

Calculation of dynamic stresses acting on wind turbine blades using finite element method

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ABSTRACT

The aim of this work is the calculation of dynamic loads and stresses acting on wind turbine blades. The prediction of the dynamic behaviour of the blades constitutes is one of the most important processes in the design of wind turbines.

Initially, the blade element theory was used to calculate aerodynamic loads applied on the rotor. This method can be also used to estimate the power coefficient and the total power extracted by the turbine. A modal analysis of the rotor was carried out in order to compute the frequencies and the mode shapes of the blades. This analysis is useful for estimating dynamic loads.

Finally, the dynamic stresses were calculated for the root region of the blades using finite element modelling. The resulting curves of stresses versus time, obtained for different wind speeds, may be utilized in a latter study for fatigue analysis, in order to make an optimal choice of blades resistant to fatigue and being energetically efficient.

Keywords: Wind Energy, structural dynamics, Aerodynamics, and Numerical Analysis.

NOMENCLATURE

A_i	cross section area of air flow at station i
a	Axial interference factor
a'	Tangential interference factor
B	blade number
C_L	lift coefficient
C_D	drag coefficient
C_p	power coefficient
D	drag force
dF_x	Differential tangential force applied on one blade.
dF_y	Differential axial force applied on one blade
L	lift force
P	extracted power

Q	aerodynamic torque
T	thrust force
V_i	wind speed at station i
W	relative wind speed
α	Angle of attack
β	Twist angle
λ	Tip speed ratio
Ω	Rotational speed of the rotor
ω	Angular velocity of the wake
ϕ	Flow angle of relative wind speed
σ	Local solidity

INTRODUCTION

The prediction of the dynamic behaviour of the blades constitutes one of the most important processes in the design of wind turbines, due to its role in estimating the energetic performance of turbine as well as in predicting the structural problems such as fatigue failure.

The analysis of this dynamic behaviour is a complicated task and it can be undertaken by a variety of techniques (Younsi, 2001).

The improvement in the design of a wind turbine can result in a substantial increase in the total energy produced as well as in a decrease of the system energy cost (Veers and Ashwill, 2003).

For small energy systems, where the cost of the energy produced is still high and site choice is often imposed, the design success is largely dependent on the dynamic and aerodynamic modelling of the rotor (Rasmunssen and Hansen, 2003).

In the first part of this work, aerodynamic modelling is made using two aerodynamic theories, the first one is the axial momentum theory and the second is the blade element theory. In the first theory, the flow is considered to be completely axial, while in the second theory the effect of wake rotation is included, assuming that the flow downstream rotates.

The momentum theory that employs the mass and momentum conservation principles cannot provide alone the necessary information for the rotor design. However, the blade element theory that uses the angular momentum conservation principal, gives complementary information about the blade geometry such as airfoil shape and twist distribution. When both theories are combined the aerodynamic loads can be obtained.

In the second part of this work, a modal analysis is performed using the finite element method, in order to calculate frequencies and mode shapes of the blades. Dynamic stresses are also computed using finite element modelling; these alternating stresses can be used latter on to estimate the blade fatigue.

AERODYNAMIC CALCULATION:

The objective of this part is to estimate the aerodynamic loads, which are essential to design of wind systems. These loads are required for predicting and analyzing wind system energetic performance and for structural design as well.

In this aerodynamic modelling two aerodynamic theories were used, the first one is the axial momentum theory and the second is the blade element theory.

THE AXIAL MOMENTUM THEORY

In this simple one-dimensional model, airflow is assumed to be incompressible, completely axial and rotationally symmetric. This model applies the principles of mass and momentum conservation on the annular control volumes surrounding the flow as shown in Fig. 1.

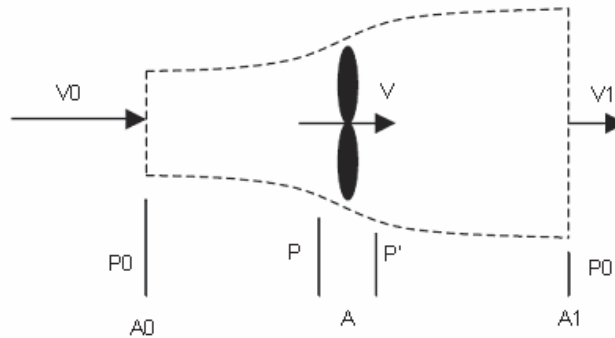


Fig.1. Annular control volume

Applying the conservation of mass to the control volume yields:

$$V_0 A_0 = V A = V_1 A_1 = V_i A_i \tag{1}$$

The thrust force at the rotor disc T can be found by applying the conservation of linear momentum to the control volume in the axial direction:

$$T = \dot{m}(V_0 - V_1) = \rho A V (V_0 - V_1) \tag{2}$$

Where ρ is the density of the air. Bernoulli's equation can be applied to obtain the pressure difference across the rotor plane:

$$p - p' = \frac{1}{2} \rho (V_0^2 - V_1^2) \tag{3}$$

The thrust is given as:

$$T = \frac{1}{2} A \rho (V_0^2 - V_1^2) \tag{4}$$

The velocity of the flow through the rotor disc is found to be the average of the upwind (free stream) and downwind velocities:

$$V = \frac{V_0 + V_1}{2} \tag{5}$$

The power P extracted from the wind by the rotor is:

$$P = \frac{1}{2} \dot{m} (V_0^2 - V_1^2) = \frac{1}{2} \rho V A (V_0^2 - V_1^2) \tag{6}$$

The power coefficient, C_P , is defined as the ratio of the power extracted by the turbine (P) to available power in the wind, i.e.:

$$C_p = \frac{P}{\frac{1}{2}\rho V_0^3 A} \quad (7)$$

Introducing the axial interference factor, a , which is defined as the fractional decrease in wind velocity between the free stream and the rotor plane:

$$V = (1-a)V_0 \quad (8)$$

The trust expression of Eqn.(4) becomes:

$$T = \frac{1}{2}\rho A V_0^2 4a(1-a) \quad (9)$$

The power extracted by the rotor is:

$$P = \frac{1}{2}\rho A V_0^3 4a(1-a)^2 \quad (10)$$

The expression of C_p becomes:

$$C_p = 4a(1-a)^2 \quad (11)$$

THE BLADE ELEMENT THEORY

This analysis, which uses the angular momentum conservation principle and the blade geometry properties to determine the forces and torque exerted on a wind turbine, is referred to as *blade element theory* (Lysen, 1983).

The control volume used in the previous one-dimensional model can be divided into several annular stream tube control volumes, which split the blade into a number of distinct elements, each of length dr (Fig. 2). In this case, the differential area of an annular ring at station i , dA_i , is defined as: $dA_i = 2\pi r_i dr_i$

In this theory, it is assumed that there is no interference between these blade elements and the blade elements behave as airfoils.

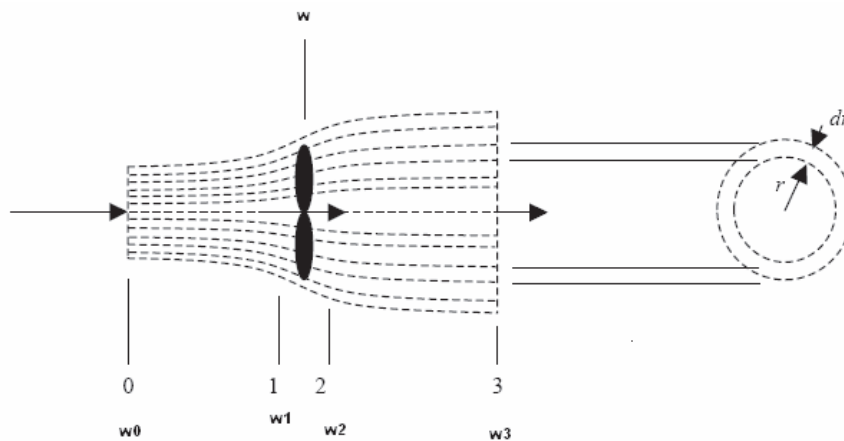


Fig. 2. Annular stream tube control volumes

The differential rotor thrust, dT , at a given span location on the rotor (at a specified radius r) can be derived from the previous theory using equation (9):

$$dT = 4a(1-a)\rho V_0^2 \pi r dr \quad (12)$$

In the previous model, it was assumed that airflow doesn't rotate. However, the conservation of angular momentum implies the rotation of the wake if the rotor is to extract useful torque. Moreover, the flow behind the rotor will rotate in the opposite direction (Jonkman, 2003), as shown in Fig. 3:

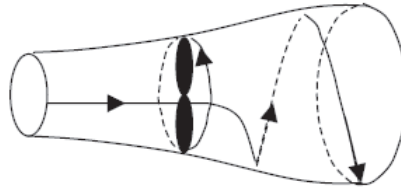


Fig.3. Wake rotation

The effect of wake rotation will be now included. In describing this effect, the assumption is made that upstream of the rotor, the flow is entirely axial and that the flow downstream rotates with an angular velocity ω . The conservation of angular momentum can be applied to obtain the differential torque at the rotor disc, dQ , resulting in:

$$dQ = 2\pi\rho V \omega r^3 dr \quad (13)$$

The total torque is:

$$Q = 2\pi\rho \int_0^R V \omega r^3 dr \quad (14)$$

The differential extracted power is given by the expression:

$$dP = 2\pi\rho\Omega V \omega r^3 dr \quad (15)$$

The total extracted power is:

$$P = 2\pi\rho\Omega \int_0^R V \omega r^3 dr \quad (16)$$

In order to calculate P and Q , the wake angular velocity ω has to be known. The tangential interference factor a' (Fig. 4) is defined as:

$$\omega = a'\Omega \quad (17)$$

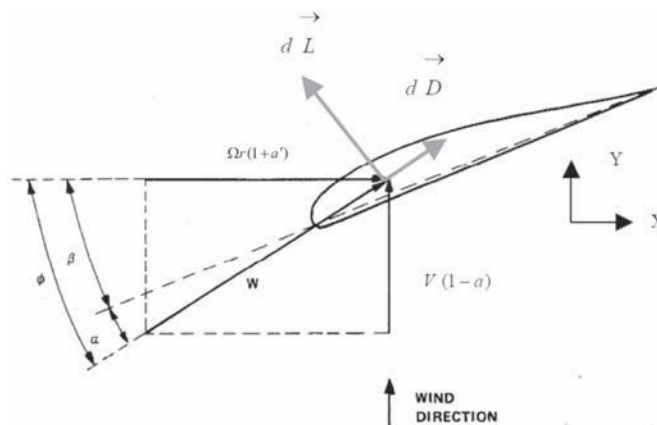


Fig. 4. Blade element section at radius r

The differential lift and drag forces are:

$$dL = C_L dq \quad (18)$$

$$dD = C_D dq \quad (19)$$

With:

$$dq = \frac{1}{2} \rho W^2 dA = \frac{1}{2} \rho W^2 c dr \quad (20)$$

Where C_L and C_D are lift and drag coefficient.

The components of the resulting force are:

$$dF_x = C_x dq, \quad (21)$$

$$dF_y = C_y dq, \quad (22)$$

where:

$$C_y = C_L \cos \phi + C_D \sin \phi \quad (23)$$

$$C_x = C_L \sin \phi - C_D \cos \phi \quad (24)$$

The following relation can be derived from Fig. 4:

$$\tan \phi = \frac{(1-a)V_0}{(1+a')\Omega r}, \quad (25)$$

where:

$$\alpha = \phi - \beta \quad (26)$$

The differential thrust and torque can now be derived as follows:

$$dT = BC_y dq = BC_y \frac{1}{2} \rho W^2 c dr \quad (27)$$

$$dQ = BC_x dq r = BC_x \frac{1}{2} \rho W^2 c r dr \quad (28)$$

Equating the thrust in equations (12) and (27) as well as the torque in Eqns. (13) and (28), will yield to the expressions of the both interference factors:

$$a = \frac{1}{\frac{4 \sin^2 \phi}{\sigma C_y} + 1} \quad (29)$$

$$a' = \frac{1}{\frac{4 \sin \phi \cos \phi}{\sigma C_x} - 1} \quad (30)$$

Where σ is the local solidity, defined as:

$$\sigma = \frac{cB}{2\pi r} \quad (31)$$

In order to estimate the loads applied on the rotor, an iterative method should be used to determine the values of the interference factors.

For each element at radius r , the following steps are carried out (Wood, 2002):

1. An initial reasonable guess of a and a' is given.
2. ϕ and α are then calculated using Eqns. (25) and (26).
3. C_L and C_D are estimated as a function of α by approximation method.

4. a and a' are finally calculated using Eqns. (29) and (30).
5. These steps are repeated till the successive values of a and a' converge.

Once the values of local (differential) thrust and torque are known they may be integrated numerically to determine the overall torque and thrust as well as the total extracted power. Table 1 lists the axial and the tangential loads and the torque at different blade stations.

Table 1. Distribution of aerodynamic loads wind speed 15 m/s profile NACA 63-421

Station (r/R)	Axial force(N)	Tangential force(N)	Torque (N.m)
0.16	86.02	221.24	206.30
0.20	85.93	286.49	263.25
0.25	81.92	351.16	305.56
0.29	73.54	411.09	323.48
0.34	73.37	466.19	372.82
0.38	38.41	535.52	316.39
0.43	57.87	586.57	467.49
0.47	208.88	556.61	1008.82
0.51	35.67	764.83	724.62
0.56	304.36	649.67	1690.88
0.60	39.33	908.37	1120.22
0.65	354.32	759.48	2370.09
0.69	83.60	998.27	1686.21
0.73	320.40	901.37	2780.43
0.78	221.50	1012.80	2591.9
0.82	217.84	1063.01	2693.02
0.87	211.31	1109.45	2746.15
0.91	191.81	1148.48	2624.72
0.96	169.79	1181.84	2434.31
1.00	140.04	1206.90	2100.65

Total axial force on one blade = 553.24 N
 Total tangential force on one blade = 4729.56 N
 Total axial force on the rotor= 1659.72N
 Torque = 8218.81 N.m

CALCULATION OF MODE SHAPES AND FREQUENCIES USING FINITE ELEMENT MODELLING

The method used in this part is a finite element modelling of a complex geometry blade using ANSYS software. The blade used is twisted, with a variable chord length and having a complex form at the root region. This blade has the following characteristics:

Profile: NACA63-421
 Length: 5m
 Maximum chord length: 0.6 m
 Average chord length: 0.4
 Maximum twist angle: 14°

After the geometrical modelling and the meshing of this blade with shell elements using ANSYS, the following figure of the blade is obtained (Fig. 5):

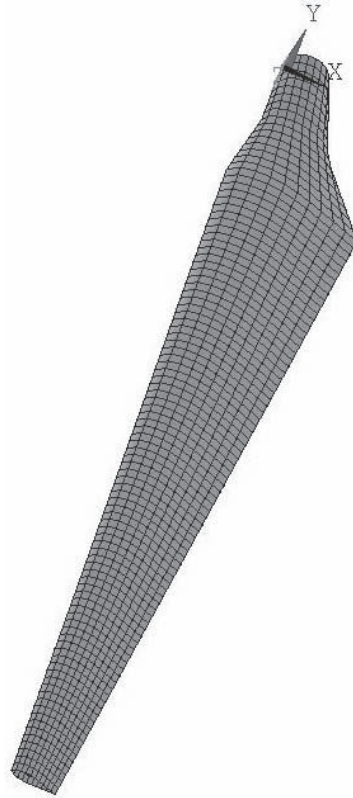


Fig. 5: Geometry modelling of the blade (using ANSYS)

The modal analysis using ANSYS gives the following results:

Table 2 gives the first three frequencies.

Table 2. Resulting blade frequencies calculated by FEM

Mode number	(Hz) ω Frequency
First mode	8.37
Second mode	14.94
Third mode	62.74

The blade mode shape deformation obtained by ANSYS software is given by Fig. 6 through Fig. 8.

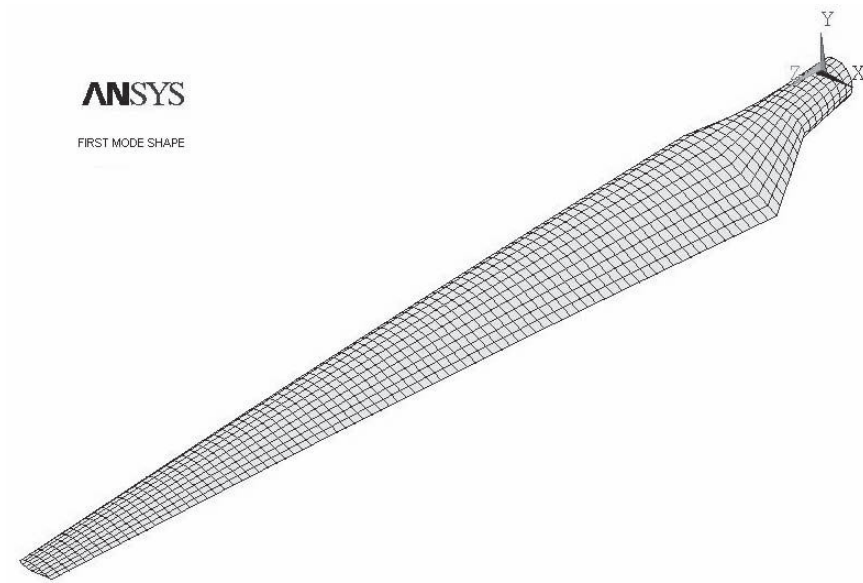


Fig. 6. First mode shape deformation

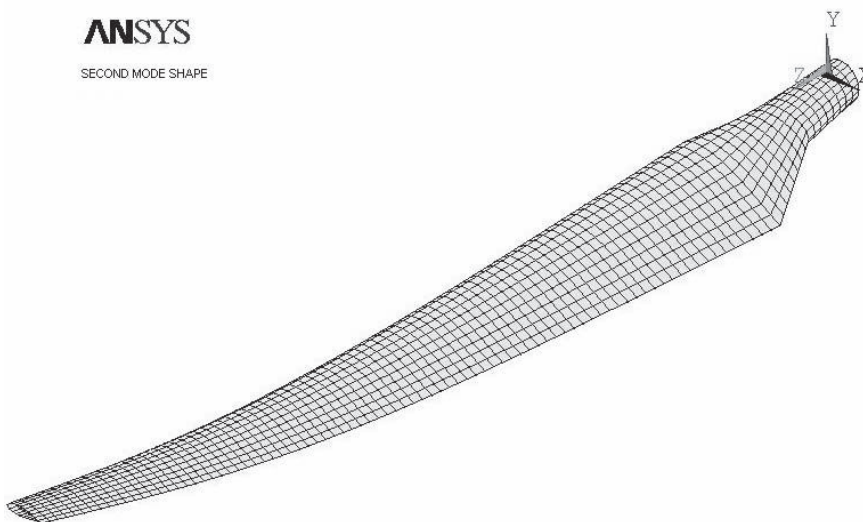


Fig. 7. Second mode shape deformation

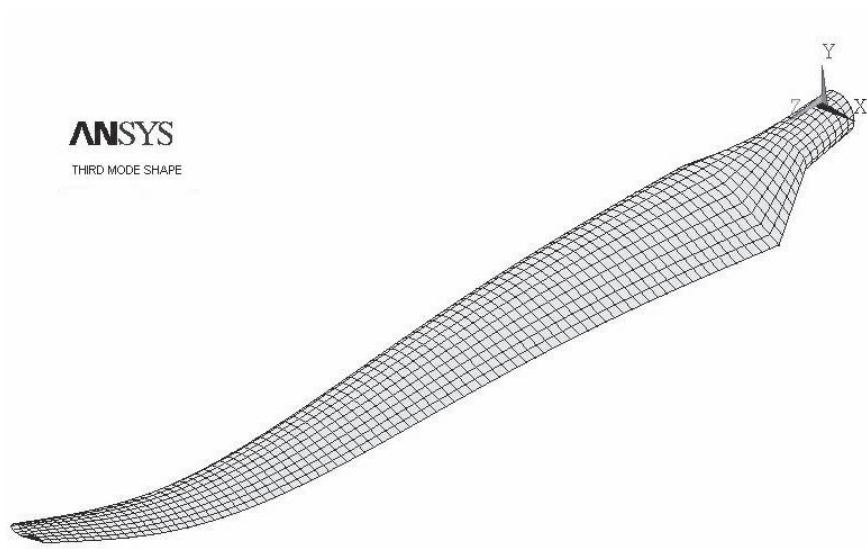


Fig. 8. Third mode shape deformation

The flapwise mode shape curves are given by Fig. 9 through Fig. 11

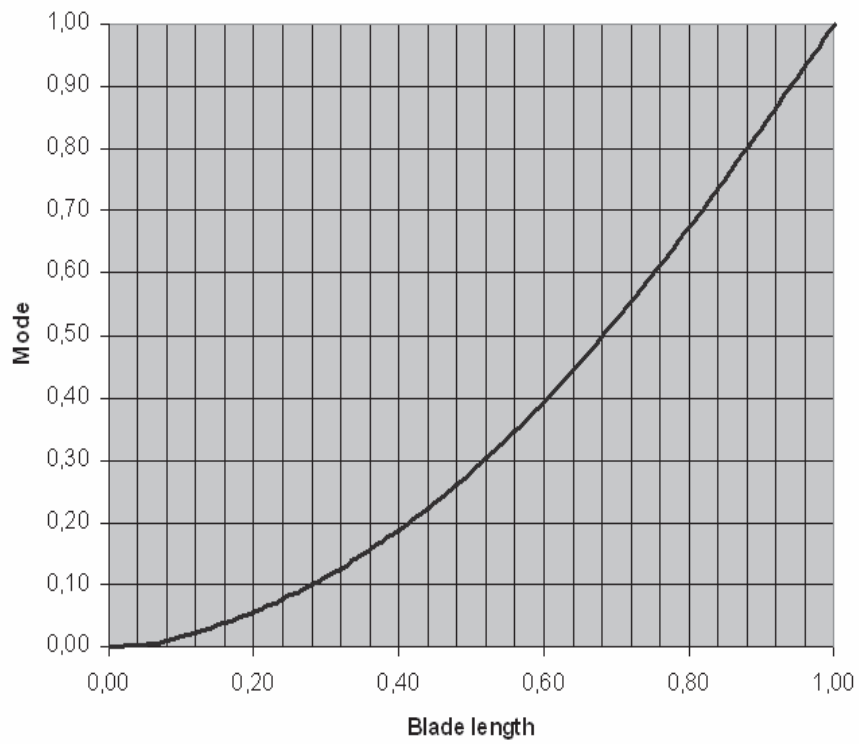


Fig.9. First flapwise mode shape by ANSYS

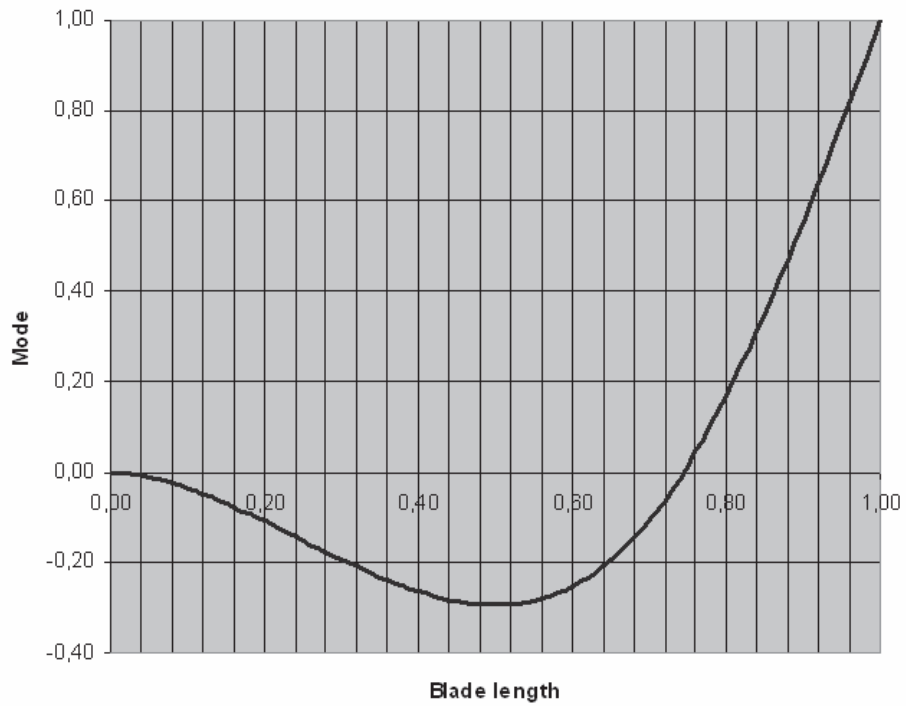


Fig. 10. Second flapwise mode shape by ANSYS

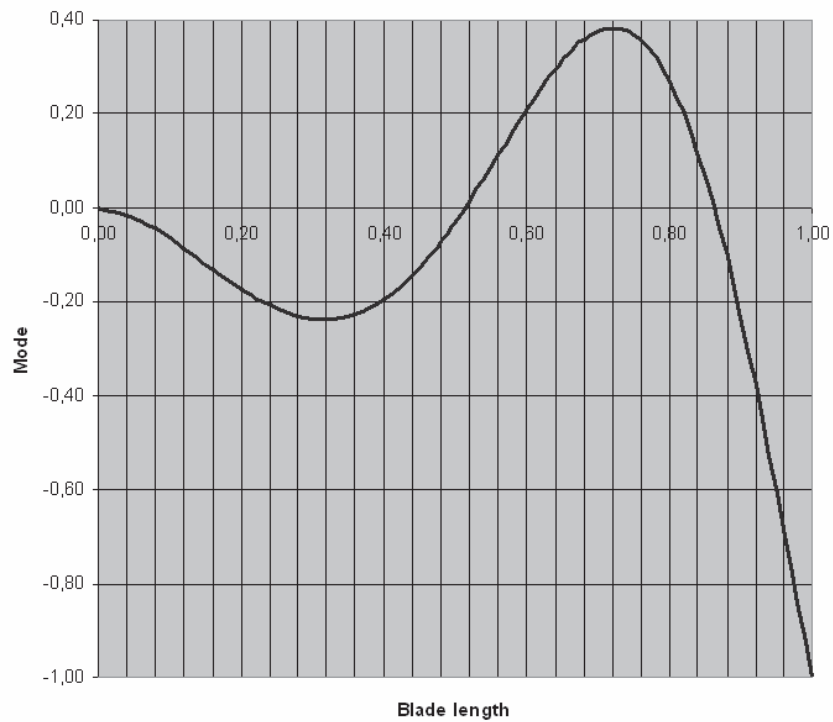


Fig. 11. Third flapwise mode shape by ANSYS

DYNAMIC STRESS CALCULATION USING FINITE ELEMENT MODELLING

The results obtained previously such as loads from aerodynamic calculations and mode shapes and frequencies from the modal analysis may be combined to compute dynamic forces. These forces are required to calculate the dynamic stress.

An ansys source program (log file) was implemented to perform the dynamic analysis using finite element modelling.

The following results of the equivalent stresses in the root region of the blade, for different wind speeds, are obtained (Fig. 12. through Fig. 16):

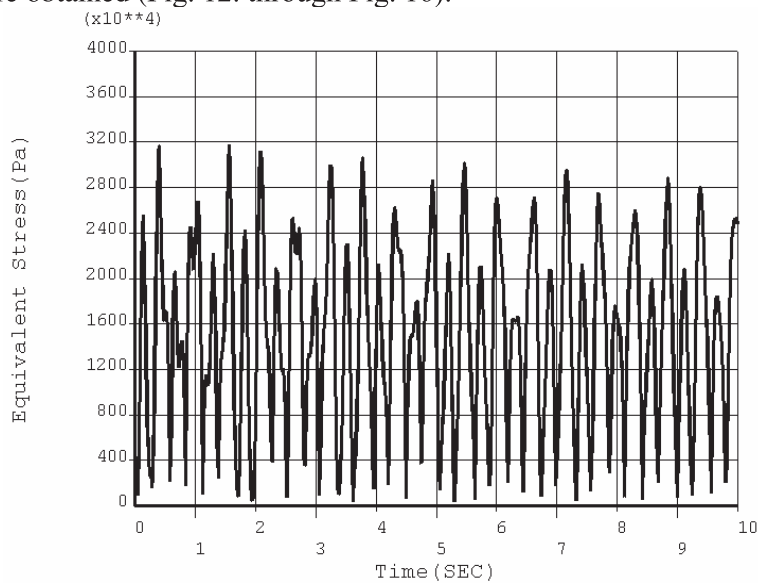


Fig.12. Equivalent alternating stress at the blade root Profile NACA63-421 Material composite (wind speed 7 m/s)

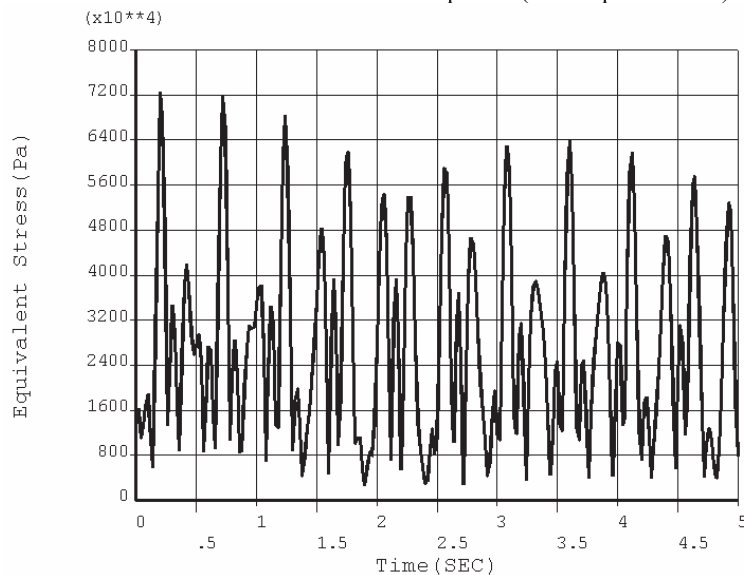


Fig. 13. Equivalent alternating stress at the blade root Profile NACA63-421 Material composite (wind speed 15 m/s)

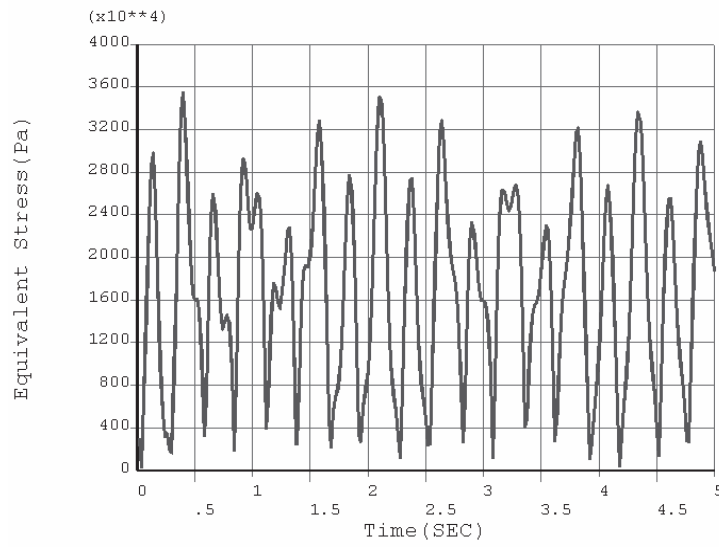


Fig. 14. Equivalent alternating stress at the blade root Profile S809 Material composite (wind speed 7 m/s)

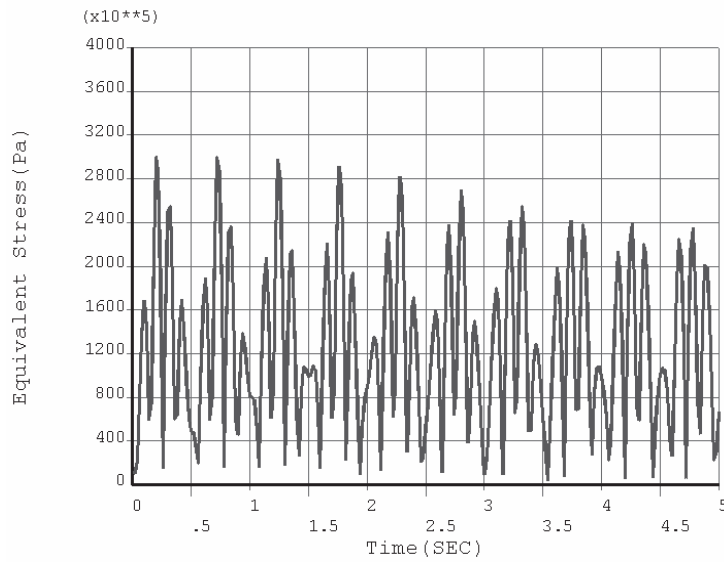


Fig.15. Equivalent alternating stress at the blade root Profile S809 Material composite (wind speed 15 m/s)

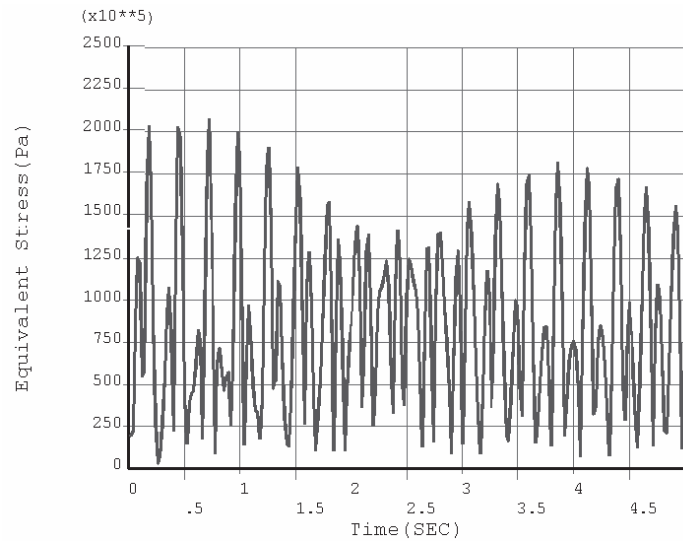


Fig. 16. Equivalent alternating stress at the blade root
Profile NACA63-421 Material Aluminium (wind speed 15 m/s)

CONCLUSION

In this work the blade element theory was used to calculate aerodynamic loads for small wind turbine blades. This method can also be used to estimate the power extracted by the turbine.

A modal analysis of the blades was also performed using a finite element modelling to compute the frequencies and the mode shapes. This analysis is essential for estimating the dynamic loads. The resulting mode shapes were compared with those obtained by Baumgart (Baumgart, 2002). The corresponding modes have very similar shapes.

Finally, dynamic stresses were calculated for the root region of the blades using finite element modelling. This root region is a highly loaded and structurally complex area.

When blades with different profiles and materials were used, the previous calculations had shown that the blade having NACA 63-421 profile and a composite material (fiber glass), can withstand higher loads resulting from a wind speed that exceeds 15 m/s.

The alternating stresses, obtained from dynamic analysis, can be used to estimate the blade fatigue by miner's rule approach (Juvinal, 1980). This fatigue analysis aims to make an optimal design of a rotor having resistant and energetically efficient blades.

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حساب الاجهادات الديناميكية المؤثرة على شفرات توربينات الرياح

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الملخص:

يهدف هذا العمل لحساب القوى والاجهادات الديناميكية التي تؤثر على شفرات توربين الرياح. تعتبر دراسة السلوك الديناميكي لشفرات هذه التوربينات من أهم مراحل تصميم هذه الآلة. بداية، استخدمت نظرية عنصر الشفرة (Blade Element Theory) لحساب القوى الانسيابية المؤثرة على الشفرات. وهذه النظرية تمكن أيضا من حساب الطاقة الكلية المحولة من طرف التوربين. و في مرحلة ثانية تم حساب الأنماط والذبذبات (التواترات) المتعلقة بها، وهذا الحساب يمكننا من تقدير القوى الديناميكية المطبقة. وأخيرا، تم تعيين الاجهادات الديناميكية بطريقة الأجزاء المنتهية.

إن هذه النتائج المتحصل عليها (منحنيات الاجهادات الديناميكية بدلالة الزمن) تمكننا في مرحلة لاحقة من حساب الإعياء من أجل تصميم أمثل لتوربينات تقاوم الإعياء وتتميز بمردود طاقة عال.