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Pricing of Application in Two-Sided Markets

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Abstract: The paper considers a two-sided market of mobile cross-platform applications. The most important elements of the market are identified. They comprise consumers of the first category who are simultaneously subscribers of a certain mobile communication operator; consumers of the second category that enter into agreements with platform manufacturers to obtain application development tools and mobile operators to ensure the operability of developed applications in the mobile operator networks; and platform-software or a mobile device in which applications can be used. The main distinction of the market under study is the interaction between the application producer and the consumer, which is always conducted not only using a certain platform but also through a certain mobile operator. The optimizing problem for the market equilibrium on the cross-platform applications market is formulated. The parametric pricing model on the two-sided mobile applications market is constructed. The pricing model takes into account the following factors: average analog price; functionality indicators; the convenience of use; the ratio between supply and demand for the product is based on the number of existing analogs; fame and prestige of the manufacturer; the number of operating systems on which the use of applications is possible.

Keywords: Pricing Model, Two-Sided Market, Business Models, Cross-Platform Applications

1. Introduction

Currently, the world society developed a program to create conditions for the transition to a digital economy. In Russia, the Non-profit Organization "Digital Economy", created by successful Russian high-tech companies, coordinates the participation of the expert and business communities in the planning of implementation, development, and evaluation of the program effectiveness. The main intermediate results of the implementation of the program of transition to the digital economy should be the creation of 5G / IMT-2020 networks and infrastructure for storing and processing data.

In addition, the roadmap contains an action plan for the development of the state digital platform information systems to provide citizens with state and municipal services in the electronic form. Moreover, by 2024 all data is planned to be made available for the use on digital platforms, including mobile phone terminals with mobile Internet.

According to the Yandex Group Company, the number of people using mobile Internet has grown three times faster than the number of fixed Internet users over the past three years. Today, 92% of users have the opportunity to be online with mobile. Nevertheless, 44% of all requests on the Internet, according to Mediascope [1], are still performed from desktops, which in our opinion is due to the imperfection of existing mobile applications and the cost of using them.

Mobile applications are software designed specifically for small portable devices, such as a pocket personal computer, smartphones, or cell phones. These applications can be native, i.e. pre-installed in the device in the production process, uploaded by the user via various platforms for distribution, or are web applications processed on the client or server-side.

Modern operating systems are developed by Apple, Microsoft, Google, and Nokia. Among the most used platforms for mobile devices are the operating systems iOS, Android, Windows Phone, and Symbian.

In 1999, the PalmOS operating system dominated, which owned 74% of the market. By the third quarter of 2001, PalmOS accounted for 51.28%, while Windows Mobile already has more than 30%. Symbian and BlackBerry OS took third and fourth places with 3.32% and 2.79% respectively.

In 2002, PalmOS and Windows Mobile began to yield to Symbian. By the first quarter of 2003, Symbian overtook Windows Mobile, losing only to PalmOS. Six months later, Symbian took first place in the ranking with a share of 35%.

By the first quarter of 2006, Symbian's share in the global market for mobile operating systems amounted to

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60.08%. In the second quarter of 2007, when the iPhone was released, the iPhone operating system (later called iOS) occupied only 0.64% of the market.

Android first appeared on the list in the fourth quarter of 2008 and debuted in sixth place. In the first quarter of 2011, Android OS took first place, which it still holds (38.77% of the market). The second place with a share of 24.51% is occupied by the Symbian operating system. In the third place is iOS (17.12%)

As part of the "Digital Economy" roadmap implementation, the idea of creating platforms for a network of Internet-connected objects capable of collecting data or exchanging data coming from embedded services, the so-called Internet of Things (IoT), has become widespread. The convenience of the Internet of things for the consumer is becoming apparent. Any Internet-connected Devices can be controlled remotely.

The importance of digital technologies in our lives has reached new heights, and more and more people spend more and more time on the Internet, solving more and more tasks there. This year, the number of Internet users in the world increased by seven percent compared to the previous year and amounted to 4.54 billion. The increase in users in January 2020 compared with January 2019 is 298 million new subscribers. The number of people using social networks is also growing. In January 2020, there were 3.80 billion social network users in the world, an increase of 9% compared to 2019. The number of subscribers using mobile communication is also growing. Today, more than 5.19 billion people use mobile phones [18].

To increase the effectiveness of the Digital Economy road map activities using cross-platform applications, a prudent pricing policy is necessary. This would ensure the maximum profit to platform producers, on the one hand, and as a result, it would lead to maximum allocations to the budget and non-budgetary funds. On the other hand, it would ensure the accessibility of the services using digital economy platforms for ordinary consumers. Therefore, the solution of the optimization problem for the cross-platform mobile applications industry and the creation on this basis of a pricing model that takes into account the interests of application manufacturers of mobile operators and end-users of the Service through these applications is very important.

2. LITERATURE REVIEW

The problem of pricing in the two-sided Internet applications market for personal computers, in contrast to the market of mobile device cross-platform applications, is widely discussed in the scientific community.

Ranging from the works on the description of the two-sided market general theory (Armstrong M., Wright J.) [2,3] to the development of pricing models in these markets as a result of the interaction of both parties, taking into account various network effects, some authors [4] use game-theoretic models to describe the processes of product pricing. Other authors [9, 10] apply models of the logit type for these purposes. King into consideration the cross-platform in two-sided markets, the research of J.P. Choi (2010) [4] seems worth mentioning. It resulted in the development of two-sided market models, where both sides have the opportunity to simultaneously use the services of several platforms. The author examined the impact of the measures taken by the platforms to "tie" users, i.e. to compel them to use additional services. In such markets, the users have no opportunity to simultaneously use several different platforms. Wright (2003) [6] considered the issues of "tying" the users to a single platform, as well. The authors analyzed from the standpoint of a general consideration of the antimonopoly policy in two-sided markets. However, their discussion was mostly informal and did not deal with the establishment of analytical or quantitative relationships.

Such authors as Roche and Tyrol (2003) [7], and Amelio and Julien (2006) [8] are notable exceptions in the study of the economic effects of "tying" users to a certain platform in two-sided markets. Roche and Tyrol carried out an economic analysis of the customers' "tying" practice, initiated by the associations of Visa and MasterCard payment cards. Amelio and Jullien (2006) made a more general study of tying in two-sided markets. They considered the situation when producers being on one side of the market wanted to set platform prices below zero to solve the problem of demand management in two-sided markets. According to their analysis, "tying" can serve as a mechanism for introducing hidden subsidies on one side of the market to tackle the problem of increasing demand in two-sided markets. As a result, the "tying" of users can benefit consumers in case of a monopoly platform. However, in the context of a duopoly, tying also has a strategic impact on competition. Amelio and Jullien have shown that the "tying" influence on consumer surplus and social welfare depends on the asymmetry degree of external factors between the two parties. The article by Amelio and Julien and the work by Choi are focused on different aspects of "tying" the users. For example, they compare the effects of tying to different market structures (monopolistic and duopolistic) but do not consider the issues of oligopoly when there are more than two participants in the market. A recent study of the problem of pricing in two-sided markets was carried out in the article [17]. The authors studied the equilibrium in the market, represented by network platforms and heterogeneous agents. They showed the optimal pricing strategy under the assumption of a monopolistic market model and in the case of a duopoly in a two-sided market with two platforms.

This work is the development of approaches to the study of pricing processes in two-sided markets [11-16] concerning cross-platform applications developed for mobile devices under the assumption of the oligopolistic market of mobile communication operators.

3. DATA AND METHODOLOGY

In this work, we study the global and Russian cross-platform applications market, which began its formation in the 2000s. The study is theoretical. We will suppose, that cross-platform applications developed for the interaction of mobile application market participants can be attributed to innovative products, and the mobile applications market itself can be referred to as two-sided markets having two groups of users between which network effects arise. The goals of using the network and the role of users in the network are markedly different. Representatives of various groups, being interdependent, outline different requirements on the functionality of a two-sided network. On the other hand, end-users of the services provided via cross-platform applications for mobile devices use mobile Internet through intermediaries (their mobile operators), who in turn establish their billing for the traffic.

In our work, we will consider the mobile application market as two-sided. To create a pricing model, we highlight the following important elements.

The first element is consumers of mobile applications that are simultaneously subscribers of a certain mobile communication operator. The second element is mobile application manufacturers or content providers that enter into agreements with platform manufacturers. They do it to obtain application development tools for mobile operators to ensure the functionality of developed applications in mobile operator networks. The third element is a platform with software or a mobile device in which applications can be used.

The majority of pricing models, which are quite acceptable to most goods and services can not be used for the pricing of mobile applications. In this case, you must use the tools of parametric pricing, where the price of goods and services is based on a formal pre-model depending on the values of the level of prices of basic consumer characteristics of these goods and services. This is a complex form of pricing, which is used in the case of advanced information and the technical base of the manufacturer.

In general, the standard-parametric method of pricing advocates an economic-mathematical method that allows calculating the price levels of the same type of products based on mathematical equations that take into account the cost of production of goods or services and their consumer properties.

For regulatory and parametric method is a group of methods that uses other than mathematical tools and expert assessment method of quality parameters of the commodity or services in the form of scoring individual consumer preferences. The normative-parametric method for the determination of prices yields reasonable price levels for similar goods.

The method of specific indicators is used to establish and analyze the product's price. This method is based on the fact that the goods or services have the main parameter. The value of this parameter determines the overall price of the product by the formula

$$P_h = \frac{P_1}{H_1} \qquad (1)$$

P_h is a price per unit of the main parameter in units of national currency. P_1 is a price of a basic product in units of national currency. H_1 is a value of the main parameter of the product in the appropriate units of measurement.

The method of pricing the new products based on the technical level coefficients. The coefficient of the technical level is calculated by comparing the technical performance of the new base model and peers. As a product-sample, we must select a real-life product that best reflects the world level progress for this type of equipment per selected period of time. In this case, the price of services is determined based on the relative technological level of the new model and the cost of similar products.

The ranking point pricing method is based on expert judgment. The price of the products depends on determining the significance of the individual characteristics of the goods to the consumer. Each product characteristic is assigned the number of points by a specific product expert. By adding up these values, we can get an integrated assessment of the technical and economic levels of the product, which is an influence on the product's price.

To develop the model estimates the cost of cross-platform applications for mobile devices, we will use parametric pricing and ranking point pricing approaches, taking into account the expert judgment and factors affecting the demand and supply in the cross-platform applications market.

4. A PRICING MODEL FOR CROSS-PLATFORM APPLICATIONS FOR MOBILE DEVICES

The functioning of the market has many features that are not inherent in any other market. The main difference is the ways of interaction between the service provider and the consumer (subscriber), which are always developed not only via a certain platform but also through a certain mobile operator [16]. In this case, the operator has the right to choose from a variety of services providing companies it will cooperate with, and what price will be set for this or that service to the end-user.

Currently, the market has more than three hundred organizations that develop services to provide subscribers with. The value of the Herfindahl-Hirschman index, calculated for the first fifty largest companies, does not exceed one-tenth (HHI = 0.049 <0.100) [14], which indicates a high intensity of competition and a low concentration of the market for producers of additional mobile services.

At the same time, the mobile operators' market, represented by the three largest companies, the total share of which in the sector of additional mobile services exceeds 90.0 percent, is oligopolistic (HHI = 0.317> .200) [14].

Competition in the provider market leads to a decrease in the price of the application, the value of which is established at the equilibrium of the sectoral demand and supply. Consumers and the operator, transporting services to subscribers via a mobile application, are interested in the reduction of prices. To increase its own profit, the operator will choose those providers that offer their services at the lowest price.

Given a high loyalty of subscribers to their operator, which is observed in the Russian mobile communications market, there is practically no redistribution of consumers among operators. Because of this, the operator, in fact, is a monopolist that can dictate its price of both basic and additional services to the subscribers who have already been connected. At the same time, attracting new subscribers requires the operator to set the minimum price for new billing plans in the oligopoly market. The pricing process is shown schematically in Figure 1.

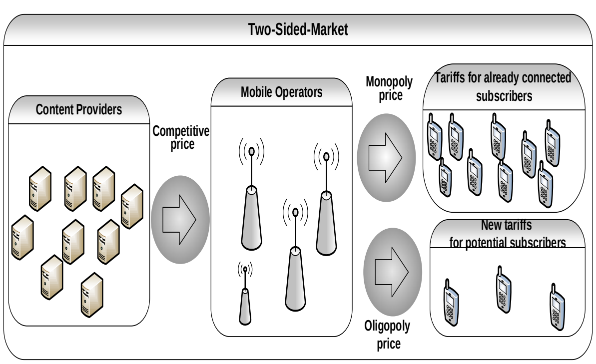

Figure 1. A two-sided market model for cross-platform applications.

To develop the pricing model of the cross-platform applications for mobile devices, we will take into account the following limitations:

- The situation will be reviewed from the perspective of a small development company that is not a market leader. This situation is typical for the market for mobile applications. Most of the companies are quite easy to enter the market due to the existence of a large set of programming tools that permit the creation of a large number of market products.
- Option monopoly price or cost accounting for the development of applications is not taken into account based on the specifics of the market.
- A new application has an equivalent on the market.
- There are no free or shareware products on the market.
- The calculations will not take into account the market share of existing product-analog due to information has a limited period of use in connection with the short product life cycle.

We will suppose that the price of a new mobile application that has its analogs should consider such factors as the average price of analog, performance functionality, the fame of the manufacturer, a number of operating systems on which it is possible to use applications.

The price of a new application can be found as the average of the market prices of existing applications.

$$P_{new} = \frac{\sum_{i=1}^{n} P_i}{n} \tag{2}$$

P_{new} is the average value of the market price of a new application based on the market price of existing products. P_i is a price of existing application i. n is quantity of existing applications on the market.

If there are no analogs on the market, we can use the price of a single commodity with similar characteristics.

For example, Table 1 shows the calculation of the base cost of the application based on the existing analogs on the market. The price is given in dollars since most of the applications implemented through the Internet, and foreign companies mainly produce applications.

TABLE I. THE BASE COST OF THE NEW APPLICATION, USD

	Analogue 1	**Analogue 2**	**Analogue 3**
Pi, Price of Analogue	6.5	4.5	7.0
Price of new application		6.0	

Functionality and usability Indicators can be determined by comparison with existing application analogs by peer review.

In assessing the functionality and usability of mobile applications, we propose the introduction of a coefficient reflecting the comparative characteristics of the product and its analogs based on a possible maximum score.

To calculate the coefficient of functionality and usability, it necessary to divide the sum of points given by all experts by the maximum number of points.

$$K_i = \frac{\sum_{j=1}^{N} K_j}{10 \cdot N} \tag{3}$$

K_j is a final value of the corresponding coefficient given by expert *j*. These factors include functionality coefficient, usability coefficient, competition coefficient, popularity coefficient, cross-platform coefficient. N is a number of experts. The maximum score is 10 points, the minimum is 1.

Table 2 provides an example of the expert evaluation of the functionality and usability of the mobile application as compared with the analogs. Estimations made by a single expert.

TABLE II. FUNCTIONAL USAGE COEFFICIENT

Parameter of estimation	**New Apps**	**Analogue 1**	**Analogue 2**	**Analogue 3**	**Max**
Functionality	9	10	8	7	10
Coefficient of functionality	0,9	1,0	0,8	0,7	1,0

Table 3 illustrates an exemplary variant of the single expert usability evaluation of mobile applications in comparison with the analogs.

TABLE III. INDICATOR CONVENIENCE OF USING A MOBILE APPLICATION

Parameter of estimation	**New Apps**	**Analogue 1**	**Analogue 2**	**Analogue 3**	**Max**
Usability	9	8	10	6	10
Usability coefficient	0,9	0,8	1,0	0,6	1,0

Table 4 shows an example of the expert evaluation of the functionality and usability of the mobile application as compared with the three product analogs. Three experts made estimations.

TABLE IV. INDICATORS OF FUNCTIONALITY AND USABILITY

Parameter of estimation	**Expert 1**	**Expert 2**	**Expert 3**	**Summary score**
Functionality				
New Apps	10	8	7	25
Analogue 1	9	7	8	24
Analogue 2	9	9	10	28
Analogue 3	8	10	9	27
Functionality coefficient				
New Apps	0,83			
Analogue 1	0,80			
Analogue 2	0,93			
Analogue 3	0,90			
Usability				
New Apps	9	8	9	26
Analogue 1	7	7	8	22
Analogue 2	7	8	8	23
Analogue 3	8	7	9	24
Usability coefficient				
New Apps	0,87			
Analogue 1	0,73			
Analogue 2	0,77			
Analogue 3	0,80			

In the most general case, the market competition leading to the equilibrium between the analogs supply and demand can be determined based on the number of existing applications and the expected number of new analogs.

The competitiveness coefficient for a new application will be the higher, the less existing and expected analogs will be on the market at the time of the release of our new application. Thus, the competitiveness coefficient can be determined by the formula

$$K_3 = \frac{1}{n} \quad (4)$$

For example, there are two similar applications on the market. We can expect the release of another 2-5 products of this type of mobile applications from different vendors. According to optimistic for us estimates, the market will be present with five applications including our new one. In this situation, the factor of the competition for our new application will be equal to 0.2.

Based on a pessimistic prognosis, we can expect eight analogs on the market. The coefficient of the competition will be 0,125.

We can identify the fame and prestige of the manufacturer by experts' opinions.

The property of cross-platform is essential for selling applications. It is proposed to estimate this coefficient based on the possibility of using a new application on different operating systems.

If the developed application will work on all operating systems existing in the market, the value of its cross-platform coefficient will be equal to one.

Suppose that there are four main platforms in the market and a new application can operate using three of them. The value of the coefficient will be 3/4=0.75. If a new product operates only on two platforms, the coefficient will be 2/4=0.5.

Operating a mobile application only on one platform is not a cross-platform factor. The coefficient equals zero.

Based on the defined criteria, it is possible to build a pricing model for the cross-platform applications for mobile devices

$$P = P_{new} \cdot \left(\frac{K_1+K_2+K_3+K_4+K_5}{N}\right) \quad (5)$$

P_{new} is the base price of the application calculated by (2) and being the average price in the market. K_1 is functionality coefficient. K_2 is a usability coefficient. K_3 is a competition coefficient. K_4 is a popularity coefficient. K_5– cross-platform coefficient. N– number of coefficients (in this case N = 5).

We proceed from the fact that the price of a new application will not exceed the price of the existing product. Otherwise, it will be not competitive in price and not be able to repay development costs.

The development cost is not included in the price formula, so the developer must focus on higher sales, but not the costs and expenses.

The values of the coefficients in the formula (5) can be administered according to a more complicated than linear dependence.

To account for the nonequivalence factors we can put weight characteristics. In this case, the pricing model represents by formula 6.

$$C = P_{new} \cdot \left(\frac{A_1 \cdot K_1 + A_2 \cdot K_2 + A_3 \cdot K_3 + A_4 \cdot K_4 + A_5 \cdot K_5}{N}\right) \quad (6)$$

where $A_1 - A_5$ weights the importance of each indicator $K_1 - K_5$ (amounting to a total of 1). In this case, it is possible to identify the most and least important factors that influence the pricing policy in the field of mobile applications.

For example, when evaluating the functionality of 0.3, the usability of 0.15, popularity of 0.05, competition of 0.1, cross-platform of 0.4 in the formula (6) takes on the following form

$$C = P_{new} \cdot \left(\frac{0{,}3 \cdot K_1 + 0{,}15 \cdot K_2 + 0{,}05 \cdot K_3 + 0{,}1 \cdot K_4 + 0{,}4 \cdot K_5}{5}\right) \quad (7)$$

It should be noted that in each case for determining the weight factors may be used various opinions as individual experts and teams.

The approbation of the developed model.

Suppose you want to estimate the cost of a mobile application with the following conditions:

- On the market there are two analog applications, the average price is $ 5.5
- We expect that there will be three of the same application including our product on the market.
- As a result of the peer review, the coefficient of functionality of the mobile application is 1.0.
- As a result of peer review, the factor usability of mobile applications is 0.9.
- As a result of peer review, the Developer popularity factor is 0.3.
- As a result of peer review, the cross-platform ratio is 1.

According to the formula (7), we can estimate the price of a new mobile application $C = 4{,}813\ (USD)$.

We also can find conditions for maximizing the profit of mobile operators and industry profits when implementing cross-platform applications for mobile devices. The final purchase price of a cross-platform application for a subscriber, p_i consists of the price formed in the providers' competitive market, p_c and the surcharge established by the mobile operator Δp_i, and $p_i = p_c + \Delta p_i$.

To estimate the operator's premium Δp_i to the price, we consider the conditions for maximizing the operator's profit, defined as the difference between the revenue and costs

$$\pi_i = y_i^d(p_i) \cdot p_i - c\left(y_i^d\right) \quad (8)$$

The revenue of the *i*-th mobile operator from the provision of applications is equal to the product of the number of applications for the period under consideration and their price. At the same time, the number of developed applications depends on their price (the law of demand) and is determined by the individual demand function of the mobile operator $y_i^d(p_i)$. The demand function of a mobile operator depends on the number of subscribers and can be estimated empirically based on statistics. [12, 13].

The costs of the *i*-th mobile operator $c\left(y_i^d\right)$ depend on the number of services rendered by using the application and determined by the demand, which, in turn, depends on the price $c\left(y_i^d(p_i)\right)$.

When determining the price level for cross-platform applications provided to subscribers through their networks (especially when launching new service provision), the *i*-th mobile operator seeks to increase its profit primarily by attracting new consumers. Therefore, setting the price of a cross-platform application, the *i*-th mobile operator is guided not only by the price of the service, formed in the competitive provider market and the demand of their subscribers but also by the prices of similar services implemented in the competitors' networks. So, the price set by the *i*-th operator will depend on the price of other mobile operators. In this case, the demand function of the *i*-th mobile operator can be represented in the form of

$$y_i^d = f\left(p_i, p_j\right);\ j = 1, \dots, N;\ j \neq i \quad (9)$$

where *j* is the number of the mobile operator, and *N* is the number of mobile operators working in the coverage area of the *i*-th operator.

Given the dependence of the price premium Δp_i, (set by the *i*-th operator) on the price premiums of their competitors, and the case of an additive dependence of the demand function of the *i*-th operator on the competitors' prices, the conditions for maximizing the profit of the *i*-th mobile operator can be put down in a general form

$$\begin{cases} \frac{\partial \pi_i}{\partial \Delta p_i} = y_i^d(p_i) \cdot \frac{\partial p_i}{\partial \Delta p_i} + p_i \cdot \sum_{j=1}^{4} \left(\frac{\partial y_i^d(p_i)}{\partial p_j} \cdot \frac{\partial p_j}{\partial \Delta p_i} - \right. \\ \left. - \frac{\partial c(y_i^d)}{\partial y_i^d} \cdot \frac{\partial y_i^d(p_i)}{\partial \Delta p_i} \right) = 0 \ ; \\ \Delta p_i = p_i - p_c \\ \frac{\partial^2 \pi_i}{\partial (\Delta p_i)^2} < 0 \ ; \quad i = \overline{1,4}. \end{cases} \quad (10)$$

The first equation in the system (10) reflects the necessary condition for the existence of an extremum, and the inequation in the system (10) is a sufficient condition for the maximum. The solution to the system (10) for each i allows estimating the price extra charges of mobile operators at which their profit is maximum.

The industry profit is determined by the amount of individual companies' profits

$$\Pi = \sum_{i=1}^{4} \pi_i \quad (11)$$

The conditions for optimizing the industry profit are given in the formula below

$$\begin{cases} \mathrm{d}\Pi(\Delta p_1, \Delta p_2, \Delta p_3, \Delta p_4) = 0 \ ; \\ A(\mathrm{d}\Delta p_1, \mathrm{d}\Delta p_2, \mathrm{d}\Delta p_3, \mathrm{d}\Delta p_4) = \sum_{i,j=1}^{4} \frac{\partial^2 \Pi \cdot \mathrm{d}\Delta p_i \cdot \mathrm{d}\Delta p_j}{\partial(\Delta p_i)\partial(\Delta p_j)} \end{cases} \quad (12)$$

The cross-platform usage leads to an increase in the demand for these applications, raising their value and functionality in the mind of a consumer. In this case, its cost will be higher than the cost of applications that run only on one platform. On the one hand, this will increase the industry profits and the corresponding deductions to the budget. On the other hand, the drop in the demand for the application due to its price rise, as well as the emergence of two-sided market network effects reflected by the presence of a negative term in the model (10) leads to the possibility of obtaining optimal price values. They take into account both the interests of application producers and the interests of consumers, subscribers of the mobile operators.

5. Conclusions

Based on the results of the study, the following conclusions can be drawn.

In the framework of this work, approaches to the formation of a pricing model for applications for mobile devices are considered. The mobile application in the framework of this work is considered as stand-alone software created for a mobile device, such as a mobile phone or tablet computer, as well as for laptops.

Applications for mobile phones and tablets today are a popular innovative product on the market and find their application both for the personal goals of users (social networks, games, media, etc.) and in organizing and conducting business. Cross-platform applications can successfully run on more than one hardware platform and/or operating system (i.e., be used in a wide range of mobile devices using software from various developers). The development of cross-platform applications for mobile devices looks attractive for manufacturers and important for consumers.

The application market for mobile devices is dynamically developing and highly competitive. The main trends in the global market for mobile applications are high demand for products for social networks, communication applications, and games. Among paid and cash applications, leadership belongs to games for mobile devices.

Summarizing the research and the results of developing a model for optimizing the industry profit of cross-platform applications for mobile devices, it should be noted that the problems of considering the innovative product pricing in the two-sided mobile application markets have not yet been properly explored either in theory or in practice. This research should be considered as beginning in the study of the question of pricing innovative products in the current conditions of increasing competition and the presence of a growing number of developers and intermediaries in the Russian market. Based on the indicated approaches, a further study and modeling of approaches to the pricing of the applications created for various mobile devices can be carried out taking into account the tendency to make use of the cross-platform opportunities. Pricing models for new types of applications and dynamic models that consider the introduction of new players like free apps will be considered in our further research.

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