University of Bahrain
Journal of the Association of Arab Universities for Basic and Applied Sciences
www.elsevier.com/locate/jaaubas
www.sciencedirect.com

حلول الموجات المسافرة لمعادلات (KdVs) باستخدام طريقة الجيب - جيب التمام (Sine-Cosine Method)

Sadaf Bibi, Syed Tauseef Mohyud-Din

Department of Mathematics, HITEC University, Taxila Cantt, Pakistan

الملخص:

في هذا البحث، تم الحصول على حلول الموجات المسافرة باستخدام طريقة الجيب – جيب التمام (Sine–Cosine Method) لمعادلات (KdV). الحلول والإجراء والنتائج التي تم الحصول عليها تؤكد كفاءة الطريقة المقترحة.

Journal of the Association of Arab Universities for Basic and Applied Sciences (2014) 15, 90–93

University of Bahrain
Journal of the Association of Arab Universities for Basic and Applied Sciences
www.elsevier.com/locate/jaaubas
www.sciencedirect.com

ORIGINAL ARTICLE

Traveling wave solutions of KdVs using sine–cosine method

Sadaf Bibi, Syed Tauseef Mohyud-Din *

Department of Mathematics, HITEC University, Taxila Cantt, Pakistan

Received 5 June 2012; revised 24 December 2012; accepted 31 March 2013
Available online 28 June 2013

KEYWORDS

Nonlinear equations;
Sine–cosine method;
Traveling wave solutions;
Generalized KdV equation

Abstract Traveling wave solutions are obtained by using the sine–cosine method for KdVs. Solution procedure and the obtained results re-confirm the efficiency of the proposed scheme.

1. Introduction

There has been an unprecedented development in the solutions of nonlinear sciences (see Abbasbandy, 2007; Abdou and Soliman, 2005; Ablowitz and Clarkson, 1991; Al-Muhiameed and Abdel-Salam, 2011; Aslan, 2011; Bekir and Boz, 2008; Bekir and Cevikel, 2011; Deakin and Davison, 2010; Feng et al., 2011; Gepreel, 2011a,b; Gepreel and Shehata, 2012; Gomez and Salas, 2010; Guo et al., 2011; He, 2006; He and Wu, 2006; Hirota, 1971; Liu et al., 2010; Ma and You, 2004; Massabo et al., 2011; Mohyud-din et al., 2010; Malfliet, 1992; Naher et al., 2011a,b; Nofel et al., 2011; Ozis and Aslan, 2010; Rogers and Shadwick, 1982; Salah et al., 2011; Salas, 2008; Salas and Gomez, 2010; Soliman and Abdo, 2009; Wang et al., 2008; Wazwaz, 2004a,b, 2007, 2011; Yildirim and Pinar, 2010; Yun, 2011; Zayed and Al-Joudi, 2010; Zayed and Gepreel, 2011; Zhang et al., 2010; Zhao et al., 2011; Zhu, 2008) during the last two decades. In the similar context, several numerical and analytical techniques including Homotopy Analysis (HAM), Perturbation, Modified Adomian's Decomposition (MADM), Variational iteration (VIM), Variation of Parameters, Finite difference, Finite volume, Backlund transformation, inverse scattering, Jacobi elliptic function expansion, tanh function have been developed to solve such equations (see Abbasbandy, 2007; Abdou and Soliman, 2005; Ablowitz and Clarkson, 1991; Al-Muhiameed and Abdel-Salam, 2011; Aslan, 2011; Bekir and Boz, 2008; Bekir and Cevikel, 2011; Deakin and Davison, 2010; Feng et al., 2011; Gepreel, 2011a,b; Gepreel and Shehata, 2012; Gomez and Salas, 2010; Guo et al., 2011; He, 2006; He and Wu, 2006; Hirota, 1971; Liu et al., 2010; Ma and You, 2004; Massabo et al., 2011; Mohyud-din et al., 2010; Malfliet, 1992; Naher et al., 2011a,b; Nofel et al., 2011; Ozis and , 2010; Rogers and Shadwick, 1982; Salah et al., 2011; Salas, 2008; Salas and Gomez, 2010; Soliman and Abdo, 2009; Wang et al., 2008; Wazwaz, 2004a,b, 2007, 2011; Yildirim and Pinar, 2010; Yun, 2011; Zayed and Al-Joudi, 2010; Zayed and Gepreel, 2011; Zhang et al., 2010; Zhao et al., 2011; Zhu, 2008) and the references therein. Most of these techniques have their inbuilt deficiencies including the evaluation of the so-called Adomian's polynomials, divergent results, successive applications of the integral operator, un-realistic assumptions, non-compatibility with

* Corresponding author.
E-mail address: syedtauseefs@hotmail.com (S.T. Mohyud-Din).
Peer review under responsibility of University of Bahrain.

ELSEVIER Production and hosting by Elsevier

1815-3852 © 2013 Production and hosting by Elsevier B.V. on behalf of University of Bahrain.
http://dx.doi.org/10.1016/j.jaubas.2013.03.006

the nonlinearity of physical problem and very lengthy calculations. Inspired and motivated by the ongoing research in this area, we apply a relatively new technique which is called sine–cosine method (Wazwaz, 2004a,b) to find traveling wave solutions of Generalized KdVs. It is to be highlighted that such an equation arises frequently in various branches of physics, applied and engineering sciences (see Abbasbandy, 2007; Abdou and Soliman, 2005; Ablowitz and Clarkson, 1991; Al-Muhiameed and Abdel-Salam, 2011; Aslan, 2011; Bekir and Boz, 2008; Bekir and Cevikel, 2011; Deakin and Davison, 2010; Feng et al., 2011; Gepreel, 2011a,b; Gepreel and Shehata, 2012; Gomez and Salas, 2010; Guo et al., 2011; He, 2006; He and Wu, 2006; Hirota, 1971; Liu et al., 2010; Ma and You, 2004; Massabo et al., 2011; Mohyud-din et al., 2010; Malfliet, 1992; Naher et al., 2011a,b; Nofel et al., 2011; Ozis and Aslan, 2010; Rogers and Shadwick,1982; Salah et al., 2011; Salas, 2008; Salas and Gomez, 2010; Soliman and Abdo, 2009; Wang et al., 2008; Wazwaz, 2004a,b, 2007, 2011; Yildirim and Pinar, 2010; Yun, 2011; Zayed and Al-Joudi, 2010; Zayed and Gepreel, 2011; Zhang et al., 2010; Zhao et al., 2011; Zhu, 2008) and the references therein. The proposed scheme is fully compatible with the complexity of such problems and is very user-friendly. It is to be highlighted that the proposed algorithm gives some solutions which are compatible with solutions obtained in the Tanh method and the Tanh–Coth method. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. Moreover, some modifications can be seen in Nofel et al. (2001).

2. Sine–cosine method

In the proposed scheme, introducing the wave variable $\xi = x - ct$, we get

$$P(u, ux, ut, uxx, uxt, utt, uxxx\ldots) = 0, \tag{1}$$

where $u(x, t)$ is the traveling wave solution. This enables us to use the following changes

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi},\quad \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \xi^2},\quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi},\quad \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}$$
$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi},\quad \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \xi^2},\quad \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi},\quad \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial \xi^2}. \tag{2}$$

One can immediately reduce the nonlinear PDE (1) into a nonlinear ODE

$$Q(u, u\xi, u\xi\xi, u\xi\xi\xi\ldots) = 0. \tag{3}$$

The ordinary differential Eq. (3) is then integrated as long as all terms contain derivatives, where we neglect integration constants. The solutions of many nonlinear equations can be expressed in the form

$$u(x,t) = \{\lambda \sin^\beta(\mu\xi), \quad |\xi| \leqslant \frac{\pi}{\mu}, \tag{4}$$

or in the form

$$u(x,t) = \{\lambda \cos^\beta(\mu\xi), \quad |\xi| \leqslant \frac{\pi}{2\mu}, \tag{5}$$

where λ, μ and β are parameters that will be determined, μ and c are the wave number and the wave speed, respectively.

We use

$$u(\xi) = \lambda \sin^\beta(\mu\xi),$$
$$u^n(\xi) = \lambda^n \sin^{n\beta}(\mu\xi),$$
$$(u^n)_\xi = n\mu\beta\lambda^n \cos(\mu\xi)\sin^{n\beta-1}(\mu\xi),$$
$$(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi), \tag{6}$$

and the derivatives of (5) become

$$u(\xi) = \lambda \cos^\beta(\mu\xi),$$
$$u^n(\xi) = \lambda^n \cos^{n\beta}(\mu\xi),$$
$$(u^n)_\xi = -n\mu\beta\lambda^n \sin(\mu\xi)\cos^{n\beta-1}(\mu\xi),$$
$$(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \tag{7}$$

and so on for the other derivatives. We substitute (6) or (7) into the reduced equation obtained above in (3), balancing the terms of the sine functions when (6) is used to otherwise balance the terms of the cosine functions. The resulting system of algebraic equations is solved by using the computerized symbolic calculations. We next collect all terms with the same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and equating their coefficients to zero to get a system of algebraic equations with unknowns λ, μ and β. All possible values of the parameters λ, μ and β are obtained.

3. Solution procedure

3.1. Generalized KdV equation

Consider the following gKdV equation

$$u_t + (n+1)(n+2)u^n u_x + u_{xxx} = 0, \tag{8}$$

We now employ the sine–cosine method. Using the wave variable $\xi = x - ct$ carries (8) into ODE

$$-cu' + (n+1)(n+2)u^n u' + u''' = 0, \tag{9}$$

Integrating (9) gives and by considering the constant of integration to be zero for simplicity, we get

$$-cu + (n+2)u^{n+1} + u'' = 0, \tag{10}$$

Substituting (7) into (10) gives

$$-c\lambda\cos^\beta(\mu\xi) + (n+2)\lambda^{n+1}\cos^{(n+1)\beta}(\mu\xi) - \lambda\mu^2\beta^2\cos^\beta(\mu\xi)$$
$$+\lambda\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\xi) = 0, \tag{11}$$

Equating the exponents and the coefficients of each pair of the cosine functions, we find the following system of algebraic equations:

$$\beta - 1 \neq 0,$$
$$(n+1)\beta = \beta - 2,$$
$$-c\lambda = \lambda\mu^2\beta^2,$$
$$(n+2)\lambda^{n+1} = -\lambda\mu^2\beta(\beta-1). \tag{12}$$

Solving the system (12) yields

$$\beta = -\frac{2}{n},$$
$$\lambda = \left(\frac{1}{2}c\right)^{\frac{1}{n}},$$
$$\mu = \frac{n}{2}\sqrt{-c}, \quad c < 0, \tag{13}$$

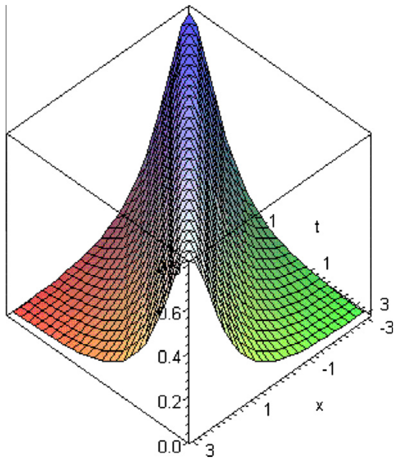

Figure 1 Soliton solution corresponding to $u_3(x,t)$ for $c = 1$, $n = 3$.

The result (13) can be easily obtained if we also use the sine method (6). We obtain the following periodic solutions for $c < 0$,

$$u_1(x,t) = \left\{ \left(\frac{1}{2}c\right) \sec^2\left[\frac{n}{2}\sqrt{-c}(x-ct)\right] \right\}^{\frac{1}{n}}, \quad |\mu\xi| < \frac{\pi}{2} \tag{14}$$

and

$$u_2(x,t) = \left\{ \left(\frac{1}{2}c\right) \csc^2\left[\frac{n}{2}\sqrt{-c}(x-ct)\right] \right\}^{\frac{1}{n}}, \quad O < \mu\xi < \pi \tag{15}$$

However, for $c > 0$, we obtain the soliton solution

$$u_3(x,t) = \left\{ \left(\frac{1}{2}c\right) \sec h^2\left[\frac{n}{2}\sqrt{c}(x-ct)\right] \right\}^{\frac{1}{n}},$$

The graphical representation of $u_3(x, t)$ is shown in Fig. 1

Fig. 1 depicts soliton solution corresponding to $u(x,t)$ for $c = 1$, $n = 3$.

3.2. mKdV equation

Consider the modified KdV equation

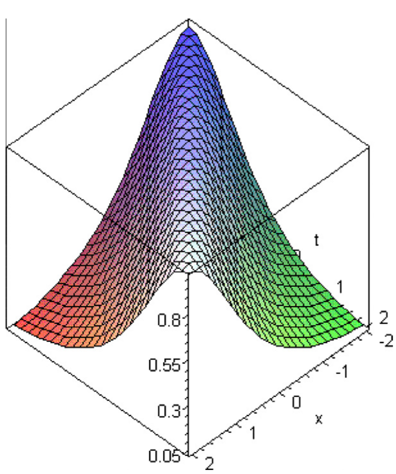

Figure 2 Soliton solution corresponding to $u(x,t)$ for $c = 1$.

$$u_t + 6u^2u_x + u_{xxx} = 0. \tag{16}$$

We now employ the sine–cosine method. Using the wave variable $\xi = x - ct$ carries (16) into ODE

$$-cu' + 6u^2u' + u''' = 0, \tag{17}$$

Integrating (17) gives and by considering the constant of integration to be zero, we get

$$-cu + 2u^3 + u'' = 0. \tag{18}$$

Substituting (6) into (18) gives

$$(-c\lambda - \mu^2\beta^2\lambda)\sin^{\beta}(\mu\xi) + 2\lambda^3\sin^{3\beta}(\mu\xi) + \mu^2\lambda\beta(\beta-1)\sin^{\beta-2}(\mu\xi) = 0, \tag{19}$$

Equating the exponents and the coefficients of each pair of the sine functions we find the following system of algebraic equations:

$$\begin{aligned} &\beta - 1 \neq 0, \\ &\beta - 2 = 3\beta, \\ &-c\lambda - \mu^2\beta^2\lambda = 0, \\ &2\lambda^3 + \mu^2\lambda\beta(\beta-1) = 0. \end{aligned} \tag{20}$$

Solving the system (20) yields

$$\begin{aligned} &\beta = -1, \\ &\lambda = \sqrt{c}, \\ &\mu = \sqrt{-c}. \end{aligned} \tag{21}$$

Consequently, we obtain the following periodic solutions $c < 0$,

$$u(x,t) = \sqrt{c}\sec[\sqrt{-c}(x-ct)], \quad |\mu\xi| < \frac{\pi}{2} \tag{22}$$

and

$$u(x,t) = \sqrt{c}\csc[\sqrt{-c}(x-ct)],. \quad O < \mu\xi < \pi \tag{23}$$

However, for $c > 0$, we obtain the soliton solution

$$u(x,t) = \sqrt{c}\sec h[\sqrt{c}(x-ct)].$$

Fig. 2 depicts soliton solution corresponding to $u(x,t)$ for $c = 1$.

4. Conclusion

The study shows that sine–cosine method is quite efficient and practically well suited for use in calculating traveling wave solutions for KdVs and other differential equations. The proposed algorithm gives some solutions which are compatible with solutions obtained in the Tanh method and the Tanh–Coth method. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

References

Abbasbandy, S., 2007. A new application of He's variational iteration method for quadratic Riccati differential equation by using Adomian's polynomials. J. Comput. Appl. Math. 207, 59–63.

Abdou, M.A., Soliman, A.A., 2005. New applications of variational iteration method. Physics D 211 (1–2), 1–8.

Ablowitz, M.J., Clarkson, P.A., 1991. Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform. Cambridge Univ. Press, Cambridge.

Al-Muhiameed, Z.I.A., Abdel-Salam, E.A.B., 2011. Generalized Jacobi elliptic function solution to a class of nonlinear Schrodinger –type equations. Mathematical Problems in Engineering 2011. http://dx.doi.org/10.1155/2010/575679 (Article ID 575679, 11 pages).

Aslan, I., 2011. Application of the exp-function method to nonlinear lattice differential equations for multi-wave and rational solutions. Math. Meth. Appl. Sci. 34, 1707–1710.

Bekir, A., Boz, A., 2008. Exact solutions for nonlinear evolution equations using Exp-function method. Phys. Lett. A 372, 1619–1625.

Bekir, A., Cevikel, A.C., 2011. The tanh–coth method combined with the Riccati equation for solving nonlinear coupled equation in mathematical physics. J. King Saud Univ. – Sci. 23, 127–132.

Deakin, A.S., Davison, M., 2010. Analytic solution for a vasicek interest rate convertible bond model. Journal of Applied Mathematics 2010. http://dx.doi.org/10.1155/2010/263451 (Article ID 263451, 5 pages).

Feng, J., Li, W., Wan, Q., 2011. Using (G'/G)-expansion method to seek traveling wave solution of Kolmogorov–Petrovskii–Piskunov equation. Appl. Math. Comput. 217, 5860–5865.

Gepreel, Khaled A., 2011a. AGeneralized (G'/G)-expansion method for finding trvelling wave solutions of nonlinear evolution equations. J. Partial Differ. Equ. 24, 55–69.

Gepreel, K.A., 2011b. Exact solutions for nonlinear PDEs with the variable coefficients in mathematical physics. J. Inf. Comput. Sci. 6 (1), 003–014.

Gepreel, Khaled A., Shehata, 2012. Exact complextion soliton solutions for nonlinear partial differential equations in mathematical physics. Sci. Res. Essays 7 (2), 149–157 (16 January).

Gomez, C.A., Salas, A.H., 2010. Exact solutions for the generalized BBM equation with variable coefficients. Mathematical Problems in Engineering, 2010. http://dx.doi.org/10.1155/2010/498249 (Article ID 498249, 10 pages).

Guo, S., Mei, L., Zhou, Y., Li, C., 2011. The extended Riccati equation mapping method for variable-coefficient diffusion-reaction and mKdV equation. Appl. Math. Comput. 217, 6264–6272.

He, J.H., 2006. Some asymptotic methods for strongly nonlinear equation. Int. J. Mod. Phys. 20 (20), 1144–1199 (10).

He, H., Wu, X.H., 2006. Exp-function method for nonlinear wave equations. Chaos Soliton Fract. 30, 700–708.

Hirota, R., 1971. Exact solution of the KdV equation for multiple collisions of solutions. Phys. Rev. Lett. 27, 1192–1194.

Liu, X., Tian, L., Wu, Y., 2010. Application of (G'/G) -expansion method to two nonlinear evolution equations. Appl. Math. Comput. 217, 1376–1384.

Ma, W.X., You, Y., 2004. Solving the Korteweg–de Vries equation by its bilinear form: Wronskian solutions. Trans. Am. Math. Soc. 357, 1753–1778.

Malfliet, W., 1992. Solitary wave solutions of nonlinear wave equations. Am. J. Phys. 60, 650–654.

Massabo, M., Cianci, R., Paladino, O., 2011. An analytical solution of the advection dispersion equation in a bounded domain and its application to laboratory experiments. Journal of Applied Mathematics, 2011. http://dx.doi.org/10.1155/2011/493014 (Article ID 493014, 14 pages).

Mohyud-din, S.T., Noor, M.A., Noor, K.I., 2010. Exp-function method for traveling wave solutions of modified Zakharov–Kuznetsov equation. J. King Saud Univ. 22, 213–216.

Naher, H., Abdullah, F., Akbar, M.A., 2011a. The (G'/G) -expansion method for abundant traveling wave solutions of Caudrey-Dodd-Gibbon equation. Mathematical Problems in Engineering, 2011. http://dx.doi.org/10.1155/2011/218216 (Article ID 218216).

Naher, H., Abdullah, F., Akbar, M.A., 2011b. New travelling wave solutions of the higher dimensional nonlinear partial differential equation by the Exp-function method 2012 (2012). http://dx.doi.org/10.1155/2012/575387 (Article ID 575387, 14 pages).

Nofel, T.A., Sayed, M., Hamad, Y.S., Elagan, S.K., 2011. The improved (G'/G)-expansion method for solving the fifth-order KdV equation. Ann. Fuzzy Math. Info. 3, 9–17.

Ozis, T., Aslan, I., 2010. Application of the (G'/G)-expansion method to Kawahara type equations using symbolic computation. Appl. Math. Comput. 216, 2360–2365.

Rogers, C., Shadwick, W.F., 1982. Backlund Transformations. Aca Press, New York.

Salah, F., Aziz, Z.A., Ching, D.L.C., 2011. New exact solutions for MHD transient rotating flow of a second-grade fluid in a porous medium. Journal of Applied Mathematics, 2011. http://dx.doi.org/10.1155/2011/823034 (Article ID 823034, 8 pages).

Salas, A., 2008. Some solutions for a type of generalized Sawada–kotera equation. Appl. Math. Comput. 196, 812–817.

Salas, A.H., Gomez, C.A., 2010. Application of the Cole-Hopf transformation for finding exact solutions to several forms of the seventh-order KdV equation. Mathematical Problems in Engineering, 2010. http://dx.doi.org/10.1155/2010/194329 (Article ID 194329, 14 pages).

Soliman, A.A., Abdo, H,A., 2009. New exact Solutions of nonlinear variants of the RLW, the PHI-four and Boussinesq equations based on modified extended direct algebraic method. Int. J. Nonlinear Sci. 7 (3), 274–282.

Wang, M., Li, X., Zhang, J., 2008. The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A 372, 417–423.

Wazwaz, A.M., 2004a. The sine–cosine method for handling nonlinear wave equations. Math. Comput. Modell. 40 (5), 499–508.

Wazwaz, A.M., 2004b. The sine–cosine method for obtaining solutions with compact and noncompact structures. Appl. Math. Comput. 159 (2), 559–576.

Wazwaz, A.M., 2007. The tanh–coth method for solitons and kink solutions for nonlinear parabolic equations. Appl. Math. Comput. 188, 1467–1475.

Wazwaz, A.M., 2011. A new (2 + 1)-dimensional Korteweg-de-Vries equation and its extension to a new (3 + 1)-dimensional Kadomtsev–Petviashvili equation. Phys. Scr. 84, 035010, http://dx.doi.org/10.1088/0031-8949/84/03/035010.

Yildirim, A., Pinar, Z., 2010. Application of the exp-function method for solving nonlinear reaction-diffusion equations arising in mathematical biology. Comput. Math. Appl. 60, 1873–1880.

Yun, B.I., 2011. An iteration method generating analytical solutions for Blasius problem. Journal of Applied Mathematics, 2011. http://dx.doi.org/10.1155/2011/925649 (Article ID 925649, 8 pages).

Zayed, M.E., Al-Joudi, S., 2010. Applications of an Extended (G'/G) -Expansion Method to Find Exact Solutions of Nonlinear PDEs in Mathematical Physics. Mathematical Problems in Engineering, 2010. http://dx.doi.org/10.1155/2010/768573 (Article ID 768573, 19 pages).

Zayed, M.E., Gepreel, Khaled A., 2011. A series of complextion soliton solutions for nonlinear Jaulent-Miodek PDEs using Ricatti ezquations method. Proc. R. Soc. Edinburgh, Sect. A: Math. 141A, 1001–1015.

Zhang, S., Ba, J., Sun, Y., Dong, L., 2010. Analytic solutions of a (2 + 1)-dimensional variable-coefficient Broer-Kaup system. Math. Meth. Appl. Sci.. http://dx.doi.org/10.1002/mma.1343.

Zhao, Y.M., Yang, Y.J., Li, W., 2011. Application of the improved (G'/G)-expansion method for the variant Boussinesq equations. Appl. Math. Sci. 5 (58), 2855–2861.

Zhu, S., 2008. The generalizing Riccati equation mapping method in non-linear evolution equation: application to (2 + 1)-dimensional Boiti–Leon–Pempinelle equation. Chaos Soliton Fract. 37, 1335–1342.