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الديناميكا الغير خطية لجسيمة تقع على قطع مكافيء يدور بواسطة مناهج عددية وشبه عددية

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الملخص:

في هذه الدراسة الديناميكا الغير خطية لجسيمة تقع على قطع مكافيء يدور تم تحليلها بواسطة مناهج عددية وشبه عددية. طريقة اتزان الطاقة (EBM) وطريقة اضطراب هوموتوبي (AFF) وصياغة السعة والتردد تم تطبيقها كنهج تحليلي وعند اذن علاقات التردد والسعة تم الحصول عليها. أن المعادلة التي تتحكم بحركة الجسيمة أيضا قد تم حلها بإستخدام طريقة التحويل التفاضية (DTM) وذلك في حالة النهج الشبه عددي. أن أثر المتغيرات الوسيطة المختلفة على معادلة التحكم قد تم تقيمها. كما إن مقارنة النتائج مع الحلول الدقيقة والعددية قد تمت فحصها. أن أداء ومقدرة كل طريقة قد تم كشفها ومناقشها.

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ORIGINAL ARTICLE

Nonlinear dynamics of a particle on a rotating parabola via the analytic and semi-analytic approaches

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Abstract In present study, nonlinear dynamics of a particle on a rotating parabola are analyzed by means of the analytic and semi-analytic approaches. The Energy balance method (EBM), homotopy perturbation method (HPM) and amplitude–frequency formulation (AFF) are applied as the analytic approaches and the frequency-amplitude relationships are obtained. The governing equation of motion is also solved by the differential transform method (DTM) as a semi-analytic approach. The effects of different parameters on the governing equation are evaluated. Comparison of results with exact and numerical solutions are investigated. the performance and capability of each method are revealed and discussed.

1. Introduction

Many phenomena in applied sciences and engineering expressed as nonlinear differential equations. This issue especially in mechanics and physics for dynamics and oscillations analysis is visible. In recent years, remarkable attention has been directed toward solutions of these nonlinear problems and researchers developed many methods. among these methods, some of them are proposed by Prof. He and called He's methods such as: energy balance method (He, 2002), homotopy perturbation method (He, 1999), amplitude–frequency formulation (He, 2008a), max–min approach (He, 2008b), Hamiltonian approach (He, 2010), variational approach (He, 2007, 2011a,b; Zhou and He, 2010), Parameter expanding method (He, 2006, 2008c) and variational iteration method (He et al., 2010). These analytic methods successfully served to analysis of nonlinear problems. for example; the oscillation of a mass attached to a stretched elastic wire (Durmaz et al., 2011; Xu, 2010), cantilever beam vibration with nonlinear boundary condition (Sedighi and Shirazi, 2012), nonlinear oscillations of a punctual charge in the electric field of a charged ring (Yildirim et al., 2011), analytical solution for magnetohydrodynamic flows of viscoelastic fluids in converging/diverging channels (Shadloo and Kimiaeifar, 2011) and many other problems (Askari et al., 2010; Belendez et al., 2009; Chen et al., 2011; Cveticanin, 2006; Ganji et al., 2010; Kimiaeifar et al., 2011; Mehdipour et al., 2010; Özis and Yildirim, 2007; Xu and He, 2010; Yazdi et al., 2010, 2012a,b; Younesian et al., 2010, 2011a,b; Zhang, 2009) are solved by carrying out the He's methods. Besides these methods, there exist other techniques for solving nonlinear problems, that one of them is the differential transform method. The Differential transform method is a semi-analytic

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method, based on Taylor expansion and does not require any linearization and small perturbation. This method has been exerted to structural dynamics (Kaya and Ozgumus, 2007; Yalcin et al., 2009), heat transfer problems (Yaghoobi and Torabi, 2011) and so on (El-Shahed, 2008; Momani and Ertürk, 2008).

In this study, the nonlinear dynamics of a particle on a rotating parabola are considered. The governing equation introduced by Nayfeh and Mook (1979):

$$u'' + 4q^2u^2u'' + \varepsilon^2 u + 4q^2uu'^2 = 0, \quad u(0) = A, \quad u'(0) = 0. \tag{1}$$

We solve Eq. (1) via the energy balance method, homotopy perturbation method, amplitude–frequency formulation and differential transform method and examine the advantages and disadvantages of each method by comparison of results with exact and fourth-order Runge–Kutta solutions.

2. Solution procedure

2.1. The energy balance method (EBM)

In energy balance method, first the variational principle obtained and then the Hamiltonian is constructed, finally by collocation method, one can yield the angular frequency.

For the nonlinear equation presented in Eq. (1), the variational principle can be obtained as:

$$J(u) = \int_0^t \left(-\frac{1}{2}\dot{u}^2(1 + 4q^2u^2) + \frac{\varepsilon^2}{2}u^2\right)dt. \tag{2}$$

Its Hamiltonian and residue$R$, therefore, can be written in the form:

$$H = \frac{1}{2}\dot{u}^2(1 + 4q^2u^2) + \frac{\varepsilon^2}{2}u^2 = \frac{\varepsilon^2}{2}A^2 \tag{3}$$

$$R(t) = \frac{1}{2}\dot{u}^2(1 + 4q^2u^2) + \frac{\varepsilon^2}{2}u^2 - \frac{\varepsilon^2}{2}A^2 = 0. \tag{4}$$

For satisfied initial condition in Eq. (1), assume the approximate solution in the form of:

$$u(t) = A\cos\omega t. \tag{5}$$

Substituting Eq. (5) into Eq. (4), yield:

$$R(t) = \frac{1}{2}(-\omega A\sin\omega t)^2(1 + 4q^2(A\cos\omega t)^2) + \frac{\varepsilon^2}{2}(A\cos\omega t)^2 - \frac{\varepsilon^2}{2}A^2 = 0. \tag{6}$$

From Eq. (6), we obtained the following result:

$$\omega = \frac{\sqrt{2}}{A\sin\omega t}\sqrt{\frac{\frac{\varepsilon^2}{2}A^2 - \frac{\varepsilon^2}{2}(A\cos\omega t)^2}{1 + 4q^2(A\cos\omega t)^2}}. \tag{7}$$

Finally collocation at $\omega t = \frac{\pi}{4}$ gives:

$$\omega_{\text{EBM}} = \frac{\varepsilon}{\sqrt{1 + 2q^2A^2}}. \tag{8}$$

2.2. The homotopy perturbation method (HPM)

Based on standard procedure of the homotopy perturbation method, we first by using Eq. (1) establish the following homotopy:

$$u'' + 1\cdot u = p[-\varepsilon^2u - 4q^2u^2u'' - 4q^2uu'^2 + u], \quad p \in [0, 1] \tag{9}$$

It is obvious that when $p = 0$, Eq. (9) becomes a linear ordinary differential equation and when $p = 1$, it becomes the original nonlinear equation. We consider u and 1 as series of p in the following form:

$$1 = \omega^2 - p\alpha_1 - p^2\alpha_2\ldots, \tag{10}$$

$$u = u_0 + pu_1 + p^2u_2\ldots, \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (9) yields:

$$(u_0 + pu_1 + p^2u_2\ldots)'' + (\omega^2)\cdot(u_0 + pu_1 + p^2u_2\ldots) = p[-\varepsilon^2(u_0 + pu_1 + p^2u_2\ldots) - 4q^2(u_0 + pu_1 + p^2u_2\ldots)^2(u_0 + pu_1 + p^2u_2\ldots)'' - 4q^2(u_0 + pu_1 + p^2u_2\ldots)(u_0 + pu_1 + p^2u_2\ldots)'^2 + (u_0 + pu_1 + p^2u_2\ldots)]. \tag{12}$$

By expanding Eq. (12) and collecting terms with same power, we can find two first linear equations with initial conditions as follows:

$$p^0 : u_0'' + \omega^2u_0 = 0, \quad u_0(0) = A, u_0'(0) = 0. \tag{13}$$

$$p^1 : u_1'' + \omega^2u_1 = -\varepsilon^2u_0 - 4q^2u_0^2u_0'' - 4q^2u_0u_0'^2 + u_0(1 + \alpha_1).$$
$$u_1(0) = u_1'(0) = 0. \tag{14}$$

Solving Eq. (13) gives:

$$u_0(t) = A\cos\omega t. \tag{15}$$

Substituting Eq. (15) into Eq. (14) yields:

$$u_1'' + \omega^2u_1 = -\varepsilon^2A\cos\omega t - 4q^2(A\cos\omega t)^2(-\omega^2A\cos\omega t) - 4q^2(A\cos\omega t)(-\omega A\sin\omega t)^2 + (A\cos\omega t)\times(1 + \alpha_1). \tag{16}$$

Avoiding secular term in u_1, requires:

$$\int_0^{\frac{2\pi}{\omega}}[-\varepsilon^2A\cos\omega t + 4q^2\omega^2A^3\cos^3\omega t - 4q^2\omega^2A^3\cos\omega t\sin^2\omega t + A(1 + \alpha_1)\cos\omega t]\cos\omega t = 0. \tag{17}$$

From Eq. (17) we obtain:

$$\alpha_1 = \varepsilon^2 - 1 - 2q^2A^2\omega^2. \tag{18}$$

Setting $p = 1$ in Eq. (10), we have:

$$\omega^2 = 1 + \alpha_1. \tag{19}$$

First-order approximate solution can be obtain by substituting Eq. (18) into Eq. (19) as:

$$\omega_{\text{HPM}} = \frac{\varepsilon}{\sqrt{1 + 2q^2A^2}}. \tag{20}$$

2.3. The amplitude–frequency formulation (AFF)

Based on standard procedure of the amplitude–frequency formulation, we consider two trial functions $u_1(t) = A\cos t$ and $u_2(t) = A\cos\omega t$, respectively. Which are the solutions of the following linear equations:

$$\ddot{u} + \omega_1^2u = 0, \qquad \omega_1^2 = 1 \tag{21}$$

$$\ddot{u} + \omega_2^2u = 0, \qquad \omega_2^2 = \omega^2. \tag{22}$$

Substituting the mentioned trial functions into Eq. (1), results in the following residuals:

$$R_1(t) = -A\cos t - 4q^2A^3\cos^3 t + \varepsilon^2 A\cos t + 4q^2A^3\cos t\sin^2 t, \quad (23)$$

$$R_2(t) = -A\omega^2\cos\omega t - 4q^2A^3\omega^2\cos^3\omega t + \varepsilon^2 A\cos\omega t + 4q^2A^3\omega^2\cos\omega t\sin^2\omega t. \quad (24)$$

According to the AFF, the residuals rewritten in the form of weighted residuals as follows:

$$\widetilde{R}_1 = \frac{4}{T_1}\int_0^{\frac{T_1}{4}} R_1(t)\cos(\omega_1 t)dt = -\frac{1}{2}A(1-\varepsilon^2+2q^2A^2), \quad (25)$$

$$\widetilde{R}_2 = \frac{4}{T_2}\int_0^{\frac{T_2}{4}} R_2(t)\cos(\omega_2 t)dt = -\frac{1}{2}A(\omega^2-\varepsilon^2+2q^2A^2\omega^2). \quad (26)$$

The original amplitude–frequency formulation reads:

$$\omega^2 = \frac{\omega_1^2\widetilde{R}_2 - \omega_2^2\widetilde{R}_1}{\widetilde{R}_2 - \widetilde{R}_1} \quad (27)$$

Finally substituting Eqs. (25) and (26) into Eq. (27), the approximate frequency obtained as:

$$\omega_{\text{AFF}} = \frac{\varepsilon}{\sqrt{1+2q^2A^2}} \quad (28)$$

2.4. The differential transform method (DTM)

The basic operations of the differential transform method tabulated in Table 1.

Applying the differential transform method to the Eq. (1), the following recurrence relation is obtained:

$$(k+2)(k+1)U(k+2) + 4q^2\left[\sum_{s=0}^{k}\sum_{m=0}^{k-s}(m+2)(m+1)U(m+2)U(s)U(k-s-m)\right] + \varepsilon^2 U(k) + 4q^2\left[\sum_{s=0}^{k}\sum_{m=0}^{k-s}(m+1)U(m+1)(s+1)U(s+1)U(k-s-m)\right] = 0, \quad (29)$$

Also initial conditions in Eq. (1) transformed as :

$$U(0) = A, U(1) = 0. \quad (30)$$

From Eq. (29), for different values of k, the following recursive relation is obtained:

Table 1 The basic operations of differential transform method.

Original function	Transformed function
$u(t) = \alpha v(t) \pm \beta w(t)$	$U(k) = \alpha V(k) \pm \beta W(k)$
$u(t) = v(t)\cdot w(t)$	$U(k) = \sum_{s=0}^{k} V(s)\cdot W(k-s)$
$u(t) = \frac{d^m v(t)}{dt^m}$	$U(k) = \frac{(k+m)!}{k!}V(k+m)$
$u(t) = \exp(t)$	$U(k) = \frac{1}{k!}$

$$k=0: 2U(2) + 4q^2[2U(0)^2U(2)] + \varepsilon^2 U(0) + 4q^2[U(0)U(1)^2] = 0, \quad (31)$$

$$k=1: 6U(3) + 4q^2[4U(0)U(1)U(2) + 6U(0)^2U(3)] + \varepsilon^2 U(1) + 4q^2[4U(0)U(1)U(2) + U(1)^3] = 0, \quad (32)$$

...

We will have:

$$U(2) = -\frac{1}{2}\frac{\varepsilon^2 A}{(1+4q^2A^2)}, \quad (33)$$

$$U(3) = 0, \quad (34)$$

$$U(4) = \frac{1}{24}\frac{\varepsilon^4 A(1-12q^2A^2)}{(1+4q^2A^2)^3}, \quad (35)$$

$$U(5) = 0, \quad (36)$$

$$U(6) = -\frac{1}{720}\frac{\varepsilon^6 A(1-168q^2A^2+720q^4A^4)}{(1+4q^2A^2)^5}, \quad (37)$$

$$U(7) = 0, \quad (38)$$

$$U(8) = \frac{1}{40320}\frac{\varepsilon^8 A(1-1620q^2A^2+43056q^4A^4-100800q^6A^6)}{(1+4q^2A^2)^7}, \quad (39)$$

$$U(9) = 0, \quad (40)$$

$$U(10) = -\frac{\varepsilon^{10}A(1-13616q^2A^2+1447264q^4A^4-16546560q^6A^6+26046720q^8A^8)}{3628800(1+4q^2A^2)^9}, \quad (41)$$

The above process to determine coefficients of power series is continuous and closed form solution finally obtained as:

$$u(t) = U(0) + U(1)t + U(2)t^2 + U(3)t^3 + U(4)t^4 + \cdots \quad (42)$$

3. Results and discussion

In this section, the results of the mentioned methods are compared with exact and numerical solutions. Regarding to the past sections, the EBM, HPM and AFF yield to a same form of solution and frequency-amplitude relationship. From Eqs. 8, 20 and 28 the analytical period of motion obtained as:

$$T_{\text{anal}} = \frac{2\pi}{\omega_{\text{anal}}} = \frac{2\pi}{\varepsilon}\sqrt{1+2q^2A^2}. \quad (43)$$

The exact period of Eq. (1) is calculated by Wu et al. (2003):

$$T_{\text{exact}} = \frac{4}{\varepsilon}\int_0^{\frac{\pi}{2}}\sqrt{1+4q^2A^2\cos^2 t}\,dt. \quad (44)$$

Previously, He (2006) and Marinca and Herisanu (2006) obtained a similar period using max–min approach and modified iteration perturbation method, respectively. The maximum relative error of the period reach 10% even when $qA \to \infty$. Also Marinca and Herisanu (2010) have determined a periodic solution for this Equation by means of optimal homotopy asymptotic method. The comparison between the analytic and semi-analytic methods in conjunction with the fourth-order Runge–Kutta method, presented in Figs. 1–4.

It can be clearly observed that for larger value of parameters the results of EBM, HPM and AFF show some discrepancies in comparison with the obtained results using the fourth-

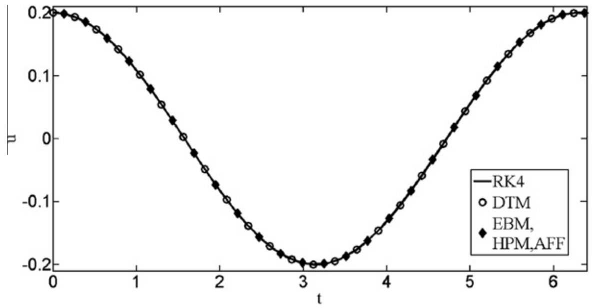

Figure 1 A comparison between the analytic and semi-analytic methods in conjunction with the fourth-order Runge–Kutta method for $\varepsilon = 1$, $q = 0.2$, $A = 0.2$.

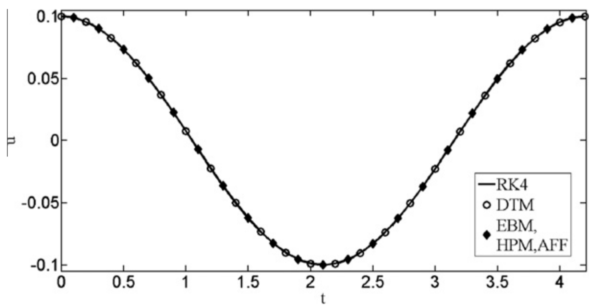

Figure 2 A comparison between the analytic and semi-analytic methods in conjunction with the fourth-order Runge–Kutta method for $\varepsilon = 1.5$, $q = 0.6$, $A = 0.1$.

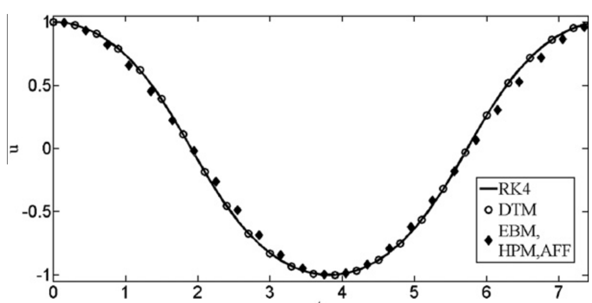

Figure 3 A comparison between the analytic and semi-analytic methods in conjunction with the fourth-order Runge–Kutta method for $\varepsilon = 1$, $q = 0.5$, $A = 1$.

order Runge–Kutta numerical method. Whereas the differential transform method can predict the solution with high accuracy even for large value of parameters. From Eq. (43), i.e., the obtained period from the governing equation of motion by the analytical approaches, we can investigate the effects of different parameters on the period. Easily be seen that when value of the ε decreases or value of the q increases and other parameters remain constant; the period increases. Also for constant value of the ε when multiple values of qA are constant, the period remains without changing. One can conclude that the ε have opposite effect on the period of motion in comparison with A, q. To verify this issue, we consider several values of the parameter and show results in Figs. 5–7. It should be noted that these results obtained using the DTM. Moreover, the influence of constant parameters on stability and phaseplane are investigated in Figs. 8 and 9.

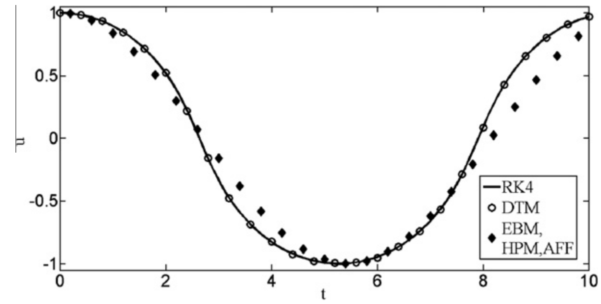

Figure 4 A comparison between the analytic and semi-analytic methods in conjunction with the fourth-order Runge–Kutta method for $\varepsilon = 1$, $q = 1$, $A = 1$.

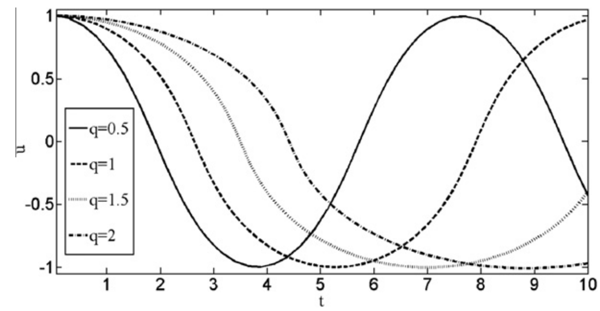

Figure 5 The effects of q on the period of motion ($\varepsilon = A = 1$).

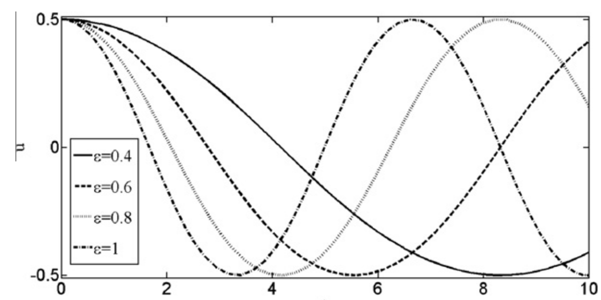

Figure 6 The effects of ε on the period of motion ($q = A = 0.5$).

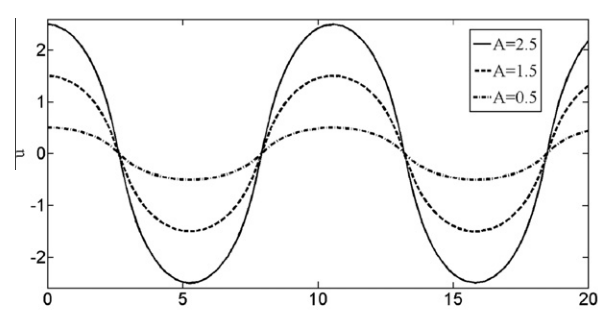

Figure 7 The effect of q and A on the period of motion ($\varepsilon = 1$, $qA = 1$).

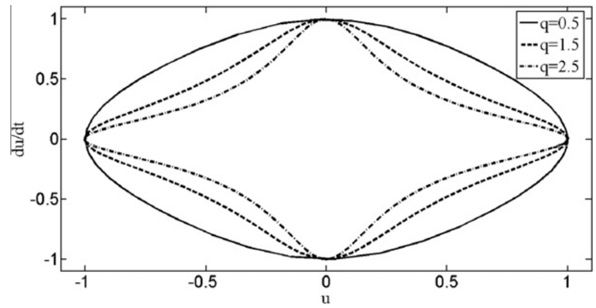

Figure 8 The effect of q on phase plane ($\varepsilon = 1$, $A = 1$).

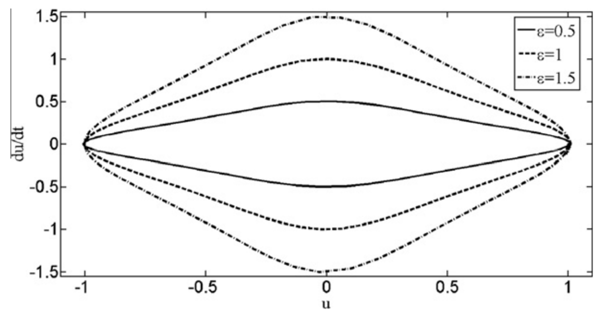

Figure 9 The effect of ε on phase plane ($q = 1$, $A = 1$).

4. Conclusion

In this paper, nonlinear dynamics of a particle on a rotating parabola are investigated. The analytical approaches are applied via the energy balance method, homotopy perturbation method and amplitude–frequency formulation, also the semi-analytical approach implemented by the differential transform method. Results show the analytic approaches cannot predict dynamics of the particle as well as the semi-analytic approach; in contrast, the analytic methods are able to product an explicit expression as the solution by a simple calculation, whereas each parameter is effected in the governing equation clearly, and its role can be investigated easily.

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