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Economic Reliability Test Plan under Hybrid Exponential Distribution

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Abstract: The question of developing Economic Reliability Test Plan is considered in this paper assuming product lifetime follows Hybrid Exponential Distribution. Economic Reliability Test Plans determine the time to terminate the experiment when the termination number, producer's risk and the sample size are given. Comparison of this test plan over the time truncated acceptance sampling plan considered by Sampath and Lalitha [15] for hybrid exponential distribution is also carried out in this present work.

Keywords: Hybrid exponential distribution, reliability test plans, time truncated acceptance sampling plan, producer's risk.

1. INTRODUCTION

In Statistical Quality Control studies, designing acceptance sampling plans suitable for different environments is an important task. Acceptance sampling plans help us to decide whether to accept or reject a submitted lot of manufactured products on the basis of pre-specified quality levels. If the quality of a product is assessed using its life time then a randomly selected lot of such products is subjected to life testing process. Here it is assumed that products which are submitted for testing is a true representative of the items being manufactured. The decision to accept or reject a lot is based on the risk associated with two types of errors, namely Type-I error (rejecting a good quality lot) and Type-II error (accepting a bad quality lot). Procedures based on such life testing process which are designed to decide whether a lot is to be accepted or rejected based on these errors are termed as 'Reliability test plans' or 'Acceptance sampling based on life test'. Designing of such test procedures (sampling plans) requires the identification of an appropriate probability model governing the life time of products. Economic reliability test plans were studied under log-logistic distribution and generalized exponential distribution by Kantam, Srinivasa Rao and Sriram [5] and Aslam and Shabaz [1] respectively. Rao, Ghitany and Kantam [10] have considered an economic reliability test plan under Marshal-Olkin extended Lomax distribution and Rao, Kantam, Rosaiah and Prasad [9] made similar studies under type II exponentiated log logistic distribution. An economic reliability test plan was developed by Rosaiah, Kantam and Santhosh [11] under Exponentiated log-logistic distribution.

It is to be noted that large volume of work has been done in developing acceptance sampling plans of different types under various probability distributions. It is well known that the efficiency of a sampling plan can be fully assessed only when prior values of the parameters involved in the probability distributions being considered are fully known. However, such values are very rarely known and often they are estimated by means of some statistical techniques. In order to handle such situations, researchers working in statistical quality control studies have started pursuing a different direction using the tools available in fuzzy set theory developed by Zadeh [17]. Sampling plans for attributes where the parameters involved in the underlying distributions are assumed to be fuzzy quantities have been studied by Ohta and Ichihashi [7], Grzegorzwski [2],[3],[4], Tong and Wang [16] etc.

Recently, Liu [6] introduced credibility theory which is parallel to probability theory for environments involving impreciseness (fuzziness). Subsequently, a new branch called Chance theory which is an integration of impreciseness and randomness has been developed by Liu [6]. Sampath [12] and Sampath and Deepa [13] have applied Chance theory for designing fuzzy acceptance sampling plans for attributes. Sampath, Lalitha and Ramya [14] have applied hybrid

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normal distribution (normal distribution where parameters are treated as fuzzy variables) towards developing a single sampling plan for variables for situations involving both impreciseness and randomness. Recently, Sampath and Lalitha [15] have considered the application of hybrid exponential distribution for developing time truncated acceptance sampling plan.

In this paper, Hybrid exponential distribution is used for constructing an economic reliability test plan. The paper is organized as follows. Description about Hybrid exponential distribution is given in section 2. The design of reliability test plan under hybrid exponential distribution is considered in section 3. A comparative study is made with time truncated acceptance plan for hybrid exponential distribution considered by Sampath and Lalitha [15] and is presented in the section 4. Conclusions drawn from the investigation carried out in this paper are given in the final section.

2. HYBRID EXPONENTIAL DISTRIBUTION

For an exponential distribution, the probability density function of the random variable X is given by $\phi(x,\theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}}, x>0, \theta>0$, where θ is the mean value of the distribution. Here we assume θ is a triangular fuzzy variable. Since randomness and impreciseness are created through the random variable X and the fuzzy variable θ respectively, the above distribution qualifies to be a hybrid distribution. Let ξ be the hybrid variable and μ be the membership function associated with θ. Then by Qin and Liu [8], for any Borel set B of real numbers, the chance $Ch(\xi \in B)$ is given by

$$Ch(\xi \in B)=\begin{cases} \sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_B \phi(x,\theta)dx\right\} & \\ \qquad \text{if } \sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_B \phi(x,\theta)dx\right\}<0.5 & \\ 1-\sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_{B^c} \phi(x,\theta)dx\right\} & \\ \qquad \text{if } \sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_B \phi(x,\theta)dx\right\}\geq 0.5 & \end{cases} \tag{1}$$

where $\phi(x,\theta)$ is the probability density function of exponential distribution. Therefore

$$Ch(\xi \in B)=\begin{cases} \sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_B \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} \quad \text{if } \sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_B \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}<0.5 & \\ 1-\sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge 1-\int_{B^c} \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} & \\ \qquad \text{if } \sup\limits_{\theta\in\Theta}\left\{\left(\frac{\mu(\theta)}{2}\right)\wedge\int_B \frac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}\geq 0.5 & \end{cases} \tag{2}$$

The fuzzy variable θ is assumed to be a triangular fuzzy variable with membership function defined over (a,b,c), namely

$$\mu(\theta)=\begin{cases}\dfrac{\theta-a}{b-a}, & if\ \mathrm{a}\le\theta\le b\\ \dfrac{\theta-b}{b-c}, & if\ \ b\le\theta\le c\\ 0, & otherwise\end{cases}$$

where a,b and c are real numbers satisfying the condition $a<b<c$.

For hybrid exponential distribution, the distribution function is given by

$$Ch(\xi\le t)=\Phi(t,\theta)=\begin{cases}\sup\limits_{\theta\in\Theta}\left\{\left(\dfrac{\mu(\theta)}{2}\right)\wedge\int_0^t\dfrac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} & if\ \sup\limits_{\theta\in\Theta}\left\{\left(\dfrac{\mu(\theta)}{2}\right)\wedge\int_0^t\dfrac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}<0.5\\ 1-\sup\limits_{\theta\in\Theta}\left\{\left(\dfrac{\mu(\theta)}{2}\right)\wedge 1-\int_0^t\dfrac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\} & \\ & if\ \sup\limits_{\theta\in\Theta}\left\{\left(\dfrac{\mu(\theta)}{2}\right)\wedge\int_0^t\dfrac{1}{\theta}e^{-\frac{x}{\theta}}dx\right\}\ge 0.5\end{cases} \quad (3)$$

Sampath and Lalitha [15] have derived an expression for the above chance distribution. The expression derived by them is given by

$$Ch(\xi\le t)=\Phi(t,\theta)=\begin{cases}0 & if\ t\le 0\\ 1-e^{-\frac{t}{\theta_1^*}} & if\ 0<t<b*\ln(2)\\ \dfrac{1}{2} & if\ t=b*\ln(2)\\ 1-e^{-\frac{t}{\theta_2^*}} & if\ t>b*\ln(2)\end{cases} \quad (4)$$

where θ_1^* and θ_2^* are the solutions of $\dfrac{\mu(\theta)}{2}=1-e^{-\frac{t}{\theta}}$, $\dfrac{\mu(\theta)}{2}=e^{-\frac{t}{\theta}}$ respectively.

3. DESIGN OF AN ECONOMIC RELIABILITY TEST PLAN FOR HYBRID EXPONENTIAL DISTRIBUTION

The operational procedure of economic reliability test plan (Kantam, Srinivasa Rao and Sriram [5]) is explained below. Let n be the number of items taken for inspection from a lot and r be the termination number. If r failures out of n items occur before the termination time t_0 then the lot will be rejected, otherwise it is accepted. The experiment is stopped as soon as the r^{th} failure is reached or the termination time t_0 is reached, whichever is earlier. One of the important factors in life test experiment is selection of sample size. The choice of sample size plays a crucial role in the performance of life test experiment. The sample size is selected in such a way that the expected waiting time to reach the decision and cost of the experiment are optimized. Following Kantam et al [5] we assume the sample size is a multiple of the termination number r. It is to be noted that the number of failures follows binomial distribution and the probability of having i failures out of n tested items is given by $\binom{n}{i} p^i q^{n-i}$. Therefore, the probability of accepting the lot is given by $\sum_{i=0}^{r-1} \binom{n}{i} p^i q^{n-i}$. Kantam et al [5] determined the value of p in their crisp version of the reliability test plan by using the cumulative distribution function of binomial distribution. In our study, in lieu of the probability distribution we obtain the value of p using the relation $p = \Phi(t,\theta)$, where $\Phi(t,\theta)$ is a hybrid distribution function considered in equation (4). If α is the producer's risk, then we have

$$\sum_{i=0}^{r-1} \binom{n}{i} p^i q^{n-i} = 1-\alpha \tag{5}$$

Here we assume $n = rk$ where k is a non-negative integer. Given the values of r, k, α, equation (5) can be solved for p using cumulative probabilities of binomial distribution. The values of p so obtained when using in equation (4) will give the corresponding value of $\frac{t}{\theta}$. This is useful for finding the value of termination time t_0 for a specified average life.

Values of $\frac{t}{\theta}$ corresponding to different choices of r and k are given in Tables 1 and 2 for $\alpha = 0.05$ and $\alpha = 0.01$ respectively. These tables can be used for deciding the termination for specific choices of design parameters. Consider the situation where we are interested in constructing a life test sampling plan with the acceptance probability 0.95 for lots with an acceptable mean life of 10000 hours, sample size 10 and termination number 5. From Table 1, we can see that against $r = 5$, under the column $2r$, the value $\frac{t}{\theta}$ is 0.2515. Since θ_0, the specified value of θ is taken as 10000 hours, we get $t_0 = \theta_0$ (0.2515) =2515.

TABLE 1: RELIABILITY TEST PLAN FOR HYBRID EXPONENTIAL DISTRIBUTION $(\alpha = 0.05)$

r\n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.0256	0.0170	0.0128	0.0103	0.0085	0.0073	0.0063	0.0056	0.0051
2	0.1027	0.0649	0.0474	0.0374	0.0309	0.0262	0.0229	0.0203	0.0182
3	0.1662	0.1028	0.0745	0.0585	0.0481	0.0408	0.0355	0.0314	0.0282
4	0.2143	0.1310	0.0945	0.0740	0.0608	0.0516	0.0448	0.0396	0.0354
5	0.2515	0.1527	0.1098	0.0858	0.0704	0.0598	0.0518	0.0458	0.0410
6	0.2813	0.1700	0.1221	0.0952	0.0781	0.0661	0.0574	0.0508	0.0454
7	0.3058	0.1840	0.1319	0.1029	0.0844	0.0715	0.0620	0.0548	0.0490
8	0.3266	0.1959	0.1403	0.1094	0.0896	0.0759	0.0658	0.0582	0.0520
9	0.3442	0.2060	0.1473	0.1148	0.0940	0.0796	0.0690	0.0610	0.0546
10	0.3594	0.2148	0.1535	0.1196	0.0979	0.0829	0.0719	0.0635	0.0568

TABLE 2: RELIABILITY TEST PLAN FOR HYBRID EXPONENTIAL DISTRIBUTION $(\alpha = 0.01)$

r\n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.0050	0.0033	0.0025	0.0020	0.0016	0.0014	0.0012	0.0011	0.0010
2	0.0428	0.0271	0.0198	0.0156	0.0129	0.0110	0.0095	0.0084	0.0075
3	0.0885	0.0548	0.0397	0.0312	0.0256	0.0217	0.0189	0.0167	0.0150
4	0.1289	0.0788	0.0569	0.0446	0.0366	0.0311	0.0270	0.0238	0.0213
5	0.1630	0.0990	0.0713	0.0557	0.0457	0.0387	0.0337	0.0297	0.0267
6	0.1919	0.1160	0.0833	0.0651	0.0533	0.0452	0.0393	0.0346	0.0310
7	0.2165	0.1305	0.0935	0.0730	0.0598	0.0507	0.0440	0.0388	0.0348
8	0.2379	0.1429	0.1024	0.0798	0.0654	0.0554	0.0480	0.0424	0.0379
9	0.2564	0.1537	0.1100	0.0858	0.0702	0.0594	0.0516	0.0455	0.0407
10	0.2730	0.1633	0.1169	0.0910	0.0745	0.0630	0.0547	0.0482	0.0432

4. COMPARISON OF TIME TRUNCATED AND RELIABILITY TEST PLANS

In this section, it is proposed to compare the time truncated acceptance sampling plan considered by Sampath and Lalitha [15] under hybrid exponential distribution and the reliability test plan considered in this paper. Table 3 gives minimum sample size needed for specific $\frac{t}{\theta}$ for different choices of $P^* = 1 - \alpha$ to assert the median life to exceed a given value, θ_m^0 with probability P^* and corresponding acceptance number c in time truncated sampling plan considered by Sampath and Lalitha [15] under situations involving both randomness and impreciseness.

TABLE 3: MINIMUM SAMPLE SIZES UNDER TIME TRUNCATED TEST PLAN

P^*	c	$\frac{t}{\theta}$ = 0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.95	0	7	5	4	3	2	2	2	1
	1	12	8	7	5	4	3	3	3
	2	16	11	9	7	6	5	4	4
	3	20	14	11	9	7	6	5	5
	4	23	17	13	11	9	7	7	6
	5	27	19	15	13	10	9	8	7
	6	31	22	17	15	12	10	9	8
	7	34	24	20	17	13	11	10	9
	8	38	27	22	18	14	12	11	11
	9	41	29	24	20	16	14	12	12
	10	44	32	26	22	17	15	14	13
0.99	0	11	8	6	5	3	3	2	2
	1	16	11	9	7	5	4	4	3
	2	21	14	11	9	7	6	5	4
	3	25	17	14	11	9	7	6	6
	4	29	20	16	14	10	8	7	7
	5	33	23	18	15	12	10	9	8
	6	37	26	21	17	13	11	10	9
	7	41	29	23	19	15	12	11	10
	8	44	31	25	21	16	14	12	11
	9	48	34	27	23	18	15	13	13
	10	52	37	29	25	19	16	15	14

Comparison of reliability test plan over the time truncated acceptance sampling plan under hybrid exponential distribution is carried out using common values for n, $c \backslash r$. It is to be noted that Tables 1 and 2 yield the values of the ratio $\frac{t}{\theta}$ for different choices of n and r, for $\alpha = 0.05$ and $\alpha = 0.01$. Further, Table 3 gives sample sizes needed under time truncated sampling plan for specific choices of $\frac{t}{\theta}$ and c for $\alpha = 0.05$ and $\alpha = 0.01$. On making use of Tables 1,2 and 3, the values of ratio $\frac{t}{\theta}$ are identified for common values of n, $c \backslash r$ under reliability test plan and time truncated test plan. The values are reported in Tables 4 and 5 for $\alpha = 0.05$ and $\alpha = 0.01$.

TABLE 4: COMPARISON OF PROPORTION OF TERMINATION TIME $(\alpha = 0.05)$

r\n	2r	3r	4r	5r	6r	7r	8r
1	0.0256 2.356	0.0170 1.571	0.0128 1.257	0.0103 0.942	-	0.0073 0.628	-
2	0.1027 2.356	-	0.0474 0.942	-	0.0309 0.628	-	-
3	0.1662 2.356	-	0.0745 1.257	-	-	-	-
4	-	-	-	0.0740 0.628	-	-	-
9	0.3442 1.571	0.2060 0.942	-	-	-	-	-
10	0.2729 1.571	-	-	-	-	-	-

TABLE 5: COMPARISON OF PROPORTION OF TERMINATION TIME $(\alpha = 0.01)$

r\n	2r	3r	4r	5r	6r	7r	8r
1	0.0050 3.927	0.0033 2.356	-	0.0020 1.571	0.0016 1.257	-	0.0012 0.942
2	0.0428 3.141	-	-	-	-	-	0.0095 0.628
3	0.0885 3.141	0.0548 1.571	-	-	-	-	-
5	0.1630 2.356	0.099 0.942	-	-	-	-	-
6	0.1919 2.356	0.1160 1.257	-	-	-	-	-
9	-	0.1537 1.571	-	-	-	-	-

In these tables, the upper and lower entry value in each cell represents the proportion of termination time of a reliability test plan and time truncated acceptance sampling plan respectively for hybrid exponential distribution. Contents of the above table clearly reveal that the termination time of this reliability test plan is uniformly smaller than the corresponding value in the time truncated test plan.

5. Conclusion

In this paper, the life time distribution of a product under consideration is treated as hybrid exponential distribution. That is, the parameter involved in one parameter exponential distribution is treated as a fuzzy variable. Reliability test plan is constructed using the hybrid exponential distribution. The results obtained under the developed sampling plan are compared with the time truncated acceptance sampling plan studied by Sampath and Lalitha [15] with reference to the same hybrid exponential distribution. It is concluded that, under hybrid exponential distribution, the reliability test plan is most economical when compared to the time truncated life test, in terms of saving cost, time and energy.

References

[1] Aslam, M. and Shabaz, M.Q. (2007), "Economic reliability test plans using the generalized exponential distribution", *Journal of Statistics*, vol. 14, 52-59.

[2] Grzegorzewski, P. (1998), "A Soft Design of Acceptance Sampling Plans by Attribute", In: *Proceedings of the 6th International Workshop on Intelligent Statistical Quality Control*, Würzburg, 29–38.

[3] Grzegorzewski, P. (2001), "Acceptance Sampling Plans by Attributes with Fuzzy Risks and Quality Levels", In: *Frontiers in Frontiers in Statistical Quality Control*, Springer, Heidelberg, vol. 6, 36–46.

[4] Grzegorzwski, P. (2002), "A Soft Design of Acceptance Sampling Plans by Variables", *Studies in Fuzziness and Soft Computing*, vol. 90, 275-286.

[5] Kantam, R.R.L., Srinivasa Rao, G. and Sriram, B. (2006), "An Economic Reliability Test Plan: Log-Logistic Distribution", *Journal of Applied Statistics*, vol. 33(3), 291-296.

[6] Liu, B. (2004), "Uncertainty Theory". Springer-Verlag, Berlin.

[7] Ohta, H. and Ichihashi, H. (1988), "Determination of single-sampling attribute plans based on membership functions", *International Journal of Production Research*, vol. 6(9), 1477-1485

[8] Qin, Z.F. and Liu, B. (2007), "On some special hybrid variables", *Technical Report*, Uncertainty Theory Laboratory, Tsinghua University, China.

[9] Rao, G.S., Kantam, R.R.L., Rosaiah, K. and Prasad, S.V. (2012), "Reliability test plans for type-II exponentiated log-logistic distribution", *Journal of Reliability and Stochastic Studies*, vol. 5, 55-64.

[10] Rao, G.S., Ghitany, M.E., and Kantam, R.R (2009), "Marshal-Olkin extended Lomax distribution: An economic reliability test plan", *Journal of Applied Mathematics*, vol. 32(1), 139-148.

[11] Rosaiah, K., Kantam, R.R., and Santhosh, K.C. (2007), "Exponentiated log-logistic distribution in economic reliability test plan", *Pakistan Journal of Statistics*, vol. 23(2), 147-156.

[12] Sampath, S. (2009), "Hybrid Single Sampling Plan", *World Applied Sciences Journal*, vol. 6, 1685-1690.

[13] Sampath, S. and Deepa, S.P. (2013), "Determination of optimal chance double sampling plan using genetic algorithm", *Model Assisted Statistics and Applications*, vol. 8, 265-273.

[14] Sampath, S., Lalitha, S.M. and Ramya, B. (2015), "Chance single sampling plan for variables", *International Journal of Fuzzy Systems Applications*, vol. 4, 43-59.

[15] Sampath, S. and Lalitha, S.M. (2015), "Time Truncated Sampling Plan under Hybrid exponential distribution", under review in *Journal of Uncertain Systems*.

[16] Tong, X. and Wang, Z. (2012). "Fuzzy acceptance sampling plans for inspection of geospatial data with ambiguity in quality characteristics", *Computers & Geosciences*, vol. 48, 256-266.

[17] Zadeh, L.A., (1965). "Fuzzy sets", Info Control, vol. 8, 338-353.