Harmonic Oscillator In An Infinite N- Dimensional Spherical well

Al- Jaber, Sami .A

Department of physics , An – Najah National University , Nablus , Palestine .

Jaber@Najah.edu

ABSTRACT

The boundary effects on a quantum system are discussed by examining an N- dimensional harmonic oscillator confined in an impenetrable spherical well. The corrections, due to the boundary and the space dimension, to the ground- state energy and wave function are calculated by using a linear approximation method which is linear in energy and by numerical method using Mathematica. Our results for the energy corrections obtained by the two methods are in very good agreement. A simple analytical expression for the asymptotic dependence of the ground – state energy on the well radius and on the dimension N is derived. Finally, the pressure needed to compress a free N-dimensional harmonic oscillator to a certain size is computed.

Keywords: Approximation Methods, Foundations, Higher Dimensions.

INTRODUCTION

The simple harmonic oscillator is a topic of utmost importance which is discussed in every standard textbook on quantum mechanics. It is applicable in different physical situations (Moshinsky and Sminov, 1996) and has the great advantage that it has closed solutions for the energy eigenvalues and eigenfunctions. Various investigations of the harmonic oscillator have been considered, time – dependent harmonic oscillator (Liu and Wang, 2007), harmonic oscillator with delta –function potential (Patil, 2006), relativistic harmonic oscillator (Nagiyev et al., 2007), anharmonic oscillator (Skala et al.,1999), and the spiked harmonic oscillator (Hall et al., 2001) . Another system which has received considerable attention is the confined one (Fröman et al., 1987), (Sako et al., 2004) and (Zang et al., 2008).

Recently, there has been some renewed interest in the confined hydrogen atom (Djajaputra and Cooper, 2002), and in the confined harmonic oscillator (AL-Jaber, 2002), (Marcilio et al., 2005) and (Ndengue and Motapon, 2008). This interest is partly motivated by the technological advances, for example in the field of semiconductor quantum dots (Jacak et al., 1998), where the computation of the electronic structure of such systems necessarily has to take into account the presence of the finite confining boundaries and their effect on the system. During the past years, the generalization of physical problems to higher dimensions received a considerable development in theoretical and mathematical physics. For example, the N- dimensional analogy of the hydrogen atom has been studied extensively over the years (Nouri, 1999), (Kirchberg et al., 2003).

The purpose of this paper is to consider the boundary corrections for an N- dimensional harmonic oscillator in a spherical cavity using an approximation method which is linear in energy and compare the results with those obtained numerically by using Mathematica. This method (Anderson, 1975) is usually used in the calculations of a wave function of a Hamiltonian with energies which are close to the energies of a known wave function. Beside its mathematical interest, the present paper presents a new approach, which has some pedagogical simplicity to the confined N- dimensional harmonic oscillator.

THE LINEAR APPROXIMATION

We examine the boundary corrections for a harmonic oscillator situated at the center of an N-dimensional spherical cavity of radius S. It is assumed that the wall of the cavity to be impenetrable. Thus the potential to be considered is:

$$V(r) = \begin{cases} \frac{1}{2}kr^2 & r<S \\ \infty & r>S \end{cases} \qquad (1)$$

The time – independent Schrödinger equation for the harmonic oscillator is

$$H\Psi(r) = \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}kr^2\right]\Psi(r) = E\Psi(r) \qquad (2)$$

Where ∇^2 is the N- dimensional Laplacian operator given by :

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{(N-1)}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\Omega^2 \qquad (3)$$

where r is the radial coordinate and Ω^2 is the Laplace operator on the unit sphere S^{N-1}. The eigenfunction $\psi(r)$ satisfies the Schrödinger equation for the harmonic oscillator for r<S and should still be regular at the origin. The only difference from the free space- case is that now we have to impose a different boundary condition. Namely, the ware function must vanish at r=S instead of at r = ∞. For S>>$r_\circ$ the changes in the ground- state wave function and energy due to the presence of the well are expected to be small. Here $r_\circ$, to be determined latter, is the radius at which the radial distribution function for the free- space case is maximum. In free space, *i.e*, in the absence of the cavity, the energy spectrum for the N- dimensional harmonic oscillator is

$$\varepsilon_n = \hbar\omega\left(n + \frac{N}{2}\right). \qquad n = 0, 1, 2, ... \qquad (4)$$

In the presence of the cavity, the energy spectrum is

$$E_n = \varepsilon_n + \Delta\varepsilon_n \qquad (5)$$

And the un normalized wave function at energy E_n is approximated by:

$$\Psi(E_n, r) = \phi(\varepsilon_n, r) + \Delta\varepsilon_n\,\dot{\phi}(\varepsilon_n, r) \qquad (6)$$

where ϕ is the wave function in the free- space case and $\dot{\phi}(\varepsilon_n, r)$ is the derivative with respect to the energy of $\phi(\varepsilon, r)$ evaluated at $\varepsilon = \varepsilon_n$. therefore,

$$\dot{\phi}(\varepsilon_n, r) = \frac{\partial}{\partial \varepsilon} \phi(\varepsilon, r) \mid \varepsilon = \varepsilon_n \qquad (7)$$

The vanishing of the eigen functions for the cavity problem at r = S implies $\Psi(E_n, S) = 0$ an thus the energy correction becomes

$$\Delta \varepsilon_n = \frac{\phi(\varepsilon_n, S)}{\dot{\phi}(\varepsilon_n, S)}. \qquad (8)$$

The Schrödinger Eq. 2 splits into radial and angular differential equations with the latter having the hyper-spherical harmonics as solutions (eigenfunctions of Ω^2), $Y_\ell^m(\{\theta_i\})$ with i = 1, 2, …., N-1, which are independent of the form of the central potential. The radial part solution R(r) satisfies.

$$\left[\frac{-\hbar^2}{2m}\left(\frac{d^2}{dr^2} + \frac{N-1}{r}\frac{d}{dr}\right) + \frac{\ell(\ell+N-2)}{2mr^2} + \frac{1}{2}m\omega^2 r^2\right]R(r) = ER(r) \qquad (9)$$

where ω^2 = k/m. Introducing the variables

$$\rho = \alpha r \ , \ \alpha = (m\omega/\hbar)^{\frac{1}{2}}, and\ \lambda = 2E/\hbar\omega,$$

and letting R(r) = U(r) /$r^{(N-1)/2}$, Eq.9 becomes

$$\left[\frac{d^2}{d\rho^2} - \frac{\left(\ell + \frac{N-3}{2}\right)\left(\ell + \frac{N-1}{2}\right)}{\rho^2} + \lambda - \rho^2\right]U(\rho) = 0 \qquad (10)$$

The above equation can be transformed into Kummer-Laplace differential equation (Bransden and Joachain, 1990) whose solution is the confluent hypergeometric function, ${}_1F_1$(a,b,z) with $a = \frac{1}{2}\left(\ell + \frac{N}{2}\right) - \frac{\lambda}{4}$ and $b = \ell + \frac{N}{2}$, and thus

$$R(r) = Ar^\ell e^{-\alpha^2 r^2/2}\ {}_1F_1\left[\frac{1}{2}\left(\ell + \frac{N}{2}\right) - \frac{\lambda}{4}, \ell + \frac{N}{2}, \alpha^2 r^2\right] \quad , \qquad (11)$$

where A is a normalization constant and the energy ε_n is given by

$$\varepsilon_n = \lambda\frac{\hbar\omega}{2} = \hbar\omega\,(n + \frac{N}{2}) \quad , \qquad (12)$$

where n = 0, 1, 2, ……

Now, as the radial function R(r) is known, we are in a position to discuss the validity of the linear approximation method in *N* dimensions. As we mentioned earlier , this method is valid whenever the radius of the cavity is much greater than some radius $r_\circ$. We choose $r_\circ$ to be the value of r at which the radial distribution function $D(r)\left(= r^{N-1}\left|R(r)\right|^2\right)$ is maximum. This occurs when the angular momentum has its largest value $\ell = n.$ In that case n_r =0 and thus the confluent hypergeometric function ${}_1F_1$is unity. Therefore, the radial distribution function, with the help of Eq. 11, is

$$D(r) \sim r^{(N-1+2n)} e^{-\alpha^2 r^2},$$

and hence D(r) exhibits a maximum at the value $r_\circ$ obtained by requiring

$\frac{d}{dr} D(r) \,|_{r=r_\circ} = 0$, which yields

$$r_\circ^2 = \left(n + \frac{N-1}{2}\right) / \alpha^2. \tag{13}$$

This shows that the linear approximation is valid when $S >> \sqrt{n + (N-1)/2} \,/\, \alpha$. We note that S increases as the dimension *N* increases. This is so because the centripetal term in the effective potential becomes more repulsive as *N* increases, which means tries to repel the particle away more as *N* increases (AL-Jaber, 1998).

The function R(r) given in Eq. 11 contains most of the aspects we are looking for. For example the $\lambda = N$ case is the ground – state wave function of the *N*- dimensional harmonic oscillator in the free space and is nodless. As λ is increased above *N*, the wave function acquires a node at $Z_\circ$, i.e the hypergeometric function ${}_1F_1$ (a, b, $Z_\circ$) vanishes . This occurs at a cavity radius S given by , (see Eq.13):

$$S = r_\circ \sqrt{z_\circ \Big/ \left(n + \frac{N-1}{2}\right)}. \tag{14}$$

For example, consider the case $\ell = 0$ and $\lambda = N + 2$ and *N*+4. By using the relation $\lambda = 2E / \hbar\omega$, we find:

For $\lambda = N + 2$, this corresponds to the energy $E = \hbar\omega\left(1 + \frac{N}{2}\right)$ which becomes the (n, ℓ)= (1,1) eigenstate of *N*- dimensional harmonic oscillator in free space.

For $\lambda = N + 4$, this corresponds to the energy $E = \hbar\omega\left(2 + \frac{N}{2}\right)$ which becomes the (n, ℓ)=(2,0) eigenstate of the *N*- dimensional harmonic oscillator in free space. For these two cases, the wave function for the cavity problem acquires a node which moves from $r = \infty$ to r(=S) given by equation (14) with n=0 , namely $S = \left(2Z_\circ / (N-1)\right)^{1/2}$, where $Z_\circ$ is the point at which the hypergeometric function ${}_1F_1$(a,b,z) vanishes. Note that in the above two cases

$a=-\frac{1}{2}$ for $\lambda = N+2$,and a =-1 for $\lambda = N+4$, and in each case there is only one zero of $_1F_1$ (a, b, z).

One can therefore obtain the ground- state wave function and energy of the harmonic oscillator in a cavity of radius S by numerically searching for the energy (via the parameter λ) which gives a wave function with a node at r =S . This is achieved by using the Mathematica software. This gives a useful comparison for our approximation.

Since the hyperspherical harmonics are independent of the energy, Eq. 8 could be written as:

$$\Delta \varepsilon_n = -\frac{\hbar\omega}{2} R(\lambda_n, S) / \left(\frac{\partial R}{\partial \lambda}\right)_{\lambda=\lambda_n} . \qquad (15)$$

Substituting the radial function $R(\lambda, r)$ in Eq.11 into Eq.15 yields an expression for $\Delta \varepsilon_n$ which should be valid for $r >> r_\circ$ given by Eq.13.

We shall consider the ground state (n=0). Here $\ell = 0$ and thus Eq.15 gives

$$R_\circ(\lambda, r) = Ae^{-\alpha^2 r^2/2}\, {}_1F_1\left[\frac{1}{4}(N-\lambda), \frac{N}{2}, \alpha^2 r^2\ \right] \qquad (16)$$

The ground state (n=0) implies that , see (Eq.12), $\lambda = N$ and thus we have

$$R_\circ(N, r) = Ae^{-\alpha^2 r^2/2}\, {}_1F_1\left(0, \frac{N}{2}, \alpha^2 r^2\right) \qquad (17)$$

Our purpose is to obtain a simple analytical expression for the correction to the ground – state energy for $S >> r_\circ$. Therefore, we calculate the limiting form of $R_\circ(\lambda, r)$ for r >> $r_\circ$. Following (Djajaputra and Cooper, 2000), the asymptotic expansion of the confluent hypergeometric function $_1F_1$ (a, b, z) for large z is (Abromowitz and Stegun, 1965).

$$\frac{{}_1F_1(a,b,z)}{\Gamma(b)} = \frac{e^{i\pi a}}{z^a \Gamma(b-a)} I_1(a,b,z) + \frac{e^z z^{a-b}}{\Gamma(a)} I_2(a,b,z) \qquad (18)$$

where

$$I_1(a,b,z) = \sum_{n=0}^{R-1} \frac{(a)_n (1+a-b)_n}{n!} \frac{e^{i\pi n}}{z^n} + O(1Z1^{-R}) \qquad (19)$$

$$I_2(a,b,z) = \sum_{n=0}^{R-1} \frac{(b-a)_n (1-a)_n}{n!} \frac{1}{z^n} + O(1Z1^{-R}) \qquad (20)$$

The Pochhammer symbol $(a)_n$ is given by [28]

$$(a)_n = a\,(a+1)\,(a+2) \ldots\ldots (a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}. \qquad (21)$$

We seek the derivative of $_1F_1$ (a,b,z) at $a = \frac{1}{4}(N-\lambda)$ with $\lambda = N$.

The dominant term comes from the derivative of $\Gamma(a)$ in the second term in Eq. 18. The first term can be neglected because it does not contain the exponential term e^z which dominates the derivative at large z. Retaining only the largest term, we get

$$\frac{\partial}{\partial a}{}_1F_1(a,b,z) \approx -e^z z^{a-b}\Gamma(b) I_2(a,b,z)\frac{\Psi(a)}{\Gamma(a)} \tag{22}$$

where $\Psi(a)$ is the digamma function defined as $\Psi(a)=\Gamma'(a)/\Gamma(a)$. The ratio $\Psi(a)$ to $\Gamma(a)$ $as\, a \to 0$ satisfies.

$$\lim_{a\to 0}\frac{\Psi(a)}{\Gamma(a)} = \lim_{a\to 0}\left(\frac{-\gamma - 1/a}{-\gamma - 1/a}\right) = -1,$$

where δ is the Euler constant. Thus

$$\frac{\partial}{\partial a}{}_1F_1(a,b,z)\,\Big|_{a=0} \approx e^z z^{a-b}\Gamma(b) I_2(a,b,z). \tag{23}$$

Using this expression and taking the first two terms in I_2(a,b,z), we get.

$$\frac{\partial}{\partial a}{}_1F_1(a,b,z) = e^z z^{a-b}\Gamma(b)\left[1+\frac{\Gamma(b-a+1)\Gamma(2-a)}{z\Gamma(b-a)\Gamma(1-a)}\right] \tag{24}$$

Using $a=\frac{1}{4}(N-\lambda)$,$b= N/2$,$Z=\alpha^2 r^2$, we get.

$$\frac{\partial R}{d\lambda} = \frac{\partial R}{\partial a}\frac{\partial a}{\partial \lambda} = -\frac{1}{4}Ae^{-\alpha^2 r^2/2}\frac{\partial\, {}_1F_1}{\partial a}\,\Big|_{\lambda=N}$$

and using (31) , we obtain

$$\frac{\partial R}{\partial \lambda}\,\Big|_{\lambda=N} = \frac{-Ae^{\alpha^2 r^2/2}}{4(\alpha r)^N}\Gamma\left(\frac{N}{2}\right)\left[1+\frac{N}{2}\frac{1}{\alpha^2 r^2}\right] \tag{25}$$

Therefore, using the above equation and equations (13) and (17) we obtain the boundary correction at r = S to the ground – state energy:

$$\Delta\varepsilon_{\circ} = \frac{2\hbar\omega e^{-\alpha^2 S^2}}{\Gamma(N/2)}(\alpha S)^{N-2}\left[\alpha^2 S^2 - \frac{N}{2}\right], \tag{26}$$

which shows its dependence on the space dimension *N*.

We must note that the expression in Eq.26 is a double approximation. It is an asymptotic form of the linear approximation method, and hence it is valid for large values of S/$r_\circ$. For small values of S/$r_\circ$, but within the linear approximation method, on has to use Eq.15 which in general does not correspond to a simple analytic expression. In actual electronic- structure calculations this does not pose a problem, since there the wave function and its derivative are computed numerically. In this paper, for pedagogical purposes, we calculated the asymptotic formula (Eq.26), which corresponds to a simple analytic expression. For numerical purposes, we choose the radius of the cavity to be S=4/α. For this value and ℓ=0 for the ground-state, in the first method, we use Mathematica to search for the value of a at which the confluent hypergeometric function ${}_1F_1(a,b,16)$ has its first zero. In this case $b=N/2$ and $a = (N - \lambda)/4$. The zeroes of ${}_1F_1$ occur at negative values of a and thus the ground-state energy for the confined oscillator can be written as $E_0 = \hbar\omega(\ 2\left|a\right| + N/2\ )$. Knowing the ground-state energy for the free harmonic oscillator ($\varepsilon_0 = \hbar\omega N/2$), we compute the energy correction $\Delta\varepsilon_0$ for the ground-state due to the presence of the boundary. These results are presented in table 1 in which the energies and the energy corrections are in units of $\hbar\omega$.

In the second method, we use the asymptotic form of the linear approximation method, given in Eq. 26, to calculate the ground-state energy correction $(\Delta\varepsilon_0)_{app}$. These results are presented in table 2, in which we also include the percentages of the energy corrections $(\Delta\varepsilon_0/\ \varepsilon_0)_{num}\%$ and $(\Delta\varepsilon_0/\ \varepsilon_0)_{app}\%$ obtained by numerical method and approximation method, respectively.

Table 1: Numerical values of $|a|\ (S = 4/\alpha)$ *and* $E_\circ$ for the ground state of the confined harmonic oscillator, $\varepsilon_\circ$ and $r_\circ$ for the ground state of the free harmonic oscillator, and energy correction $\Delta\varepsilon_\circ$ for different dimensions.

N	$\lvert a\rvert$	E_0	$\varepsilon_\circ$	$\Delta\varepsilon_\circ$	$r_\circ$
2	1.68 X10^{-6}	1.00000336	1	3.36 X10^{-6}	0.7071
3	7.302 X10^{-6}	1.5000146	1.5	1.46 X10^{-5}	1
4	2.489 X 10^{-5}	2.00004978	2	4.978 X 10^{-5}	1.22474
5	7.19 X 10^{-5}	2.5001438	2.5	1.438 X 10^{-4}	1.41421
6	1.83 X 10^{-4}	3.000366	3	3.66 X 10^{-4}	1.58113
7	4.21 X 10^{-4}	3.500842	3.5	8.42 X 10^{-4}	1.73205
8	8.89 X 10^{-4}	4.001778	4	1.778 X 10^{-3}	1.87082
9	1.745 X 10^{-3}	4.50349	4.5	3.49 X 10^{-3}	2
10	3.21026 X 10^{-3}	5.0064205	5	6.4205 X10^{-3}	2.12132
11	5.579 X 10^{-3}	5.511158	5.5	0.011158	2.23606
12	9.213 X 10^{-3}	6.018426	6	0.018426	2.3452
13	0.014535	6.52907	6.5	0.02907	2.44948
14	0.022	7.044	7	0.044	2.5495
15	0.0322	7.5644	7.5	0.0644	2.6457
16	0.04546	8.09092	8	0.09092	2.7386
17	0.06243	8.62486	8.5	0.12486	2.8284
18	0.0835	9.167	9	0.167	2.91547
19	0.10922	9.71844	9.5	0.21844	3
20	0.14	10.28	10	0.28	3.0822

It is instructive to plot a graph between the ground state energy correction, calculated by numerical and linear approximation methods, and the space dimension N. This is shown in Fig.1. It is clear that the results for the energy correction using the linear approximation method are in excellent agreement with those obtained numerically.

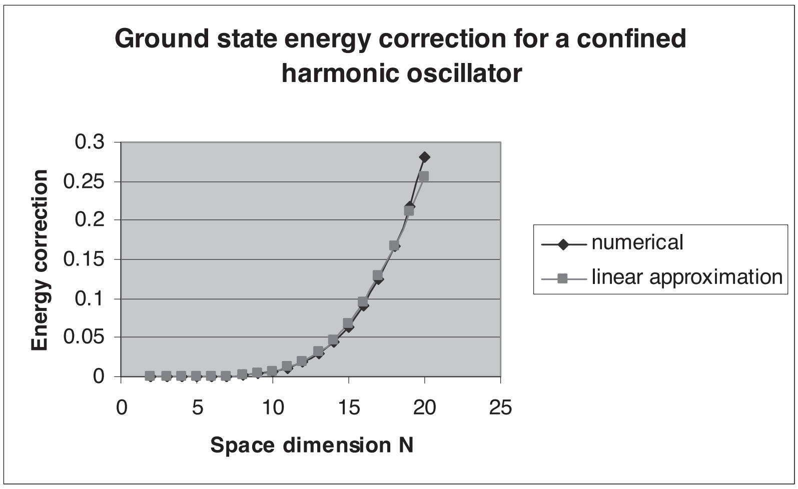

Fig. 1: Ground-state energy correction for a confined harmonic oscillator vs. space dimension N, by numerical and linear approximation methods.

Table 2: Energy correction $\Delta\varepsilon_\circ$ by linear approximation and the percentages $\left(\frac{\Delta\varepsilon_\circ}{\varepsilon_\circ}\right)num\%$ and $\left(\frac{\Delta\varepsilon_\circ}{\varepsilon_\circ}\right)\%$ app. for different dimensions for a cavity radius, $(S = 4/\alpha)$.

N	$(\Delta\varepsilon_\circ)_{app}$	$\left(\frac{\Delta\varepsilon_\circ}{\varepsilon_\circ}\right)num\%$	$\left(\frac{\Delta\varepsilon_\circ}{\varepsilon_\circ}\right)_{app}\%$
2	3.376×10^{-6}	3.36×10^{-4}	3.376×10^{-4}
3	1.473×10^{-5}	9.73×10^{-4}	9.82×10^{-4}
4	5.04112×10^{-5}	2.49×10^{-3}	2.52×10^{-3}
5	1.4628×10^{-4}	5.75×10^{-3}	5.85×10^{-3}
6	3.74516×10^{-4}	1.22×10^{-2}	1.248×10^{-3}
7	8.6686×10^{-4}	2.4057×10^{-2}	2.4767×10^{-2}
8	1.84377×10^{-3}	4.45×10^{-2}	4.609×10^{-2}
9	3.64578×10^{-3}	7.7555×10^{-2}	8.101×10^{-2}
10	6.7605×10^{-3}	1.2841×10^{-1}	1.1267×10^{-1}
11	1.18356×10^{-2}	0.20287	2.1519×10^{-1}
12	1.9667×10^{-2}	0.3071	0.3277
13	3.11517×10^{-2}	0.44723	0.4792
14	4.7200×10^{-2}	0.62857	0.67428
15	6.86094×10^{-2}	0.85867	0.91478
16	9.58996×10^{-2}	1.1365	1.1987
17	0.129147	1.4689	1.5193
18	0.167824	1.8555	1.8647
19	0.210687	2.2993	2.2177
20	0.2557	2.8	2.557

It is noticed that as the dimension N increases the energy correction $\Delta\varepsilon_\circ$ increases, which can be explained as follows: $r_\circ$ increases with N as $\sqrt{(N-1)/2}$, as Eq.13 yields for n=0, therefore, one expects the effect of the boundary on the energy correction increases as *N* increases.

It is interesting to calculate the pressure needed to compress an *N*- dimensional harmonic oscillator in its ground state to a certain size. This is of interest in dealing with studies on quantum systems enclosed in boxes, like the fabrication of semiconductor quantum dots (Varshni, 1997). The pressure p(s) needed to compress the system is

$$P(s) = -\frac{\partial}{\partial v}\Delta \varepsilon_{\circ} = -\frac{\partial}{\partial S}\left(\Delta \varepsilon_{\circ}\right)\frac{\partial S}{\partial V}$$

The volume, in *N*- dimensional space, of a sphere of radius S is

$$V = \frac{\pi^{N/2} S^N}{\Gamma(1+N/2)}$$, and thus

$$\frac{\partial V}{\partial S} = \frac{N\pi^{N/2} S^{N-1}}{\Gamma(1+N/2)} \Rightarrow \frac{\partial S}{\partial V} = \frac{\Gamma\left(1+N/2\right)}{N\pi^{N/2} S^{N-1}} = \frac{\Gamma(N/2)}{2\pi^{N/2} S^{N-1}}$$

Therefore,

$$P(s) = \frac{-\Gamma(N/2)}{2\pi^{N/2} S^{N-1}}\frac{\partial}{\partial S}\left(\Delta \varepsilon_{\circ}\right) \tag{27}$$

Using Eq.33, we get

$$\frac{\partial}{\partial S}\left(\Delta \varepsilon_{\circ}\right) = \frac{2\hbar\omega}{\Gamma\left(\frac{N}{2}\right)}\left[-2\alpha^{N+2} S^{N+1} - \frac{N}{2}(N-2)\alpha^{N-2} S^{N-3} + 2N\alpha^N S^{N-1}\right]e^{-\alpha^2 S^2} \tag{28}$$

The substitution of Eq. 27 into the above expression gives:

$$P(S) = \frac{\hbar\omega}{\pi^{N/2}}\alpha^N\left[2\alpha^2 S^2 + \frac{N}{2}\frac{(N-2)}{\alpha^2 S^2} - 2N\right]e^{-\alpha^2 S^2} \tag{29}$$

For numerical purposes, again we choose S= 4/α, to calculate the pressure needed to compress an *N*- dimensional harmonic oscillator in the ground state. This is shown in table 3 below, in which p(s) is in units of $\hbar\omega\alpha^N$.

It is noticed that as the dimension *N* increases the pressure decreases, which is again due to the increase of r_0. This means that it becomes easier to compress the free oscillator to a given size as the dimension *N* increases.

It is interesting to plot a graph that shows the dependence of the pressure P on the space dimension N for the cavity radius S=4/α. This is shown in Fig. 2, which shows that pressure decreases as the space dimension N increases.

Table 3 : Pressure *P(s)* to compress a free harmonic oscillator to a size $S = 4/\alpha$ in different dimensions.

N	*P(s)*
2	1.003×10^{-6}
3	5.2765×10^{-7}
4	2.785×10^{-7}
5	1.4454×10^{-7}
6	7.5310×10^{-8}
7	3.9098×10^{-8}
8	2.0217×10^{-8}
9	1.0408×10^{-8}
10	5.3322×10^{-9}
11	2.7166×10^{-9}
12	1.37539×10^{-9}
13	6.91366×10^{-10}
14	3.4465×10^{-10}
15	1.7014×10^{-10}
16	8.30207×10^{-11}
17	3.9939×10^{-11}
18	1.8876×10^{-11}
19	8.7193×10^{-12}
20	3.90546×10^{-12}

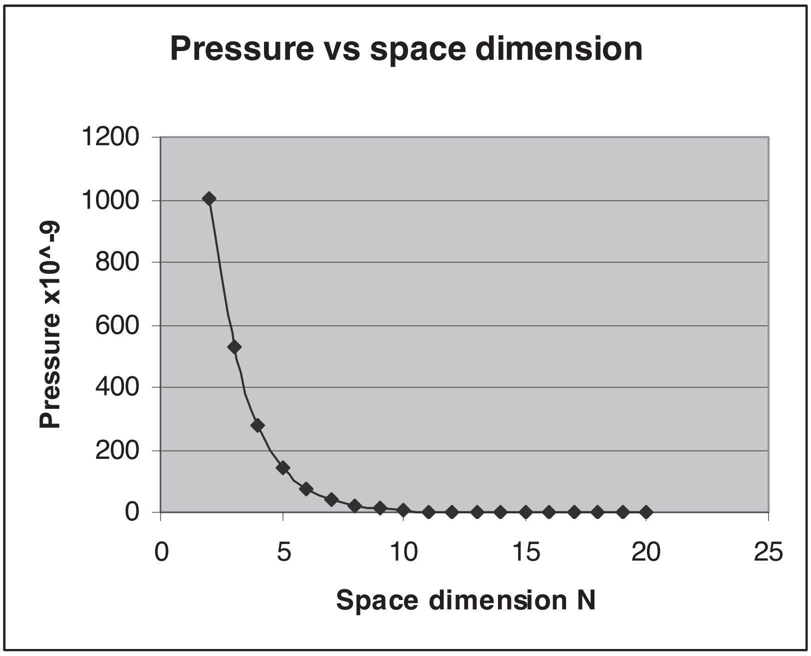

Fig. 2: Pressure needed to compress a free harmonic oscillator to a size S=4/α vs. space dimension N.

SUMMARY AND CONCLUSIONS

In this paper, a linear approximation method has been used to calculate the asymptotic dependence of the ground- state energy of an *N*- dimensional harmonic oscillator confined in a spherical cavity on the radius of the cavity and the space dimension *N*. The asymptotic formula derived in this paper, for pedagogical purposes, does correspond to a simple analytical expression which is valid for $S >> r_\circ$, that is when the radius of the cavity is much greater than the radius at which the radial distribution function for the free- space case is maximum. The ground-state energy, E_0, in the presence of the cavity was calculated by two methods for a selective value of the radius S of the cavity, namely $S=4/\alpha$. In the first method, E_0 was calculated numerically by using Mathematica. In the second method, E_0 was found by the linear approximation method. The energy corrections $\Delta\varepsilon_0$ and their percentages in both methods were calculated by comparing E_0 to the ground-state energy ε_0 for the free harmonic oscillator. It was demonstrated that these energy corrections obtained by the linear approximation method are in very good agreement to those obtained numerically. Our results also showed that the energy correction increases as the dimension *N* increases, which is due to the increase of r_0 at which the distribution function for the free oscillator has it maximum. In addition, the pressure needed to compress a free *N*-dimensional harmonic oscillator in its ground-state to a certain size has been calculated. It was shown that this pressure decreases as the dimension *N* increases which is due to the increase of r_0, at which the ground-state distribution function of the free harmonic oscillator has its maximum.

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متذبذب توافقي في منخفض لا نهائي كروي في بعد نوني

جبـــــر، سامـــي

قسـم الـفيــزيـــاء ،جامعة النجاح الوطنية، نابلـس ، فلسطيـــن

الملخص

في هذا البحث تم مناقشة التأثيرات الحدودية على نظام كمي، وذلك بالتمعن في متذبذب توافقي محاط في منخفض لا نهائي كروي في فراغ متعدد الأبعاد. وتم حساب التصحيحات لطاقة الحالة الدنيا ودالتها الموجبة الناتجة عن التأثيرات الحدودية والبعد الفراغي باستخدام طريقة التقريب الخطي والطريقة العددية باستخدام برنامج ماثماتيكا حيث نحصل على نتائج متقاربة. إضافة إلى ذلك، تم اشتقاق تعبير تحليلي بسيط لاعتماد طاقة الحالة الدنيا على نصف قطر المنخفض والبعد النوني. أخيراً تم حساب الضغط اللازم لضغط متذبذب توافقي حر في بعد فراغي نوني إلى حجم معين.