



PAPR Reduction of Localized Single Carrier FDMA using Partial Transmit Sequence in LTE Systems

Ahmed J. Jameel

Department of Telecommunication Engineering, Ahlia University, Manama, Kingdom of Bahrain

Received: 3 Sep. 2016, Revised: 12 Dec. 2016, Accepted: 20 Dec. 2016, Published: (1 January 2017)

Abstract: In this paper, the partial transmit sequence (PTS) technique is used to decrease the peak-to-average power ratio (PAPR) of the localized single-carrier frequency division multiple access (LFDMA) in the uplink of LTE systems. For a clipping rate of 0.1% with 64 user subcarriers and 256 system subcarriers with four different phase sequences; the gain of PAPR for the suggested PTS based LFDMA system is 6.56 dB and 3.06 dB, respectively when compared with the orthogonal frequency division multiple access (OFDMA) and the LFDMA uplink systems with QPSK modulation.

Keywords: Partial transmit sequence (PTS), Peak-to-Average Power Ratio (PAPR), Localized Frequency Division Multiple Access (LFDMA).

1. INTRODUCTION

The requirements of high data rate for wireless and mobile multimedia services have oriented the interest to high speed broadband wireless mobile technologies. Orthogonal frequency division multiplexing (OFDM) as a multi-carrier communication technique, has been adopted in many applications because of its performance against the broadband mobile wireless communications' frequency selective fading effect [1]. Orthogonal frequency division multiple access (OFDMA) represents the natural extension of OFDM to support many simultaneous users. The main disadvantage of OFDM and OFDMA is their high peak-to-average power ratio (PAPR) [2].

There are many solutions already proposed to decrease the PAPR values for the multicarrier systems. These solutions may be categorized into signal scrambling, such as selective mapping (SLM) and partial transmit sequence (PTS), and signal distortion methods such as clipping. If clipping based PAPR reduction method is used, then the signal with high peak will be clipped which results in loss of useful information. Because of this loss of useful information, the clipping based method produces in-band distortion as well as out-of-band radiation. Hence, the clipping technique is not suitable for PAPR reduction. On the other hand, the PTS based PAPR reduction technique does not create any in-band distortion or out-of-band radiation. Therefore, the PTS based technique is always better than the clipping based technique [3].

Single-carrier frequency division multiple access (SC-FDMA) represents a technique that is derived from

OFDMA with similar throughput, performance, and with comparable complexity. The main advantage of

the SC-FDMA technique over OFDMA is that it has lower PAPR because of its single carrier structure [4]. SC-FDMA is a good alternative to OFDMA particularly in the uplink communications, where lower PAPR noticeably improves the performance of the mobile device in terms of the transmit power efficiency. For this reason, it has been used for uplink multiple access scheme in Long Term Evolution (LTE) of the 3rd Generation Partnership Project (3GPP) [5]. LTE has been developed by 3GPP to meet the requirements of the International Telecommunication Union (ITU), which include: high data rate, increased coverage and capacity, and better Quality of Service (QoS).

There are two different techniques for subcarriers selection of SC-OFDMA signals transmission: the distributed subcarrier mapping method, at which the DFT outputs of the required input data are assigned over the total bandwidth while the zeros are padded in unused sub-carriers. However, successive sub-carriers are filled with the DFT outputs of the input data in the localized sub-carrier mapping method of SC-FDMA, which is called localized FDMA (LFDMA). The case for the distributed method with equal distances between filled sub-carriers is defined as interleaved frequency division multiple access (IFDMA) [4].

However, using single carrier frequency division multiple access (SC-FDMA) and frequency domain equalization at receiver, the transmitted localized SC-FDMA signals still have higher (PAPR) than their relevant interleaved signals [1]. Although, IFDMA

PAPR is less than that of LFDMA, the LFDMA is used for practical application because subcarriers assignment with equidistance over the total bandwidth is very hard to be implemented, as IFDMA needs additional requirements such as guard-band between adjacent subcarriers and pilots [6].

It seems that there is still some space for PTS implementation for the Localized SC-FDMA uplink in the LTE systems to reduce the PAPR. Therefore, in this paper, Partial Transmit Sequences (PTS) method will be introduced, which divides the normal OFDM stream into several sub-streams and multiplying each sub-stream by different weights, then the optimum value will be selected to be transmitted with low PAPR [7].

The rest of this paper is arranged as follows: section 2 presents the multicarrier signals characteristics, in section 3, the peak-to-average power ratio techniques are described using selective mapping and partial transmit sequence techniques, while the PAPR distribution is presented in section 4, the proposed PTS based LFDMA uplink system model for PAPR reduction is analyzed in section 5, section 6 presents the computer simulation results and section 7 concludes the paper.

2. MULTICARRIER SIGNALS CHARACTERISTICS

A multi-carrier signal is generated by adding many independently modulated sub-carriers. Let's consider a block of N symbols, $\mathbf{X} = \{X_k, k = 0, 1, \dots, N-1\}$ is made with every symbol modulating one of the set of subcarriers $\{f_k, k = 0, 1, \dots, N-1\}$, where N represents the number of subcarriers. The N subcarriers are selected to be orthogonal, i.e., $f_k = k \cdot \Delta f$, where $\Delta f = \frac{1}{NT}$ and T represents the original symbol period. Then, a continuous-time baseband representation of a multicarrier signal is given by:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp(j2\pi f_k t), \quad 0 \leq t \leq NT \quad (1)$$

The base-band discrete-time multicarrier signal x_n sampled by the Nyquist rate $t = nT$ may be represented by the following equation:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp(j2\pi \frac{k}{N} n), \quad n = 0, 1, \dots, N-1 \quad (2)$$

Let $x_L = [x_{0,L} \ x_{1,L} \ \dots \ x_{LN-1,L}]^T$, be the oversampled discrete-time multicarrier signal vector, where $x_{n,L}$

represents the oversampled discrete-time multicarrier signal sampled with $t = \frac{nT}{L}$ described as follows:

$$x_{n,L} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X'_k \exp(j2\pi \frac{k}{LN} n), \quad n = 0, 1, \dots, LN-1 \quad (3)$$

$$\text{where } X'_k = \begin{cases} X_k, & 0 \leq k \leq N-1 \\ 0, & N \leq k \leq LN-1 \end{cases}$$

The base-band continuous-time multicarrier signals may be approximately described with L -times oversampled base-band discrete-time signals. It is proved in [5], that selecting $L=4$ is enough for representation of the peak value of continuous-time multicarrier signals.

3. PEAK-TO-AVERAGE POWER RATIO TECHNIQUES

In this paper, the scrambling technique will be used to scramble the input data block of the OFDM symbols and transmit one of them with the minimum PAPR so that the probability of obtaining high PAPR can be reduced. Two techniques will be presented, the Selective Mapping (SLM), and Partial Transmit Sequence (PTS).

A. Selective Mapping

In this method, at the beginning, N independent sequences are generated from the input OFDM signal, each sequence will be multiplied by different phase sequence to generate another N independent sequences $\mathbf{X} = [X_1, X_2, \dots, X_N]$ and these sequences are then forwarded into IFFT operation simultaneously to generate the vectors: $\mathbf{x} = [x_1, x_2, \dots, x_N]$. The vector x_d with minimum PAPR will be selected for transmission as shown in Figure 1.

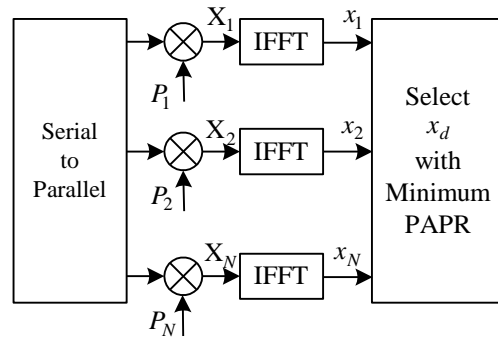


Figure 1. Block diagram of SLM technique

B. Partial Transmit Sequence (PTS)

Partial Transmit Sequence (PTS) algorithm is an approach used to improve the basic stats of a multicarrier signal. The main function of the PTS method is that it divides the input OFDM signal into

many sub-signals, each sub-signal is then multiplied by a specific, deferent weight, and this function is repeated until the needed value is reached. Figure 2 displays the block diagram of the OFDM Downlink system based on the PTS algorithm. The input signal of M symbols is divided into V sub-signals as $\mathbf{X}=[X_1, X_2, \dots, X_v]$ where X_i sub-signals are successively located and are of equal sizes.

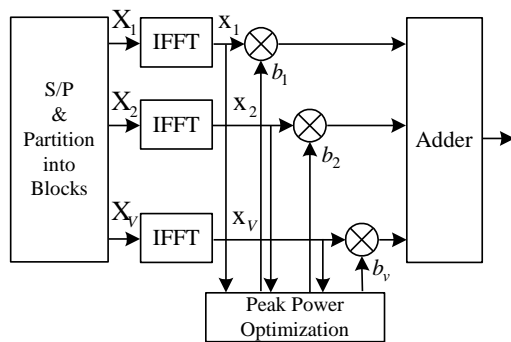


Figure 2. Pre-coding based PTS-OFDM downlink system.

The signal is obtained through applying the IFFT operations on X in the time domain, and that's defined by:

$$\mathbf{x} = IFFT(\mathbf{X}) = \sum_{v=1}^V b_v \cdot IFFT(\mathbf{X}_v) = \sum_{v=1}^V b_v \cdot \mathbf{x}_v \quad (4)$$

A suitable factor combination needs to be selected where $\mathbf{b}=[b_1, b_2, \dots, b_v]$ and the results in the output reaching perfection, this could be defined through:

$$\mathbf{b} = [b_1, b_2, \dots, b_v] = \arg \min_{(b_1, b_2, \dots, b_v)} (\max_{1 \leq n \leq N} |\sum_{v=1}^V b_v \cdot x_v|^2) \quad (5)$$

In the above equation, $\arg \min (\cdot)$ equals the output of the least value. Using this method, the best possible \mathbf{b} can be reached to improve the PAPR efficiency. The new costs can be calculated using $V - 1$ multiplied by the number of the IFFTs operation.

4. DISTRIBUTION OF PAPR

The peak-to-average power ratio (PAPR) of the base-band discrete-time multi-carrier signal can be represented as the ratio of the maximum peak power over the average power of the multicarrier signal [8], i.e.:

$$PAPR(x_n) = \frac{\max_{0 \leq n \leq N-1} |x_n|^2}{\frac{1}{N} \sum_{n=0}^{N-1} E\{|x_n|^2\}} \quad (6)$$

where, $E\{\cdot\}$ represents the expected value.

When all of the subcarriers in an OFDM system have the same phases, it's possible to reach the maximum power which results in $mean |s(t)|^2 = 1$, this makes the $PAPR = N$ which means the maximum power equals N times the mean power. At the same time, the probability that the maximum power happening becomes smaller when the number of subcarriers is increased.

According to the central limit theorem and if the number of subcarriers N is large enough, the real and imaginary parts of the time domain signal samples follow the Gaussian distribution function with zero mean and variance $1/2$. The cumulative distribution function (CDF) of the amplitude of a signal is given by [9]:

$$F(z) = 1 - \exp(-z) \quad (7)$$

The complementary CDF (CCDF) for the PAPR is one with the most frequently used performance measure for PAPR reduction methods. The CCDF of the PAPR denotes the probability that the PAPR of a data block surpasses a specified threshold. The CCDF of the PAPR for a data block with Nyquist rate sampling is defined as follows:

$$P(PAPR > z) = 1 - P(PAPR \leq z) = 1 - F(z)^N = 1 - (1 - \exp(-z))^N \quad (8)$$

5. PTS BASED LFDMA UPLINK SYSTEM

Partial Transmit Sequence (PTS) algorithm is a method used to improve the statistics of a multi-carrier signal. The main idea of PTS method is to partition the main OFDM sequence into many sub-sequences, and then each sub-sequence is multiplied with a different weights until the optimum value with lowest PAPR is achieved.

Figure 3 displays the block diagram of the suggested PTS based LFDMA for uplink system. The input data block of M symbols is divided into V disjoint sub-blocks as $\mathbf{x}=[x_1, x_2, \dots, x_v]^T$, where x_i are the sub-blocks that are sequentially located and also are of equal size and each sub-block consists of M/V nonzero elements and set the rest part to zero. Then, each partitioned sub-block is inserted through DFT precoder block to yield $\mathbf{X}=[X_1, X_2, \dots, X_v]^T$. Assuming that the sub-blocks have exactly the same size and there is no gap between them, the sub-block vector can be defined as follows:

$$\hat{\mathbf{X}} = \sum_{v=1}^V b_v \tilde{\mathbf{X}}_v \quad (9)$$

where $b_v = e^{j\varphi_v}$ ($\varphi_v \in [0, 2\pi]$) $\{v=1,2,\dots,V\}$ represents a weighting factor used for phase rotation. Then, the sub-carrier mapping of the precoded signal is done in the localized mode. The sub-carrier mapping of the localized-mode for PTS-LFDMA uplink system would be defined as:

$$\tilde{X}_k = \begin{cases} X_k, & k = 0, \dots, \frac{M}{2} - 1 \\ 0, & k = \frac{M}{2}, \dots, N - \frac{M}{2} - 1 \\ X_k, & k = N - \frac{M}{2}, \dots, N - 1 \end{cases} \quad (10)$$

where N is the number of subcarriers in the system, M is the number of user subcarriers.

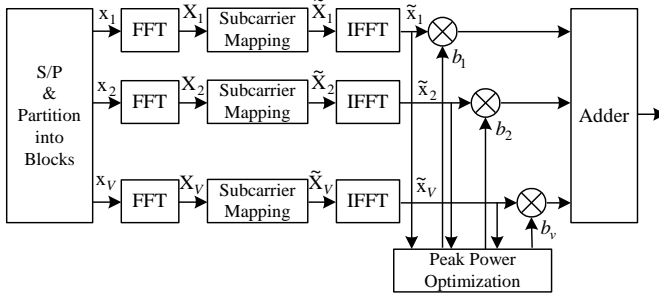


Figure 3. Pre-coding based PTS-LFDMA uplink system.

The time domain representation of the signal is obtained by applying IFFT operation on X_v , i.e.,

$$\hat{\mathbf{x}} = IFFT(\hat{\mathbf{X}}) = \sum_{v=1}^V b_v \cdot IFFT(\tilde{\mathbf{X}}_v) = \sum_{v=1}^V b_v \cdot \tilde{\mathbf{x}}_v \quad (11)$$

Select one appropriate combination factor $\mathbf{b} = [b_1, b_2, \dots, b_v]$ which lets the result achieve optimum. The combination can be represented by:

$$\mathbf{b} = [b_1, b_2, \dots, b_v] = \arg \min_{(b_1, b_2, \dots, b_v)} \left(\max_{1 \leq n \leq N} \left| \sum_{v=1}^V b_v \cdot \tilde{x}_v \right|^2 \right) \quad (12)$$

where $\arg \min (\cdot)$ is the condition that output the minimum value of function. Accordingly, we can select the best \mathbf{b} so as to optimize the PAPR performance. The additional cost we have to pay can be calculated via $V-1$ times IFFTs operation.

The IFFT output sequence \tilde{x}_n can be represented as follows:

$$\tilde{x}_n = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}_k e^{j2\pi \frac{n}{N} k} \quad (13)$$

It has been shown in [1] that using pulse shaping does not improve the localized FDMA system. Therefore, PAPR without pulse shaping can be represented by:

$$PAPR = \frac{\max_{k=0,1,\dots,N-1} |\tilde{x}_k|^2}{\frac{1}{N} \sum_{k=0}^{N-1} |\tilde{x}_k|^2} \quad (14)$$

6. SIMULATION RESULTS

Simulations are carried out to check the performance of the new suggested PTS-LFDMA system. The new system is evaluated with $V = 4$. The data is generated randomly, after that, it is modulated with QPSK, 16QAM and 64QAM, respectively. Then, the PAPR is calculated by using the complementary cumulative distribution (CCDF) technique. The CCDF for the PAPR of the LFDMA uplink signal has been used to represent the probability of being larger a specific value of threshold $PAPR_0$ ($CCDF = \text{Prob}(PAPR > PAPR_0)$). The simulation results of the suggested system are compared with the OFDMA, the SLM based LFDMA system, and the localized SC-FDMA (LFDMA) uplink systems, respectively.

Figure 4 displays the CCDF comparisons for PAPR of the OFDMA, the LFDMA, the SLM based LFDMA, the PTS based LFDMA uplink systems with $V = 4$, for $M = 64$ and $N = 256$ with QPSK modulation. For a CCDF of 0.1%, the PAPR is 10.7, 7.5, 5.9 and 4.7 dB for the OFDMA, the LFDMA, the SLM based LFDMA, and the PTS based LFDMA uplink systems, respectively.

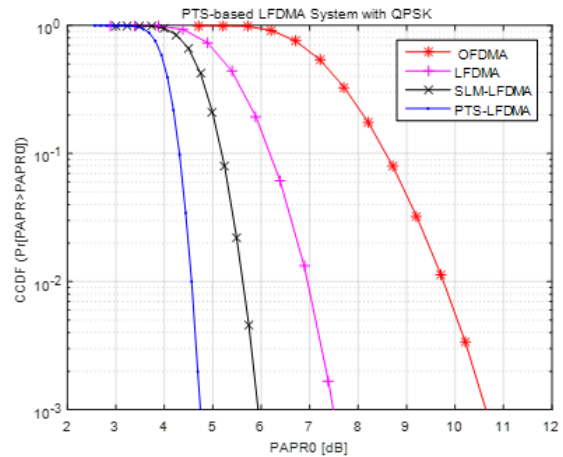


Figure 4. CCDF comparison of PAPR for the OFDMA uplink system, the LFDMA uplink system, the SLM based LFDMA, and the PTS based LFDMA uplink system with $V = 4$, respectively.

Figure 5 displays the CCDF comparisons for PAPR of the OFDMA, the LFDMA, the SLM based LFDMA, the PTS based LFDMA uplink systems with $V = 4$, for $M = 64$ and $N = 256$ with 16QAM modulation. For



a CCDF of 0.1%, the PAPR is 10.7, 8.5, 6.6 and 5.6 dB for the OFDMA, the LFDMA, the SLM based LFDMA, and the PTS based LFDMA uplink systems, respectively.

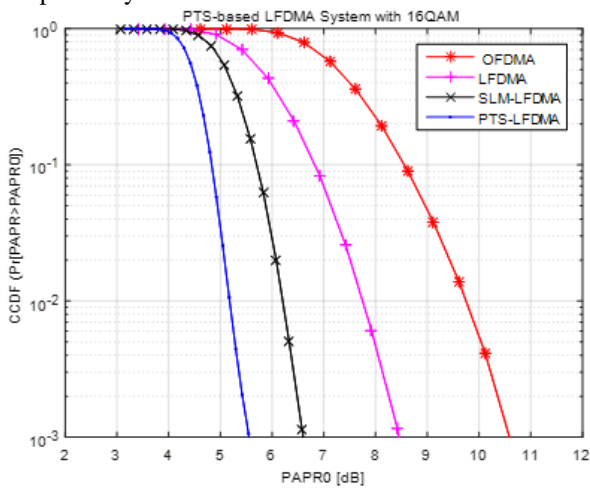


Figure 5. CCDF comparison for PAPR of the OFDMA uplink system, the localized LFDMA uplink system, the SLM based LFDMA, the PTS based LFDMA uplink system.

Figure 6, displays the CCDF comparisons for PAPR of the OFDMA, the LFDMA, the SLM based LFDMA, the PTS based LFDMA uplink systems with $V = 4$, for $M = 64$ and $N = 256$ with 64QAM modulation. For a CCDF of 0.1%, the PAPR is 10.7, 8.7, 6.6 and 5.8 dB for the OFDMA, the LFDMA, the SLM based LFDMA, and the PTS based LFDMA uplink systems, respectively.

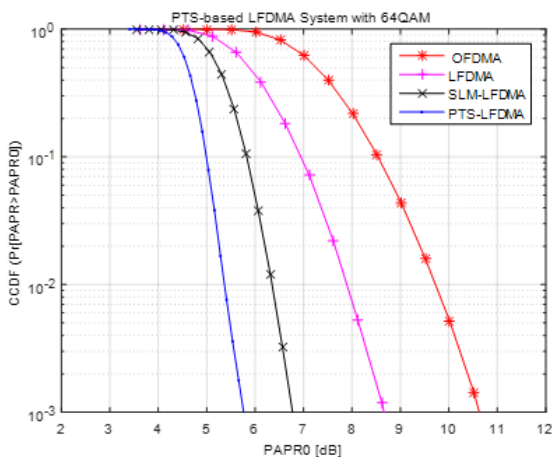


Figure 6. CCDF comparison for PAPR for the localized OFDMA, the localized LFDMA, the SLM based LFDMA, the PTS based LFDMA uplink systems.

In Table I, we summarize the PAPR analysis of the suggested PTS based LFDMA, the SLM based LFDMA, the OFDMA, and the LFDMA uplink systems, respectively. For a CCDF of 0.1%, it is clear from Table 1 that the suggested system has less PAPR

than the SLM based LFDMA, the LFDMA and the OFDMA uplink systems.

Table I At CCDF of 0.1%, the PAPR comparisons of the OFDMA, the LFDMA, the SLM based LFDMA, the PTS based LFDMA with $V = 4$.

Uplink Transmission Scheme	PAPR [dB]		
	4QAM	16QAM	64QAM
OFDMA	10.7	10.7	10.7
LFDMA	7.5	8.5	8.7
SLM-LFDMA	5.9	6.6	6.8
PTS-LFDMA	4.7	5.6	5.8

7. CONCLUSIONS

In this paper, the PTS based LFDMA uplink system is presented for PAPR reduction. Simulation results have shown that, the suggested system has less PAPR than the OFDMA, the LFDMA and the SLM based LFDMA uplink systems, respectively. The PAPR for the new system may be reduced more if the size of V is increased. However, if we increase of V size, it will increase the suggested system complexity. Therefore, the values of V must be selected carefully. Because of the low PAPR values, the new PTS based LFDMA uplink system is very suitable for the LTE mobile standard release 10 LTE-Advanced than the LFDMA uplink system which is already used in the release 8 LTE.

References

- [1] Myung, H. G., Lim, J., Goodman, D. J., (2006) "Peak-To-Average Power Ratio of Single Carrier FDMA Signals with Pulse Shaping", *IEEE 17th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 06)*, pp. 1-5.
- [2] H. Sari, G. Karam and I. Jean Claude, (1994) "Frequency-Domain Equalization of Mobile Radio and Terrestrial Broadcast Channels," *Proc. IEEE GLOBECOM*, pp. 1-5, San Francisco, CA.
- [3] Muller, St. H., Huber, J. B., (1997) "OFDM with Reduced Peak-to-Average Power Ratio by Optimum Combination of Partial Transmit Sequences", *Electronics Letters*, Vol. 33, No. 5, pp. 368-369.
- [4] Hyung G. Myung, Junsung Lim, and David J. Goodman, "Single Carrier FDMA for Uplink Wireless Transmission." *IEEE Vehicular Technology*, Sept 2006.
- [5] 3rd Generation Partnership Project (3GPP); Technical Specification Group Radio Access Network; Physical Layer Aspects for Evolved UTRA, <http://www.3gpp.org/ftp/Specs/html-info/25814.htm>
- [6] ChoY. Soo, Kim, Jaekwon, Yang W. Young, Kang, Chung G. (2010) "MIMO-OFDM Wireless Communications with MATLAB", *Wiley*, Hoboken.
- [7] I. Baig, M. Ayaz, V. Jeoti, (2013) "A SLM based localized SC-FDMA uplink system with reduced PAPR for LTE-A", *Journal of King Saud University – Engineering Sciences*, 25, 119-123.



- [8] Muller, St. H., Bauml, R.W., Fischer, R.F.H., Huber, J.B., (1997) "OFDM with Reduced Peak-to-Average Power Ratio by Multiple Signal Representation", *Annals of Telecommunications*, Vol. 52, No. 1-2, pp. 58-67.
- [9] S. H. Han and J. H. Lee, (2005) "An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multicarrier Transmission," *IEEE Wireless Communication*, vol. 12, no. 2, pp. 56–65.



Ahmed J. Jameel was born in 1963 in Baghdad, Iraq. He received his B.Sc. and M.Sc. degrees in Electrical Engineering and Electronics and Communications Engineering from University of Baghdad, College of Engineering in 1985 and 1994, respectively and his Ph.D. degree in Communication and Information Systems, from Southeast University, Nanjing, China in 2003.

He joined Ahlia University in 2008 as an Assistant Professor of Telecommunication Engineering. From 2005-2008, he worked as an Assistant Professor in the Department of Electronics and Communication Engineering of Al-Ahliyya Amman University, Jordan. From 2004 – 2005, he was a Post-Doctoral Research Associate at the Department of Communication Engineering, Southeast University, Nanjing, China.

His research interests include digital communications, error-correcting codes, Coded Multi-Input Multi-Output Systems for the B3G/4G Mobile Communications, and speech coding for mobile communications.