Y} - \frac{\delta_{yxm}}{\overline{X}}$

5. EFFICIENCY COMPARISON

In this section, we compare the efficiency of proposed estimators w. r. to other relevant estimators discussed in literature.

1.) From Eq. (3.1) and Eq. (4.4),

$MSE_{min}(P_1^*) < Var(\phi_1)$, if

$Var(\phi_1) - MSE_{min}(P_1^*) \geq 0$

$$\frac{\delta_{yxm}^2}{\delta_{xm}} \geq 0$$

2.) From Eq. (3.2) and Eq. (4.4),

$MSE_{min}(P_1^*) < Var(\phi_2)$, if

$Var(\phi_2) - MSE_{min}(P_1^*) \geq 0$

$$\frac{\overline{Y}^2}{\overline{X}^2}\delta_{xm} - 2\frac{\overline{Y}}{\overline{X}}\delta_{yxm} + \frac{\delta_{yxm}^2}{\delta_{xm}} \geq 0$$

3.) From Eq. (3.3) and Eq. (4.4),

$MSE_{min}(P_1^*) < Var(\phi_3)$, if

$Var(\phi_3) - MSE_{min}(P_1^*) \geq 0$

$$\frac{\overline{Y}^2}{\overline{X}^2}\delta_{xm} + 2\frac{\overline{Y}}{\overline{X}}\delta_{yxm} + \frac{\delta_{yxm}^2}{\delta_{xm}} \geq 0$$

4.) From Eq. (3.4) and Eq. (4.4),

$MSE_{min}(P_1^*) < Var(\phi_4)$, if

$Var(\phi_4) - MSE_{min}(P_1^*) \geq 0$

5.) From Eq. (3.5) and Eq. (4.4),

$MSE_{min}(P_1^*) < Var(\phi_5)$, if

$Var(\phi_5) - MSE_{min}(P_1^*) \geq 0$

$$\frac{\sigma_y^2}{n}\left[1 - \frac{C_x}{C_y}\left(\rho - \frac{C_x}{4C_y}\right)\right] + \frac{1}{n}\left[\frac{\mu_y^2}{4\mu_x^2}\sigma_v^2 + \sigma_u^2\right] - \delta_{ym} + \frac{\delta_{yxm}^2}{\delta_{xm}} \geq 0$$

6.) From Eq. (3.6) and Eq. (4.4),

$MSE_{min}(P_1^*) < Var(\phi_6)$, if

$Var(\phi_6) - MSE_{min}(P_1^*) \geq 0$

$$\mu_Y^2 - \frac{b_3b_4^2}{b_1b_3 - b_2^2} - \delta_{ym} + \frac{\delta_{yxm}^2}{\delta_{xm}} \geq 0$$

7.) From Eq. (3.1) and Eq. (4.9),

$MSE_{min}(P_2^*) < Var(\phi_1)$, if

$Var(\phi_1) - MSE_{min}(P_2^*) \leq 0$

$$\frac{BC^2 + AD^2 - 2CDE}{E^2 - AB} \leq 0$$

8.) From Eq. (3.2) and Eq. (4.9),

$$MSE_{min}(P_2^*) < Var(\phi_2), \text{ if}$$
$$Var(\phi_2) - MSE_{min}(P_2^*) \geq 0$$
$$\frac{\bar{Y}}{\bar{X}}\left[\frac{\bar{Y}}{\bar{X}}\delta_{xm} - 2\delta_{yxm}\right] - \frac{BC^2 + AD^2 - 2CDE}{E^2 - AB} \geq 0$$

9.)From Eq. (3.3) and Eq. (4.9),

$$MSE_{min}(P_2^*) < Var(\phi_3), \text{ if}$$
$$Var(\phi_3) - MSE_{min}(P_2^*) \geq 0$$
$$\frac{\bar{Y}}{\bar{X}}\left[\frac{\bar{Y}}{\bar{X}}\delta_{xm} + 2\delta_{yxm}\right] - \frac{BC^2 + AD^2 - 2CDE}{E^2 - AB} \geq 0$$

10.) From Eq. (3.4) and Eq. (4.9),

$$MSE_{min}(P_2^*) < Var(\phi_4), \text{ if}$$
$$Var(\phi_4) - MSE_{min}(P_2^*) \geq 0$$
$$\frac{\delta_{yxm}^2}{\delta_{xm}} - \frac{BC^2 + AD^2 - 2CDE}{E^2 - AB} \geq 0$$

11.) From Eq. (3.5) and Eq. (4.9),

$$MSE_{min}(P_2^*) < Var(\phi_5), \text{ if}$$
$$Var(\phi_5) - MSE_{min}(P_2^*) \geq 0$$
$$\frac{\sigma_y^2}{n}\left[1 - \frac{C_x}{C_y}\left(\rho - \frac{C_x}{4C_y}\right)\right] + \frac{1}{n}\left[\frac{\mu_y^2}{4\mu_x^2}\sigma_v^2 + \sigma_u^2\right] - \delta_{ym} - \frac{BC^2 + AD^2 - 2CDE}{E^2 - AB} \geq 0$$

12.) From Eq. (3.6) and Eq. (4.9),

$$MSE_{min}(P_2^*) < Var(\phi_6), \text{ if}$$
$$Var(\phi_6) - MSE_{min}(P_2^*) \geq 0$$
$$\mu_Y^2 - \frac{b_3 b_4^2}{b_1 b_3 - b_2^2} - \delta_{ym} - \frac{BC^2 + AD^2 - 2CDE}{E^2 - AB} \geq 0$$

13.) From Eq. (3.1) and Eq. (4.14),

$$MSE_{min}(P_3^*) < Var(\phi_1), \text{ if}$$
$$Var(\phi_1) - MSE_{min}(P_3^*) \geq 0$$
$$\frac{B_1 C_1{}^2 + A_1 D_1{}^2 - 2C_1 D_1 E_1}{E_1{}^2 - A_1 B_1} \geq 0$$

14.) From Eq. (3.2) and Eq. (4.14),

$$MSE_{min}(P_3^*) < Var(\phi_2), \text{ if}$$
$$Var(\phi_2) - MSE_{min}(P_3^*) \geq 0$$
$$\frac{\bar{Y}}{\bar{X}}\left[\frac{\bar{Y}}{\bar{X}}\delta_{xm} - 2\delta_{yxm}\right] - \frac{B_1 C_1{}^2 + A_1 D_1{}^2 - 2C_1 D_1 E_1}{E_1{}^2 - A_1 B_1} \geq 0$$

15.) From Eq. (3.3) and Eq. (4.14),

$$MSE_{min}(P_3^*) < Var(\phi_3), \text{ if}$$
$$Var(\phi_3) - MSE_{min}(P_3^*) \geq 0$$
$$\frac{\bar{Y}}{\bar{X}}\left[\frac{\bar{Y}}{\bar{X}}\delta_{xm} + 2\delta_{yxm}\right] - \frac{B_1 C_1{}^2 + A_1 D_1{}^2 - 2C_1 D_1 E_1}{E_1{}^2 - A_1 B_1} \geq 0$$

16.) From Eq. (3.4) and Eq. (4.14),

$$MSE_{min}(P_3^*) < Var(\phi_4), \text{ if}$$

$$Var(\phi_4) - MSE_{min}(P_3^*) \geq 0$$

$$\frac{\delta_{yxm}^2}{\delta_{xm}} - \frac{B_1C_1^{\ 2} + A_1D_1^{\ 2} - 2C_1D_1E_1}{E_1^{\ 2} - A_1B_1} \geq 0$$

17.) From Eq. (3.5) and Eq. (4.14),

$$MSE_{min}(P_3^*) < Var(\phi_5), \text{ if}$$

$$Var(\phi_5) - MSE_{min}(P_3^*) \geq 0$$

$$\frac{\sigma_y^2}{n}\left[1 - \frac{C_x}{C_y}\left(\rho - \frac{C_x}{4C_y}\right)\right] + \frac{1}{n}\left[\frac{\mu_y^2}{4\mu_x^2}\sigma_v^2 + \sigma_u^2\right] - \delta_{ym} - \frac{B_1C_1^{\ 2} + A_1D_1^{\ 2} - 2C_1D_1E_1}{E_1^{\ 2} - A_1B_1} \geq 0$$

18.) From Eq. (3.6) and Eq. (4.14),

$$MSE_{min}(P_3^*) < Var(\phi_6), \text{ if}$$

$$Var(\phi_6) - MSE_{min}(P_3^*) \geq 0$$

$$\mu_Y^2 - \frac{b_3b_4^2}{b_1b_3 - b_2^2} - \delta_{ym} - \frac{B_1C_1^{\ 2} + A_1D_1^{\ 2} - 2C_1D_1E_1}{E_1^{\ 2} - A_1B_1} \geq 0$$

6. EMPIRICAL STUDY

In this section, we demonstrate the efficiency of proposed estimators with respect to usual unbiased, usual ratio, usual product, usual unbiased difference and other relevant estimators in literature. For illustration purpose, we generate three different populations from Normal distribution for different choices of parameters using R programming language (Appendix I). The auxiliary information on X has been considered to be drawn from N (5, 10) and the population size is taken to be N= 5000, wherein n=300 (sample size).

Population I:

$$\overline{X} = 5.156873, \overline{Y} = 5.16866, \sigma_Y^2 = 126.333, \sigma_X^2 = 123.2275, \sigma_u^2 = 24.98276, \sigma_v^2 = 25.26869, \rho_{yx} = 0.7948761$$

Population II:

$$\overline{X} = 4.81693, \overline{Y} = 4.827768, \sigma_Y^2 = 108.4171, \sigma_X^2 = 106.4468, \sigma_u^2 = 8.996378, \sigma_v^2 = 8.928008, \rho_{yx} = 0.9114169$$

Population III:

$$\overline{X} = 4.94121, \overline{Y} = 4.940325, \sigma_Y^2 = 122.159, \sigma_X^2 = 120.6866, \sigma_u^2 = 25.13468, \sigma_v^2 = 24.69794, \rho_{yx} = 0.7909159$$

TABLE I . TABLE SHOWING MSES AND PRES OF PROPOSED AND OTHER ESTIMATORS W. R. TO USUAL ESTIMATOR (WITH AND WITHOUT MEASUREMENT ERROR).

Estimator	Population I		Population II		Population III	
	MSE(PRE)		*MSE(PRE)*		*MSE(PRE)*	
	With error	*Without error*	*With error*	*Without error*	*With error*	*Without error*
ϕ_1	0.5(100)	0.42(100)	0.39(100)	0.36(100)	0.49(100)	0.41(100)
ϕ_2	0.34(148.81)	0.17(240.67)	0.15(260.72)	0.08(448.42)	0.34(146.42)	0.17(240.61)
ϕ_3	1.66(30.31)	1.45(28.09)	1.66(23.63)	1.59(22.78)	1.62(30.39)	1.45(28.09)
ϕ_4	0.28(177.86)	0.16(271.61)	0.11(342.21)	0.06(590.60)	0.28(175.64)	0.15(267.06)
ϕ_5	0.30(169.63)	0.19(218.28)	0.14(274.07)	0.10(351.10)	0.29(168.12)	0.19(216.96)

ϕ_6	0.28(179.74)	0.15(273.19)	0.11(343.89)	0.06(592.15)	0.28(177.66)	0.15(268.73)
P_1^*	0.28(177.86)	0.16(271.61)	0.11(342.21)	0.06(590.60)	0.28(175.64)	0.15(267.06)
P_2^*	0.28(179.74)	0.15(273.19)	0.11(343.89)	0.06(592.15)	0.28(177.66)	0.15(268.73)
P_3^*	0.08(614.64)	0.04(986.58)	0.03(1154.90)	0.021(985.9)	0.08(599.39)	0.04(964.22)

a. Numbers in brackets represents respective PRE's.

7. CONCLUSION

Based on theoretical and empirical results obtained, it can be inferred that proposed estimator P_3^* is found to be more efficient than all other estimators discussed, P_1^* is more efficient than all discussed estimators except ϕ_6 and equally efficient to ϕ_4 and P_2^* is more efficient all other estimators and equally efficient to ϕ_6. It is a common phenomenon to observe that MSE with error is more than MSE without error which is similar as observed in Table I, i.e., MSE of proposed estimators is less than other discussed estimators (for both with and without measurement error). Correspondingly, from PRE values given in Table I it turns out that among all proposed estimators, P_3^* is more efficient than usual unbiased, ratio, product, difference and other estimators discussed in literature.

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APPENDIX I

Algorithm for data generation through R (the same computation can also be implemented using MS Excel).

1) Generate random number of size N with different choices of parameters using R language.

2) The auxiliary information on X is generated using N (μ,σ^2) i.e. X=N(μ,σ^2)

3) Obtain study variable Y as Y=X+ N (μ,σ^2).

4) Draw samples of size n from Y and X respectively as y and x.

5) Obtain error of measurement on y and x as u_i and v_i.

6) Obtain all the parameters required for computation using suitable formulation. (mean ,variance etc).