An Adaptive Load-Balancing Approach for Sharded Blockchains

David Halim Daou and Ramzi A. Haraty Department of Computer Science and Mathematics Lebanese American University Beirut, Lebanon Email: David.daou@lau.edu, rharaty@lau.edu.lb

Abstract:

Blockchains were introduced as an innovative means of storing and processing transactions securely and in a decentralized manner. They function by recording transactions in a sequential chain of blocks, wherein each block incorporates the cryptographic hash of its preceding block. However, the emergence of blockchains as a way to organize and protect user data across the internet has come with some concerns, mainly how to deal with the issue of scalability while still maintaining the security standards as well as the decentralized nature inherent to blockchains. Many different implementations were offered. This paper investigates the concept of sharding, which offers a promising solution by partitioning a blockchain into smaller clusters to optimize performance through efficient load balancing. First, we will explore the existing literature on the subject and related algorithms. Then provide a detailed explanation of the functioning of the existing centralised and decentralised algorithms, as well as the one proposed by this thesis. Next, we will elicit the settings and conditions of the simulation environment, both in data collection and preparation. Finally, we will provide the results obtained as well as a comparative analysis of the tested algorithms and give an overview of possible future endeavours in the advancement of load-balancing algorithms concerning sharded Blockchains.

Keywords: Blockchain, Sharding, Consensus Protocol, Load Balancing, Centralized Algorithms, and Decentralized Algorithms

1. Introduction

Blockchains were introduced in 2008 [1] as a new way to store and process transactions in a secure and decentralized manner. They consist of storing transactions in a chain of blocks where each block contains the hash of the previous block. To ensure validity of a block, a consensus algorithm called Proof-of-Work must be executed by all nodes in the network that have the role of "miners". This algorithm consists of pitting the different miners against each other where a computationally expensive puzzle has to be solved, each miner will have to provide a candidate block containing new incoming transactions and only one block will be chosen based on whether the majority of nodes in the network agree with the validity of the block and the transactions it contains. This ensures that as long as more than half of the network population consists of honest nodes, the resulting chain will maintain its correctness. Through the compensation of the "winning" miner in digital currency, nodes are incentivized to participate in the Proof-of-Work algorithm. As such, the larger a blockchain becomes, the harder it becomes to compromise its security; thus, bypassing the single point of failure financial institutions can be vulnerable to.

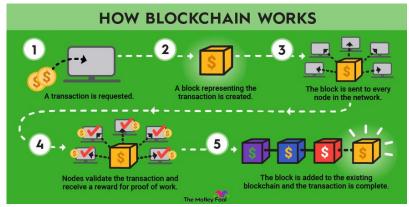


Figure 1: Blockchain Overview

Although the concept of blockchains started with the goal of supporting the Bitcoin network, new blockchains such as Ethereum [2] in 2013 would emerge, taking advantage of the decentralized and secure manner of the concept to build new cryptocurrency platforms as well as other applications such as healthcare.

Even though blockchains have significant advantages in terms of security thanks to their decentralized nature, they suffer from scalability issues [3] and are outperformed in terms of throughput per second for read and write operations as well as data storage volume by other systems such as Visa and PayPal. The fact that Proof-of-Work is extremely computationally expensive and lacks efficiency contributes to this slowdown. As such other

consensus algorithms were proposed to alleviate the negative effects of using the blockchain concept that started with the Bitcoin network.

1.1 Blockchain Consensus Algorithms

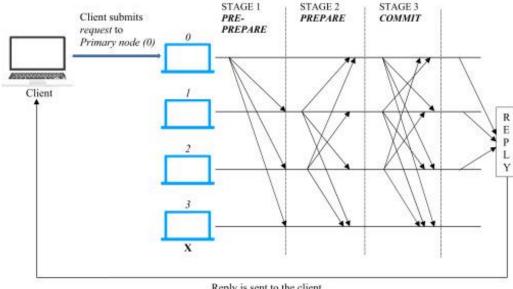
Proof of Stake:

The Ethereum network introduced a new consensus algorithm called Proof-of-Stake in 2022 [4] to improve the energy efficiency and performance of the blockchain. It relies on a staking mechanism to ensure that dishonest validators are penalized for potential fraudulent behavior thus disincentivizing this type of behavior. A block is then proposed by a randomly selected validator.

The validity of the proposed block is then verified by all other validators participating in the creation of the next block. This system not only helps securing the system through further decentralization and easier participation for nodes but also significantly increases throughput improving upon the scalability and efficiency of the blockchain. In Ethereum, a currency used called "gas" is expended on each transaction, the price and quantity required depends on the computational complexity of the transaction with a floor minimum of 21000 gas [5]. Validator nodes that are responsible for processing the transactions into blocks are then rewarded with a share of the transaction fees and distribute some of those fees to all other participating nodes in the staking process. To discourage malicious intent and enforce efficiency, validators that provide invalid transactions or who happen to be offline after their election as validator receive a penalty to their reward as a percentage of the staked amount.

Practical Byzantine Fault Tolerance:

The Practical Byzantine Fault-Tolerance consensus algorithm [6] relies on a voting protocol consisting of designated primary "leader" nodes that provide a block which must be validated through the voting of other nodes. This is achieved through 3 phases. If at least two thirds of the voter nodes agree on the validity of a block, then it is committed; thus, allowing a margin of failure based on majority voting. The significant multicasting and communications required result in significant communication overhead which itself results in bad scalability despite the increased energy efficiency. As such the Practical Byzantine Fault Tolerance algorithm is used in conjunction with Proof-of-Work in blockchain networks such as Ziliqa.



Reply is sent to the client Figure 2: Practical Byzantine Fault Tolerance

1.2 Sharding

To improve upon the scalability of blockchain systems, some networks such as Ethereum [7], Chainspace [8], RapidChain [9] and OmniLedger [10] are considering the concept of sharding in future releases [11]. Sharding consists of partitioning a database into smaller regions that operate on local nodes. In the case of blockchains studied in this work, this consists of partitioning the entire network of account nodes into smaller groups as shown in Figure 2. Each group would then independently achieve consensus. This would ideally allow parallel execution and processing of new transactions; thus, significantly improving the performance of the network.

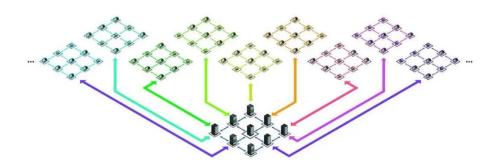


Figure 3: Blockchain Sharding

Despite its apparent advantages, sharding can suffer from issues related to the algorithm with which it is partitioned [12]. Improper balancing of computational loads among the different shards can lead to a significant decrease in performance despite the addition of parallelism. In Ethereum, transactions have varying computational complexities denoted by their gas expenditure [5], as such the gas spent by each successful transaction can be used as a means to calculate the complexity of each outgoing transaction per account; thus, denoting the total computational expenditure of each account in the blockchain.

1.3 Problem Statement

The problem we will be addressing in this work is to design and implement an improved version of the diffusion load-balancing algorithms by tuning different parameters and provide a comparative analysis of the proposed algorithm, basic load-balancing algorithm and the regular diffusion algorithm proposed in the paper by Toulouse et al. [13]. We will be using data from Ethereum crawled with the online tool BitQuery Builder [14], and using the gas consumption of the transactions as a measure of load expended by each transaction.

1.4 Contributions of the Work

In this work, we provide an implementation of the distributed diffusion algorithm as well as an algorithm to handle account migration. Both algorithms have parameters and heuristics introduced to achieve better performance. We will then be providing a comparative analysis of the proposed algorithm variant and the unmodified one. We will also show how modifying tuning parameters as well as the introduced heuristics affect the performance of the proposed algorithms.

To summarize our contributions:

- a) Adaptive diffusion algorithm implementation with both old and new topology.
- b) Optimized migration algorithm implementation.
- c) Experimental results and analysis.

The rest of the paper is organized as follows: Section 2 provides an overview of the load-balancing algorithms. Section 3 presents our model. Section 4 discusses our experimental results, and section 5 offers the conclusion and future work.

2. Literature Review

In this section, we will be reviewing the literature based on three criteria related to the proposed model. First, we present load-balancing algorithms used in multiple different contexts, such as multiprocessor task scheduling or application client traffic as well as sharded databases. This will mainly provide an explanation for the different and most common types of algorithms used in the simulation for performance comparison, specifically the centralized algorithms as defined later. This will allow us to understand how these algorithms function, how fast they converge, which conditions make convergence possible

and which prevent it, and what advantages and disadvantages do they have in terms of performance, both in terms of efficiency of the balancing and the time complexity of the process.

Load-balancing algorithms

There are two types of load-balancing algorithms, static and dynamic [13]. Static algorithms assume that all account data is known and assigns them to the corresponding shards before run-time. Dynamic algorithms on the other hand take into consideration the current state of the network and assign the nodes to the shards based on this state. A hybrid approach of those types would be to query the current state periodically and apply load-balancing when needed.

We will next present the most relevant algorithms used in the literature for load-balancing, irrespective of the context.

2.1. Common Load-balancing Algorithms:

Common static algorithms used for load-balancing include Round-Robin, First-Come-First-Served and Shortest-Load-First [15]. The main issue with those algorithms is their dependency on prediction of loads beforehand and as such are unsuitable for the case of blockchains. This applies to most static load-balancing algorithms and as such, unless they are called periodically over lapses of time, they would not perform proper load-balancing.

2.2. Longest Processing Time (LPT):

The Longest Processing Time algorithm [16] is an approximation algorithm that was shown to be optimal under certain assumptions on the input. It consists of sorting all loads in decreasing order while keeping track of the current state of the recipient shards, it then iterates through the loads assigning them to the least loaded recipients. It manages to do so with a worst-case running time of O(nlogn) since it also requires sorting which uses the bulk of the algorithm's running time.

2.3. MULTIFIT Algorithm:

The MULTIFIT algorithm [17] was initially developed for identical multiprocessors task partitioning but can also be used to balance loads on a network of servers and thus on the sharding problem considered in this thesis. It partitions the set of values in n subsets such as the largest subset sum is as small as possible and does so by using the First-Fit-Decreasing (FFD) bin-packing algorithm repeatedly until it finds the smallest capacity for the bin-packing to fit into the n required subsets. Similarly, to the Longest Processing Time algorithm its execution would have to be done on a single shard. It has a worst-case running time of O(nlogn).

Review of load-balancing in the context of sharded blockchains:

In the work by A. Mizrahi and Rottenstreich [18], an approach is proposed that relies on clustering accounts based on the space requirements of each shard, selecting only the accounts with most frequent transactions for the clustering process. Then based on the mapping, the accounts are migrated to the corresponding shard. Although better performance in transaction processing time is obtained, the authors underline the issue of balancing both the load and the number of accounts per shard which can conflict with each other in the case of skewed data where

some accounts are responsible for most of the transactions and thus take a larger chunk of load in each shard, posing a risk to shard security by having a smaller shard account population.

In their paper, Sangyeon Woo et al. [19], a heuristic is used that depends on the fact that the quantity of gas expended in an Ethereum transaction represents its computational complexity and thus can be used to balance the loads instead of the number of transactions per account. At the time of writing, it was shown to improve upon existing account relocation mechanisms such as S-ACC and D-TX in terms of throughput and makespan.

In a subsequent publication authored by the same individuals, a dynamic load-balancing algorithm is employed, leveraging the previously mentioned heuristic of utilizing gas consumption as a gauge of workload [20]. Given the dynamic nature of this load-balancing algorithm, it depends on forecasting transaction workloads by accounts, employing an aging weight that diminishes over time as an account remains inactive in initiating transactions. Subsequently, the algorithm is periodically invoked to ascertain requisite redistributions. Superior performance metrics were achieved in both transaction throughput and makespan compared to the S-ACC algorithm, exhibiting an average enhancement of 9%.

In the paper by Li et al. [21], a prediction algorithm using Long Short-Term Memory (LSTM) is used to determine the load of incoming transactions in each shard as well as each account, considering only the accounts with most frequent transactions while grouping remaining ones into aggregate accounts, due to the infeasibility of doing so on every account. In a second step, the accounts are relocated with the goal of minimizing the variance between the loads of each shard. The accounts are sorted in decreasing order of the predicted loads thus moving from the most prolific nodes to the least, migrating the accounts while the variance obtained from doing so is smaller.

The work of Okanami et al. [22], propose a novel approach which consists of integrating a load balancing element in the validation process of the consensus algorithm of the blockchain. It does so by hosting a competitive game which focuses on solving an optimization load-balancing problem where accounts are the participants. To incentivize participation, a reward is offered to the account that submitted the best solution and to disincentivize malicious behavior a fee must be paid prior to participation thus punishing the malicious players who would provide an imbalanced solution. This approach has shown to significantly mitigate the consequences of selfish migrations which occur when a user decides to move to a less loaded shard for better throughput, resulting in widespread imbalance over the different shards hampering its overall performance.

In Krol et al. [23], the authors implement a prototype that provides account migration from one shard to the other and adds a recommendation system for these migrations. It does so by assigning to each account an alignment vector, which shows how strongly a vector is aligned to a specific shard, this vector as well as the vector of the receiving address of the transaction are increased by the cost of the transaction. The alignment vectors of a shard's accounts are then provided to all miners located on this shard. Those vectors are only maintained for an interval of 100 blocks to the execution and only include the most active accounts. The prototype then selects a shard to which migrations will be made, opting for the least loaded shard with an account involved in the current transaction, in the case of new accounts choosing the least loaded shard in the network. The prototype was shown through testing to improve upon overall transaction throughput of the blockchain while remaining comparatively lightweight due to its integration with existing protocols used by miners.

In [24], the authors attempt to solve the load-balancing problem in sharded blockchain with the goal of minimizing cross-shard transactions, which result in significant overhead, by periodically executing two different allocation algorithms with different frequencies. The former optimizes the account allocation based on all historical account data whereas the latter is executed more frequently and executes allocation based on the former's results focusing only on incoming transactions. The first algorithm turns the optimization problem into the community detection problem and uses the Louvain method to solve it in O(nk + nlog) time complexity. The second is an adaptive algorithm, that instead of considering the entire historical data, only considers the last account to shard mapping and the incoming transaction, thus executing faster than the former with constant O(1) time complexity. Through simulation, the authors showed that the number of cross-shard transactions was reduced from 98% to 12% thus improving overall throughput.

Review of diffusion algorithms in the context of load-balancing.

Diffusion algorithms were first introduced by Cybenko [25] in 1989 as a way to perform dynamic loadbalancing on multiprocessors. Each processor is represented by a node in a connected graph so that every 2 processors are linked, the edges between each processor are stored in an adjacency matrix of the network. It consists of using the following formula to calculate load transfers that must be made from one processor to its neighbors:

$$w_i^{t+1} = w_i^t + \sum_{j}^{N} \propto_{ij} (w_j^t - w_i^t)$$

Equation 1: Computing load Transfers

The variable w_i^t represents the workload of processor *i* at epoch t, the summation iterates over every neighbor of the currently examined processor *i* and α_{ij} represents a weight matrix fulfilling the condition that $\alpha_{ij} = 0$ only if the 2 processors are not connected to each other. Should w_i^{t+1} be positive then processor *i* should transfer that specific workload w_i^{t+1} to processor *j*, otherwise it is processor *j* that should send this workload to processor *i* to achieve balance between the 2. This algorithm is decentralized by nature as each processor is responsible for its own load transfers. Repeatedly applying the algorithm should eventually lead to convergence though it may suffer from the step problem which occurs when the loads are not infinitely divisible. This type of algorithms would be shown to be stable by Berenbrink et al. [26], thus the total load of the system is bounded over time with an asymptotically tight upper bound. The main condition for this stability is that the graph must be connected.

The problem diffusion algorithms try to solve is known as the Distributed Averaging Consensus problem, which is mainly relevant to large-scale decentralized networks such as blockchains. The following two papers by Xiao et al. [27] [28] focus on the selection of the weight matrix used by the algorithm so as to ensure convergence. The first paper proposes the use of the Metropolis-Hastings matrix defined in the following formula:

$$W_{ij}(t) = \begin{cases} \frac{1}{(1 + \max\{di(t), dj(t)\})} & \{i, j\} \text{ are neighbors} \\ 1 - \sum_{k \in Ni(t)} W_{ik}(t) \text{ if } & i = j \\ 0 & \text{otherwise} \\ Equation 2: Metropolis Hastings Matrix} \end{cases}$$

The weight of each edge is 1 over the max degree of the 2 vertices + 1, the matrix is therefore stochastic as the sum of each row is equal to 1 and the condition specified earlier, $\alpha_{ij} = 0$, is respected as the weight at index ij is equal to 0 when i and j are not connected by an edge. In the second paper by Xiao et al., they propose solving the least-mean-square consensus problem to determine the weight matrix's values instead of the Metropolis-Hastings matrix. The Metropolis-Hastings matrix was shown to have slower convergence than the resulting matrix on average.

To solve the step problem that can occur in diffusion algorithms, [29] Jeannot & Vernier propose an approach using simulated annealing which can detect whether the algorithm would converge thus eliminating the step problem. The convergence detection algorithm was shown to provide, in the best case, a 100% performance increase while not degrading the diffusion algorithm's performance.

In [30] Lopes et al., provide an analysis is provided for diffusion algorithms on distributed networks, mainly the effect cooperation between individual nodes has on the performance of distributed averaging estimation-wise. The authors implement an algorithm they describe as an adaptive version of the diffusion algorithm. They do so by implementing local combiners that aggregate the weight on the scale of a node and its' neighbors, the combiners are then fused with an adaptive network layer that detects changes in the neighborhood of each combiner, updating the values of the combiners accordingly. T

The paper by Shi and Johansson [31] explores the conditions surrounding finite-time consensus in averaging algorithms by comparing it with consensus in maximization and minimization algorithms. The graph considered for the analysis has a time-varying topology, which means that some edges might be added or removed, similar to a network losing its connection to a node because of external events. They then demonstrate that there exists necessary conditions for reaching a global finite-time consensus. The first is that the initial values of each node

belong to a set of integers they call the consensus manifold which is one-dimensional. This means that reaching a global finite-time consensus is almost impossible. The second condition is that the graph is quasi-strongly connected, which means that the graph contains a node that is linked that can reach every other node in the network.

In Schwarz et al. [32], the authors attempt to determine the sufficient and necessary conditions for convergence of average consensus algorithms using a Metropolis-Hastings weight matrix. Instead of arbitrarily adding one to the denominator, the variable ε is summed with the maximum degree of both nodes. By modifying this variable, they identify both sufficient and necessary conditions for convergence on various topologies. They show that $\varepsilon > 0$ is a sufficient condition for convergence regardless of the topology. They also show that convergence is guaranteed with $\varepsilon = 0$ assuming that the graph is not regular and bipartite. In the case of regular and bipartite graphs, they show that the probability of convergence decreases exponentially as the total size of the graph increases. They also show that the optimum is roughly reached for these topologies with ε inversely proportional to the number of nodes in the network.

$$W_{ij}(\varepsilon) = \begin{cases} \frac{1}{(\varepsilon + \max\{di(t), dj(t)\})} & \{i, j\} \in \varepsilon \\ 1 - \sum_{k \in Ni(t)} W_{ik}(\varepsilon) & if \quad \{i, j\} \notin \varepsilon \text{ and } k \neq i \\ 0 & otherwise \\ Equation 3: Generalized Metropolis Hastings Matrix \end{cases}$$

In Toulouse et al. [33], diffusion algorithms are for the first time considered to tackle the load-balancing problem of sharded blockchains. They define the problem as such: the network is represented by an undirected graph where each shard corresponds to a vertex and edges correspond to direct connections between graphs. The network topology considered is a ring network, a 2-regular graph, where each vertex has exactly 2 neighbors. D-regular graphs are graphs where every vertex has a degree of D and the ring network topology used is an example of a 2-regular matrix. The Metropolis-Hastings matrix defined earlier is used as the weight matrix with which the calculation of required load transfers is made, in the case of a ring network, it can be determined that each row would contain exactly 3 entries that are not 0, those entries would be equal to 0.33 on the condition that $a_{ij} = 1$ or i = j. As such the matrix obtained is row-stochastic and fulfills the condition that a weight is equal to 0 only if both vertices are not connected. They provide the following pseudocode that is executed in parallel by each shard:

The variable Δi represents the transfer vector of shard *i*, *Ni* represents the number of neighbors of *i*, W represents the weight matrix, t represents number of iterations in which the transfer vector is filled and the resulting load from the transfer calculated. The authors assume convergence is reached when the sum of all initial loads is divided by the number of shards, so the total load of the system is over the number of shards. Positive entries in Δi mean that shard *i* should transfer a load of size $\Delta i[j]$ to neighbor *j*, negative ones mean that shard *i* should instead be receiving this load amount from neighbor *j*. The authors then test the algorithm and show that the approach results in similar output to other load-balancing algorithms such as LPT and MULTIFIT while being faster, though they note several shortcomings such as the heuristic used to select and migrate the account nodes that can result in different values than the ones computed in the diffusion algorithm, they also note that the topology of the network may negatively affect the migration process as shards can only migrate to neighboring ones in the topology.

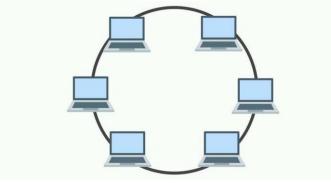


Figure 4: Ring network topology example

In Toulouse et al. [13], the authors try to improve upon the last attempt, mainly by using actual blockchain data rather than artificially generated datasets for the experiments, and propose another version of the diffusion

algorithm that computes the loads in an optimal number of iterations. They use a migration algorithm based on a heuristic that consists of sorting in decreasing order the account balances on each shard and migrating every shard that is smaller or equal to the transfer vector. The implementation is dynamic in the sense that it attempts loadprediction to add new transactions in real time using long short-term memory (LSTM) neural networks, instead of periodically redistributing the accounts among the shards. The network topologies used in the experiments are 2-regular ring networks, torus networks and line networks. The results obtained show that both algorithms are not successful in the improvement of the load-balancing among the shards.

3. Model Description

In this section, we will first be providing a comprehensive description of the proposed model and how the algorithms used are implemented, and providing explanations for some decisions in its design, specifically the numerous parameters added to regular diffusion algorithms in order to facilitate convergence and improve performance. Then in a second part, we will be presenting a use case in which the model is applied to an imbalanced sharded blockchain and walk through the steps required to apply it. It is worth noting that the model in question assumes that account location and migration within the shards are not in the control of individual nodes, as such, malicious behavior may mainly manifest from extremely rare cases where more than 51% of a shard's load belongs to a group of colluding malicious accounts which could potentially take over the shard. This decision also prevents selfish behavior from account nodes attempting to move to other shards for better throughput and rewards. Due to this assumption, it is fair to assume that potential collusion would have to take place after colluding nodes have both been added to the same shard and not be premeditated or spontaneously provoked by voluntary migration. Also, account nodes which would attempt shard takeover would have to attempt.

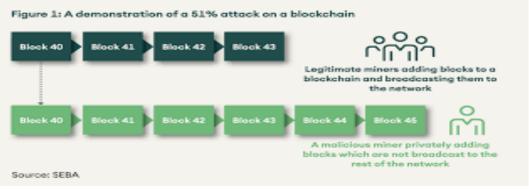


Figure 5: 51% attack example

Another assumption is that the model is expected to be called periodically, the addition of new transactions and accounts is not implemented, though we assume that newly created accounts are added to the shard with the least number of nodes. This is to ensure that when the model is used, the resulting model is as balanced as possible in terms of number of nodes per shard, and thus to avoid the possibility that some shards are much less populous than others increasing risks of a 51% attack from malicious accounts.

3.1 Model Overview:

The model will first preprocess Ethereum transaction data extracting the address of the sender and the gas consumed by a transaction. The transactions will then be grouped per their account address. This will then give us the account balance of each account and the total load an account has on the network. We then normalize the gas values by dividing them by 21000 which is the minimum amount of gas that must be provided by an Ethereum transaction.

We then decide upon the number of shards to be used to partition the network. Since each shard is expected to be relatively large, we can assume that all shards are directly connected to each other, so we can assume that the graph is (n-1)-regular for n shards. The reason for this assumption is that it only takes one account of a shard to be reachable from any one node from another shard for both shards to be considered connected. In the case of large, decentralized peer-to-peer networks such as Ethereum, this assumption is evident, and the possibility that a shard is disconnected from any other shard in the blockchain is extremely improbable. This topology is therefore a more accurate representation of load-balancing sharded blockchains such as Ethereum. A possible downside of

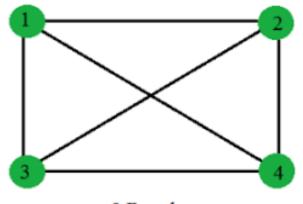
that approach is the overhead that would result from making load access for transfer vectors mutually exclusive, possibly slowing down the algorithm for the sake of consistency and correctness.

To properly simulate and demonstrate the effectiveness of the algorithm, the accounts are randomly split in n shards of equal account capacity. As such, the shards are balanced account-wise but since not all accounts have the same load, the shards obtained are not necessarily balanced. This provides us with a good way to test the enhanced diffusion algorithm.

Diffusion algorithm(i, Ni, n, W, workload of shard i)					
1.int Loadi(0) = workload of shard i					
2.int $t = 0$					
3.float $\Delta i[n] = 0$					
4.while no convergence:					
5. for each neighbor of i:					
6 $.\Delta i[j](t+1) = \Delta i[j](t) + wij (Loadi(t) - Loadj (t))$					
7. Loadi $(t + 1) = \text{Loadi}(t) - \sum j \in \text{Ni wij} (\text{Loadi}(t) - \text{Loadj}(t))$					
8. $t = t + 1$					

Algorithm 1: Pseudocode for regular diffusion algorithm [33]

The pseudocode defined earlier by Toulouse et al. is used, though to establish convergence, we make use of the standard deviation of the shard distribution as an additional tuning parameter. The reason for this decision is because the amount of data and the dependence of diffusion algorithms on the initial loads, convergence is not always guaranteed on the global average of the network. The standard deviation is therefore a good measure of proximity to the average of the network.



3 Regular

Figure 6: Example of d-regular graph.

We use the Generalized Metropolis-Hastings matrix to obtain the weights, to do so we take the adjacency matrix of the network and compute the weights accordingly. To maximize the probability of convergence we set $\varepsilon = \frac{1}{n}$, where n is the number of shards as was shown in [32] to be roughly equal to the optimum needed to guarantee convergence in regular graphs of larger size.

$$W_{ij}(1/n) = \begin{cases} \frac{1}{(1/n + \max\{di(t), dj(t)\})} & \{i, j\} \text{ are neighbors} \\ 1 - \sum_{k \in Ni(t)} W_{ik} (1/n) \text{ if } & i = j \text{ and } k \neq i \\ 0 & \text{otherwise} \\ Equation 4: Weight Matrix used for n-regular network.} \end{cases}$$

By doing so we allow each shard to transfer load directly to shards with a load too small regarding the threshold given by the distance of the standard deviation with the global average. This makes convergence faster and significantly more frequent than selecting the average, based on the randomness of initial shard loads, convergence in the average may not be possible.

We also added two shard selection algorithms, one checks which shards are overloaded and the other one which are underloaded, by applying the convergence rule defined earlier. This means that the diffusion and migration algorithms have to go through at least two epochs, one for the smaller loads and one for the larger ones. As such we added a parameter that handles the alternation between both types of epochs. To properly apply this parameter, we flip the value after each selection, where a value of 0 would mean that larger loads must be selected whereas 1 would mean the smaller loads have to be selected. This decision was made to avoid unnecessary overhead caused by creating additional threads for shards that have already converged with the global average, it will also be beneficial to greatly decrease the migration process by only selecting the shards that have to send loads to neighboring ones.

We then call the diffusion algorithm on each selected shard in parallel and fill their transfer vectors, thus designating how much load must be migrated from one shard to the other. Once the transfer vectors have been computed we can then start migrating nodes from overloaded shards to less loaded ones to achieve overall better balance. To do the migration, each shard must select candidate accounts for transfer, the heuristic used to do so is to sort each shard's accounts in decreasing order of load and selecting a node whenever it is smaller or equal to the transfer required, the node is then migrated and both shards updated. Contrarily to the migration algorithms, parallel diffusion is possible since the transfer vectors computed are stored in different parts of a greater list, each dedicated to its shard's required transfers, thus it is impossible for more than a single entity to read and write to the same transfer vector location in memory and causing a race condition in the process.

This approach to migration can be an issue, as in the case where the transfer is extremely small, the migration would take more time since it would have to iterate over every account, hampering overall performance. To alleviate this, we decided to add a tuning parameter, similar to the one used for convergence which is to place a margin of acceptable imbalance based on a fraction of the global average. Although this makes perfect balance impossible, it also avoids the significant decrease in performance required to achieve it on distributed systems, and accelerates consensus while guaranteeing an upper bound of imbalance of our choice, which in extreme cases would revert to parameter-less migration.

We have also implemented a second migration algorithm for testing that uses a different heuristic. The heuristic used consists of sorting each shard's nodes in decreasing order of size, only instead of iterating over the entire shard, it fetches the first transaction with a load of the size of the transfer to be made. This proves useful in the case of smaller transfer loads as it avoids iterating through all nodes preceding it. We will also be comparing the performance both in terms of accuracy and time taken of both heuristics for migration and show the cases in which one would be preferable to the other.

A helper function has also been added in case the number of designated epochs is more than necessary to reach load-balance. It consists of verifying before execution if all shards converged within a margin bounded by the distance to the initial standard deviation of the model that was defined before the first epoch. This parameter helps us regulate the total imbalance of the network while making sure that the algorithm finishes execution. The reason for this choice is that using the latest standard deviation could result in values between 0 and 1 which would be impossible to achieve and would prevent convergence.

3.2 Model Use Example:

To show how the model works we will be presenting the case of a sharded Blockchain similar in its functioning to Ethereum. As such, there exists a resource expended on transactions proportional to the computational expenditure of the processing of the transaction in the network such as the gas resource does in the Ethereum network. The Blockchain shards in question are also in need of balancing and the approach taken is account-based, which means that we are not only trying to balance the loads of the accounts but the number of accounts as well. We decided to divide the network into 5 shards for this example.

We assume that in its initial state, the sample Blockchain is sharded in such a way that every shard has approximately the same number of accounts, but accordingly, the total load of the accounts located on each shard is imbalanced above a certain pre-defined ratio. We decided to arbitrarily define the measure of tolerable imbalance as a fraction of the initial standard deviation instead of trying to eliminate all load imbalance. In this example we choose 0.1 of the initial standard deviation.

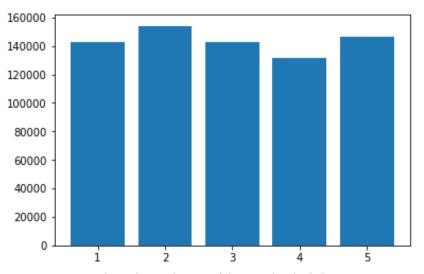


Figure 7: Initial Load Distribution of the sample Blockchain:

We then compute the Generalized Metropolis-Hastings matrix as defined earlier in [32] to get the weights of a (n-1)-regular graph in which each vertex corresponds to a shard itself pointing to the accounts within it. We decided as per the recommendation of the paper introducing it, to define $\varepsilon = \frac{1}{n} = \frac{1}{5}$, this is to increase the chance of convergence of the diffusion algorithm.

The diffusion and migration process will iterate over a finite number of epochs alternating between shards which have a higher load than the maximum tolerable one and those that have a smaller load than the minimum tolerable load, until the total imbalance of the network among all shards is within the tolerable threshold set by the initial standard deviation.

On each iteration (or epoch), we first check which specific shards need balancing and apply diffusion to these shards in parallel. We also compute the current standard deviation which decreases on each iteration, this should allow us to reach more accurate results with each epoch until the fraction of the initial standard deviation is reached. This fraction can be modified based on the minimum imbalance tolerated by the Blockchain in regard to its shards.

The diffusion algorithm will then attempt to calculate the transfer vector of the current shard, to determine how much to send to the other shards. To avoid sending accounts to already overloaded shards, the shards are sorted from the lowest loaded to the highest loaded in case the current shard is overloaded and from the highest loaded to the lowest loaded in case the current shard is underloaded. This optimization is used to ensure that the neediest shards are selected first on each iteration of the loop. The transfer vector values from the current shard to the other ones are computed by applying the previously defined weights to the current loads of the concerned shards, the currently examined neighbor and the shard itself.

Once the current shard is shown to converge, the loop ends, and the final transfer vector is computed. The next step will consist of converting the negative values within the vectors to positive ones for the corresponding shards. As such migration will be able to focus on sending rather than implementing both sending and receiving tasks.

In the final step, we iterate over every shard and apply the migration procedure, transferring excess loads to less loaded shards until the transfer vectors of each shard consists only of zeroes. Thus, on each successive iteration of the diffusion/migration process, the total imbalance of the network decreases until it reaches the tolerated threshold. In the specific example given, the process resulted in reaching a total imbalance of 1346 among the different shards as opposed to the initial imbalance of 26502. The (n-1)-regular topology also proved to be faster than the 2-regular topology, both in terms of number of iterations and running time (0.37 seconds vs 0.25 seconds, 7 epochs vs 20 epochs).

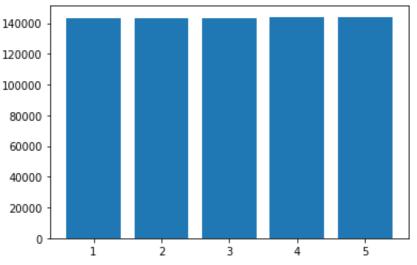


Figure 8: Final load distribution of the sample Blockchain.

4. Experimental results

In this section, we will start by eliciting the software used in the simulation. Then we will move on to how the testing data was gathered and processed for use in the experiment, as well as underline the different algorithms and settings used in the comparative analysis. We will then finalize the chapter by showing the obtained results using bar graphs.

4.1 Simulation software used:

The Jupyter Notebook web interface was used to conduct the simulation and tests in Python 3. The choice of Python was based on the assumption that the algorithm showcased would require sufficient data processing capabilities and quality of life features to reliably be tested and compared. Since Python libraries such as Pandas and Numpy offer such functionalities, it proved a better choice than taking advantage of the Object-Oriented capabilities of Java which was initially chosen for the simulation.

4.2 Data preparation:

To reliably conduct the simulation, data was obtained through two different methods. The first consists, of crawling Etherscan [34] which is a platform used to explore Ethereum blocks and transactions. To simplify the crawling process, BitQuery Builder [14] was used to make the API calls and extract the transactions within specific intervals. The crawling was therefore done three times each underlining the 25,000 first transactions to be processed within each month of a three-month period. The second method was to obtain static data from Kaggle [35] containing 100,000 last transactions, the main advantage of this method is the size of the obtained dataset as BitQuery does not allow retrieval of more than 25,000 transactions per call.

After obtaining the data, we filter out transaction data irrelevant to the simulation and keep the gas expenses and address of the sender of each transaction. We then group the total gas expenses per account address. Any outstanding account with more than 0.2. ether of total gas expenses is removed to avoid bias and for experimentation's sake.

To further simplify the data presented, we decided to divide all gas expenses by the minimum amount of gas expenses allowed which is 21,000 gwei; thus, 0.000021 ether, so that accounts that only did a single transaction would have a gas expenditure of 1. This will be useful to improve upon the performance of the second migration algorithm proposed.

4.3 Algorithms and simulation metrics:

The algorithms used and implemented for comparison are Longest Processing Time, 2-regular ring network diffusion and (n-1)-regular network diffusion. In addition, to properly conduct comparisons, both migration algorithms are tested and included in both diffusion algorithms. We will be testing each algorithm on 5, 6, 7, 8, 9 and 10 shards on all four datasets. The parameters used for convergence will be set to 0.2 of the initial standard deviation of the tested allocation for cases where the number of shards is equal to or higher than 8 shards and 0.1 for strictly lower than eight. We take this measure to maximize chances for convergence. The metrics used to determine the performance are the total load imbalance among shards multiplied by the execution time of the

process. A lower product would therefore correspond to better performance. We will also be using the total account number imbalance between the shards as a metric and try to derive correlations with the load imbalance as well as the number of shards.

The results for each dataset will be shown separately in their own graph, the x-axis will represent the number of shards used in the test and the y-axis will represent the total load imbalance for this specific shard on both algorithms. Therefore, we will be able to compare side by side the overall performance of both algorithms on a given number of shards, we will also be providing dataset characteristics such as the total number of accounts, the total load of accounts as well as the ideal global average for this shard configuration. It should be taken into consideration that there is a significant dependency of the result of a shard configuration on the initial shuffling and distribution of accounts on each shard. Theefore, results might be inconsistent between different shard numbers, though we will be underlining major trends across the tests by focusing on each algorithm's performance on the same initial distribution. For the purpose of this experiment, we conducted and recorded a total of 48 test configurations.

4.4 Results:

In this section we will show the results of the simulations each dataset at a time. Each subsection will specify the characteristics of the examined dataset, eliciting the dataset size, both in number of accounts and load and the ideal average for the specified shard size. Then we will be providing a table for each dataset showing the results obtained for each shard configuration with each algorithm. A line diagram will be given to further visualize the results and check for specific patterns across the different datasets.

4.4.1. Dataset 1:

The first dataset contains data from 48857 Ethereum accounts with a total load of 718312 across all accounts. The ideal average load for shard counts of 5, 6, 7, 8, 9 and 10 are respectively, 143662, 119719, 102616, 89789, 79812 and 71831.

Topology 🛛 🔽	# shards 🛛 💌 E	pochs 🛛 💌 E	execution Time 🛛 💌	Score 🗾 💌	Account Imbalanc
2-regular	5	4	0.38	320	14
2-regular	6	4	0.69	1328	15
2-regular	7	5	0.56	684	17
2-regular	8	4	0.73	4855	49
2-regular	9	8	1.47	8867	44
2-regular	10	5	0.57	2173	47
(n-1)-regular	5	3	0.17	348	40
(n-1)-regular	6	3	0.26	787	61
(n-1)-regular	7	3	0.36	515	87
(n-1)-regular	8	3	0.37	2028	109
(n-1)-regular	9	3	0.37	2112	110
(n-1)-regular	10	3	0.38	1716	133

Figure 9: Dataset 1 table.

As can be seen the (n-1)-regular diffusion algorithm outperforms the 2-regular one on all shard sizes in term of total execution time. It also manages to get a better score on every shard number aside from a size of 5 shards. It is worth noting that although better score and execution times are obtained using (n-1)-regular diffusion, we obtain increased imbalance in terms of account numbers in every shard size. We also notice that some shard numbers obtained considerably worse scores on certain shard configurations with both algorithms, though that might be due to the initial distribution of the accounts among the shards and the fact that perfect account number balance might not be reachable for every shard size which results in the initial distribution to start slightly overloaded or underloaded in terms of account number.

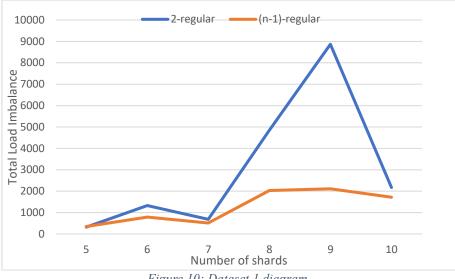


Figure 10: Dataset 1 diagram.

4.4.2. Dataset 2:

The second dataset used consists of 9407 accounts with a total load of 350924. The ideal target load across the shards for each number of shards are 70185, 58487, 50132, 43866, 38992 and 35092, for 5, 6, 7, 8, 9 and 10 shards respectively.

Topology	# shards 🛛 💌	Epochs 📃 💌	Execution Time 🛛 💌	Score 🗾 💌	Account Imbalance 💌
2-regular	5	4	0.32	486	26
2-regular	6	6	0.29	230	31
2-regular	7	5	0.39	762	43
2-regular	8	4	0.35	764	23
2-regular	9	4	0.41	885	24
2-regular	10	4	0.81	9686	61
(n-1)-regular	5	5	0.17	381	56
(n-1)-regular	6	3	0.15	319	67
(n-1)-regular	7	3	0.22	407	89
(n-1)-regular	8	3	0.25	962	109
(n-1)-regular	9	3	0.22	705	100
(n-1)-regular	10	3	0.39	2471	187

Figure 11: Dataset 2 table.

With this dataset, the observations that can be made are that the (n-1)-regular diffusion algorithm outperforms the 2-regular in terms of execution time on every test. However, on some specific shard sizes such as 6 and 8, the score is higher on 2-regular diffusion, this would mean that the imbalance is sufficiently higher on n-1-regular diffusion so that the extra execution time is compensated for by this imbalance on the 2-regular execution. Another remark to be made is that the higher the number of shards the worse the score becomes. Though it should be taken into consideration that this is the smallest dataset, therefore the small number of accounts means that dividing into a higher number of shards would intuitively be harder since not all accounts have the same load which would result in imbalance due to this inequality of load among accounts and the way account-based sharded blockchain work. For the purpose of this study it is worth noting that score for n-1-regular diffusion grows at a numerically slower rate than 2-regular diffusion (2471 and 9686 for size of 10 shards). Account number imbalance though is much higher on the n-1-regular model. We theorize this could possibly be due to the small dataset and significant inequalities in account loads, as additional accounts might have to be displaced to compensate for the most loaded ones resulting in multiple accounts being moved between shards.

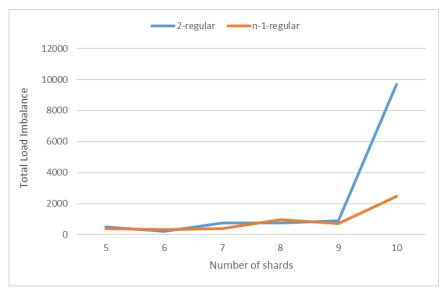


Figure 12: Dataset 2 diagram.

4.4.3. Dataset 3:

The third dataset consists of 24984 accounts with a total load of 821224, the ideal load to reach for sizes of 5, 6, 7, 8, 9 and 10 shards are respectively, 164245, 136871, 117318, 102653, 91247 and 82122.

Topology 🛛 🔽	# shards 🛛 💌 🛙	Epochs 🛛 💌 E	xecution Time 🛛 💌	Score 🗾 💌	Account Imbalance 💌
2-regular	5	4	0.83	1671	19
2-regular	6	6	0.53	999	20
2-regular	7	6	0.49	649	13
2-regular	8	6	0.84	5676	38
2-regular	9	6	0.65	4360	40
2-regular	10	5	0.51	2396	44
(n-1)-regular	5	3	0.21	1089	43
(n-1)-regular	6	3	0.29	456	62
(n-1)-regular	7	3	0.23	606	86
(n-1)-regular	8	3	0.33	1616	92
(n-1)-regular	9	3	0.31	1696	110
(n-1)-regular	10	3	0.33	1422	126

Figure 13: Dataset 3 table.

We can make the following observations from this dataset. First, (n-1)-regular diffusion performs better than 2-regular diffusion on all tests in terms of execution time, on the other hand, account imbalance is higher with (n-1)-regular diffusion due to the larger number of migrations enacted to reach the final loads. Another observation to be made, is that (n-1)-regular also performs better in terms of score though the significance of that improvement seems largely dependent on the initial distribution as well as the shard count used. In the above test, for example, on a shard count of 7 and the same initial distribution for both diffusion algorithms, (n-1)-regular diffusion performs only slightly better than 2-regular diffusion with a score of 606 against a score of 649, while the account imbalance is much higher on the former than the latter. In this specific test, it can be safely assumed that 2-regular diffusion is overall more effective than (n-1)-regular. In other tests however, such as 8 and 9 shard tests, (n-1)-regular diffusion performs significantly better than 2-regular diffusion in terms of score while having a marginally worse account imbalance. So far, with the observations made in this dataset, there seems to be a tradeoff between using 2-regular diffusion and (n-1)-regular diffusion, in terms of whether the blockchain network in question would favorize better balance in terms of load for better performance or better security through better balancing of individual accounts throughout the shards.



Figure 14: Dataset 3 diagram.

4.4.4. Dataset 4:

The last dataset consists of 24988 accounts with a total computational load of 820239 among them. The ideal loads are respectively, 164048, 136707, 117177, 102530, 91138 and 82024, for 5, 6, 7, 8, 9 and 10 shards.

Topology	🖌 # shards 🔤 🔽	Epochs 🗾 💌	Execution Time 🛛 💌	Score 🗾 💌	Account Imbalance 💌	
2-regular	Ľ	5 4	0.36	474	14	
2-regular	6	5 5	0.34	762	20	
2-regular		7 5	0.4	802	20	
2-regular	5	3 4	0.96	5532	30	
2-regular	9) 7	2.23	31907	72	
2-regular	10	0 10	1.83	15169	72	
(n-1)-regular	Ľ	5 3	0.15	478	36	
(n-1)-regular	6	5 5	0.25	479	76	
(n-1)-regular		7 3	0.25	446	90	
(n-1)-regular	5	3 3	0.27	1917	102	
(n-1)-regular	ç) 3	0.42	5246	152	
(n-1)-regular	10) 3	0.32	3155	174	
Figure 15: Dataset A table						

Figure 15: Dataset 4 table.

Similarly, as on the previous tests, execution time is shorter throughout the tests on (n-1)-regular diffusion. On the other hand, on the size of 5 shards, 2-regular diffusion performs better in terms of score and account imbalance, though as the number of shards increases, (n-1)-regular diffusion becomes preferable in terms of score culminating with a score of 5246 on (n-1)-regular diffusion against a score of 31907 on 2-regular diffusion for a shard number of 9. An important observation to be made is that shard size 10 results in a better score and execution time than shard size 9 with both diffusion algorithms, implying that the number of shards has a significant impact on the performance of diffusion algorithms, mainly when the total load divided by the number of shards results in an infinite quotient which would only be attainable approximately due to rounding which would result in a margin of error. It would therefore seem that selecting round numbers by which any possible load is divisible is preferable when selecting the number of shards for a blockchain.



Figure 16: Dataset 4 diagram.

4.5. Overall observations:

Throughout the conducted experiments, some general observations can be made about the relative performance of our (n-1)-regular diffusion algorithm with the 2-regular diffusion that has been considered to do load-balancing on blockchains such as Ethereum. Firstly, the (n-1)-regular variant is consistently more performant in terms of execution time throughout all datasets and shard counts. Secondly, as the number of shards increases in the network, the improvements introduced by the (n-1) variant, both in terms of load-balancing and execution time, become much more significant, whereas 2-regular diffusion performance greatly decreases. Lastly, in terms of account balancing, the improvement of the load-balancing comes at a higher cost. Due to the greater number of migrations caused by the topology where every shard is interlinked to all others, the account balance is significantly worse. Thus, there seems to be a tradeoff between using both algorithms.

Another slight observation to be made is the effect of the specific number of shards on all diffusion algorithms' performance, with some higher numbers performing significantly better than lower ones. We theorize that the number of shards a blockchain network is to be divided into should be carefully selected as not to cause rounding errors due to infinite quotients as ideal averages.

5. Conclusion and Future Work

In this paper, we presented an alternative implementation and imagining of the load-balancing diffusion algorithms for sharded blockchains. The solution proposed retains the main advantage in its distributed, decentralized nature while improving upon the performance of the load-balancing procedure by considering a different network topology whereby all shards are connected to each other, to reduce the number of iterations the diffusion algorithm would need to execute the load-balancing.

Despite the apparent advancements, these enhancements entail potential consequences concerning account balance, and in certain extreme scenarios characterized by disparities in account workloads, they may render shards more susceptible to 51% attacks. We suggest future work on the matter of sharded blockchains should introduce mechanisms to detect such risky situations and to alleviate the effects by increasing the threshold under which a load imbalance is tolerable, thus sacrificing performance to maintain a dependable level of security. Another possible approach to this problem would be to introduce a tolerance threshold mechanism for the number of accounts per shards, this variable would then make sure that the minimum number of accounts over the shards stays above a certain value to guarantee the security of the shard. Upon breaching this threshold, the algorithm would then revert to its pre-diffusion state and increase the load threshold.

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