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Probabilistic Analysis of a Single Unit Model with Controlled and Uncontrolled Demand Factor and Inspection Policy Available in the System

Raeesa Bashir[1], J P Singh Joorel[2] and Ranjeet Kour[3]

[1,2,3] Department of Statistics, University of Jammu, Jammu-180006, J&K (India)

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Abstract: The present paper deals with reliability analysis of a system comprises of a single unit. Initially system is in operative state with controlled demand factor i.e. when demand is less or equal to production then it may transit to the state with uncontrolled demand factor i.e. when demand is greater than production. The system remains in upstate with demand factor controlled and uncontrolled while in down state on failure of the unit. On failure of the system, an inspection is carried to detect the type of failure. There are three types of failures, i.e. failure due to poor design of component, improper selection of material and human error. Single repair facility is available to repair the system. The failure time distributions of the unit are taken as exponential and inspection time of the unit is also taken as exponential. The distribution of time to repair of unit is assumed to be general. The various important measures of system effectiveness have been obtained and studied.

Keywords: Mean sojourn time, Mean time to system failure, Availability, Busy Period, Expected number of Visits, Profit Analysis, Graphical study of Model.

1. INTRODUCTION

Reliability is an ability of a system to consistently perform its intended or required function on demand and without any degradation. It deals with the development of new techniques for increasing the system effectiveness by reducing the frequency of failures and minimizing the high maintenance costs. The main objective is to make a system as profitable as possible. The contribution of various researchers in the field of reliability modeling has made the field very rich. Single-unit system models have been analyzed widely in the literature by several authors. Chandler and Bansal [8] discussed the concept of Profit analysis of single unit reliability models with repair at different failure modes. Taneja and Malhotra analyzed [1] Cost-benefit analysis of a single unit system with scheduled maintenance and variation in demand. Rizwan, Khurana and Taneja [10] investigated Modeling and optimization of single unit PLCs' system. Taneja, Tyagi and Bhardwaj [2] worked on Profit analysis of a single unit programmable logic controller (PLC). Sharma and Agarwal [11] put forth the concept of effectiveness analysis of a system with a special warranty scheme. Further, Srinivasan and Gopalan [9] analyzed the probabilistic analysis of a two-unit system with a warm standby and a single repair facility. Bhatia, Roosel and Gulshan [6] worked on probabilistic analysis of an automatic power factor controller with variation in Power Factor. Masahir and Shunji [5] discussed about Performance/ reliability character for multi-processor system with computational demands. Gopalan and Naidu [4] put forth the concept of cost-benefit analysis of a one-server system subject to inspection. Recently, Malhotra and Taneja [7] discussed reliability and availability analysis of a single unit system with varying demand. Murari and Ali [3] worked on one unit reliability system subject to random shocks and preventive maintenances.

E-mail address: raisabashir52@gmail.com; joorel@rediffmail.com; ranjeet.johal04@gmail.com

The objective of this paper is to analyze a single unit system model with controlled and uncontrolled demand factor having single repair facility and inspection policy available in the system. The system comprises of single unit A. Initially system is in operative state with controlled demand factor then it may transit to the state with uncontrolled demand factor. When the unit fails, inspection is carried out to check the cause of failure both in case of demand less or equal to production (controlled demand factor) and demand greater than production(uncontrolled demand factor). The model is analyzed stochastically and the expressions for the various reliability measures of system effectiveness are computed. Using regenerative point technique the following important reliability characteristics of interest are obtained:

- Transition probabilities in transient and steady state
- Mean sojourn time
- Mean time to system failure (MTSF).
- Point wise and steady-state availabilities of the system.
- Expected busy period of the repairman during (0, t].
- Expected number of visits for the repair facility.
- Profit analysis of system.
- Graphical study of model.

2. ASSUMPTIONS AND SYSTEM DESCRIPTION

- The system comprises of single unit A. Initially system is in operative state with controlled demand factor i.e. when demand is less or equal to production ($d \leq p$). Then it may transit to the state with uncontrolled demand factor i.e. when demand is greater than production ($d > p$).
- The system remains in upstate with demand factor controlled and uncontrolled.
- On failure of the system, an inspection is carried to detect the type of failure. There are three types of failures, i.e. failure due to: poor design of component, improper selection of material and human error.
- Single repair facility is available to repair the system.
- The failure time distributions of unit-A is taken exponential and inspection time of A is also taken as exponential.
- The distribution of time to repair of unit-A is assumed to be general.

3. NOTATION AND STATES OF THE SYSTEM

*A) **Notations** :*

λ : Constant failure rate of unit-A.

λ_1 : Rate with which demand factor changes from controlled mode to uncontrolled mode.

λ_2 : Rate with which demand factor changes from uncontrolled mode to controlled mode.

λ_3 : Inspection rate of unit-A when unit is in failed state.

d : Demand of manufactured product.

p : Production of goods.

p_1 : Probability of type 1(poor design of component) when $d \leq p$

q_1 : Probability of type 2 (improper selection of material) when $d \leq p$

r_1 : Probability of type 3 (human error) when $d \leq p$

p_2 : Probability of type 1(poor design of component) when $d > p$

q_2 : Probability of type 2 (improper selection of material) when $d > p$

r_2 : Probability of type 3 (human error) when $d > p$

β(x), f(x) : Rate of repair and corresponding p.d.f. of repair time of unit-A with controlled demand factor s.t

$$f(x) = \beta(x) \exp[-\int_0^x \beta(u)du]$$

μ(x), g(x) : Rate of repair and corresponding p.d.f. of repair time of unit-A with uncontrolled demand factor s.t

$$g(x) = \mu(x)\exp[-\int_0^x \mu(u)du]$$

$P_j(t)$: Probability that the system is in state S_j at time t.

$Q_k(x,t)$: Probability that the system is in state S_k at epoch t and has sojourned in this state for duration between x and x+dx.

B) Symbols for the States of the System :

A_0 : Unit-A is in normal (N) mode and operative with demand less or equal to production.

A_1 : Unit-A is in normal (N) mode and operative with demand greater than production.

A_{f0} : Unit-A is in failed state with controlled demand factor

A_{f1} : Unit-A is in failed state with uncontrolled demand factor

A_{cr1} : Unit-A is under repair in case of failure of type 1 (poor design of component) with controlled demand factor

A_{cr2} : Unit-A is under repair in case of failure of type 2 (improper selection Of material) with controlled demand factor

A_{cr3} : Unit-A is under repair in case of failure of type 3 (human error) with controlled demand factor

A_{ucr1} : Unit-A is under repair in case of failure of type 1 (poor design of component) with uncontrolled demand factor

A_{ucr2} : Unit-A is under repair in case of failure of type 2 (improper selection Of material) with uncontrolled demand factor

A_{ucr3} : Unit-A is under repair in case of failure of type 3 (human error) with uncontrolled demand factor

With the help of above symbols and keeping in view the assumptions, the possible states of the system along with the transitions between the states and transition rates are shown in Fig. 1

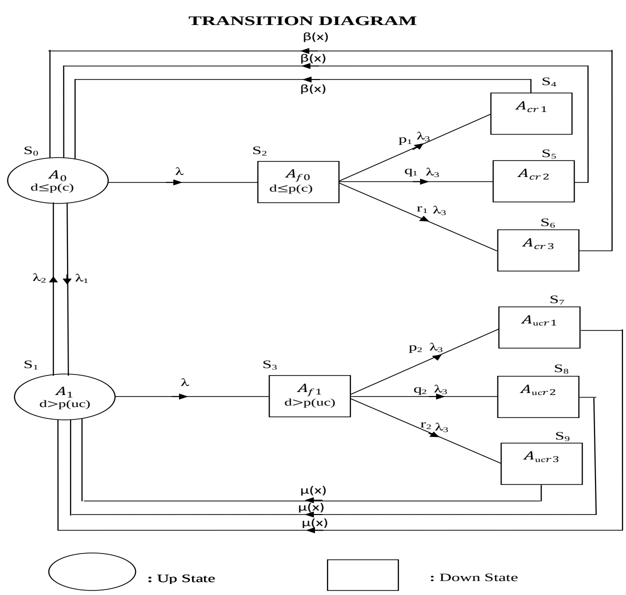

Fig.1

C) *States of the System*

The possible states of the system are:

$S_0 = A_0$ $S_1 = A_1$

$S_2 = A_{f0}$ $S_3 = A_{f1}$

$S_4 = A_{r1}$ $S_5 = A_{r2}$

$S_6 = A_{r3}$ $S_7 = A_{r1}$

$S_8 = A_{r2}$ $S_9 = A_{r3}$

The states $S_O,\ and\ S_1$ are up states while $S_3,\ S_4, S_5\ S_6, S_7, S_8\ and\ S_9$ are down states. Further, all the seven state are regenerative states. The transition diagram along with all transitions is shown in Fig. 1.

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A) Steady State Transition Probabilities:

By taking the limit as $t \to \infty$, in transition probabilities, we get the steady state probabilities defined as;

$P_{01} = Q_{ij}(\infty) \int_0^{\infty} Q_{ij}(t)$

The following expressions for the non-zero elements are obtained.

$P_{01} = \frac{\lambda_1}{\lambda_1+\lambda}$ $\quad P_{02} = \frac{\lambda}{\lambda_1+\lambda}$

$P_{10} = \frac{\lambda_2}{\lambda_2+\lambda}$ $\quad P_{13} = \frac{\lambda}{\lambda_2+\lambda}$

$P_{24} = p_1$ $\quad P_{25} = q_1$

$P_{26} = r_1$ $\quad P_{37} = p_2$

$P_{38} = q_2$ $\quad P_{39} = r_2$

$P_{40} = P_{50} = P_{60} = 1$

$P_{71} = P_{81} = P_{91} = 1$

Here it can easily be verified that $\sum_j P_{ij} = 1$; for all possible values of i.

B) Mean Sojourn Time:

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$\Psi_i = E[T_i] = \int P(T_i > t)dt$

By simple probabilistic arguments, the expressions for mean sojourn time in different states are given as:

$\Psi_0 = \frac{1}{(\lambda_1+\lambda)}$ $\quad \Psi_1 = \frac{1}{(\lambda_2+\lambda)}$

$\Psi_2 = \frac{1}{\lambda_3}$ $\quad \Psi_3 = \frac{1}{\lambda_3}$

$\Psi_4 = \Psi_5 = \Psi_6 = \int_0^{\infty} \bar{\beta}(t)\ dt$

$\Psi_7 = \Psi_8 = \Psi_9 = \int_0^{\infty} \bar{\mu}(t)\ dt$

5. MEAN TIME TO SYSTEM FAILURE

Let the random variable T_i denotes the time to system failure when $E_o = E_i \in E$ and $\pi_i(t)$ is the c.d.f. of the time to system failure for the first time when the system starts operation from state S_i. To obtain the expressions of $\pi_i(t)$ for different values of i, the arguments of regenerative point processes has been used. Taking the Laplace transform and solving the resultant set of equations for $A_0^*(s)$, we have

$\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)}$

where

$N_1(s) = \tilde{Q}_{01}(s)\tilde{Q}_{13}(s) + \tilde{Q}_{02}(s)$

$D_1(s) = 1 - \tilde{Q}_{01}(s)\tilde{Q}_{10}(s)$

On taking $s \to 0$ in (6) and (7) and using the relation $\check{Q}_{ij}(s) \to p_{ij}$,we have

$$\tilde{\pi}_0(0) = \frac{N_1(0)}{D_1(0)} = 1$$

Thus $N_1(0) = D_1(0)$ showing that $\tilde{\pi}_0(0) = 1$. hence $\pi_0(t)$ is a proper cdf.
Therefore, mean time to system failure when the initial state is $S_{0,}$ is given by

$$E(T) = -\frac{d\tilde{\pi}_0(s)}{ds}\bigg|_{s=0} = \frac{D_1^{'}(0)-N_1^{'}(0)}{D_1(0)}$$

$$D'_1(0) - N'_1(0) = \Psi_0 + P_{01}\Psi_1$$

$$D_1(0) = 1 - P_{01}P_{10}$$

Therefore, the mean time to system failure is found to be

$$\mathrm{M.T.S.F} = \frac{\Psi_0+P_{01}\Psi_1}{1-P_{01}P_{10}}$$

Now put value of $\Psi_i{}'s$ and $p_{ij}{}'s$ in above equation, we get

$$D'_1(0) - N'_1(0) = (\lambda_2 + \lambda) + \lambda_1$$

$$D_1(0) = (\lambda_1 + \lambda)(\lambda_2 + \lambda) - \lambda_1\lambda_2$$

6. AVAILABILITY ANALYSIS

We define $\mathrm{A_i(t)}$ as the probability that the system is up at epoch 't' when it initially starts from regenerative state $\mathrm{S_i}$. It is also called pointwise availability of the system. To obtain recurrence relations among different point wise availabilities $\mathrm{A_i(t)}$, we use simple probabilistic arguments. Taking the Laplace transform and solving the resultant set of equations for $\mathrm{A_0^*(s)}$, we have:

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

Where

$$N_2(s) = \{1 - q_{13}^*(q_{37}^*q_{71}^* + q_{38}^*q_{81}^* + q_{39}^*q_{91}^*)\}M_0^* + q_{01}^*M_1^* \quad (1)$$

$$D_2(s) = \{1 - q_{02}^*(q_{24}^*q_{40}^* + q_{25}^*q_{50}^* + q_{26}^*q_{60}^*)\}\{1 - q_{13}^*(q_{37}^*q_{71}^* + q_{38}^*q_{81}^* + q_{39}^*q_{91}^*)\} - q_{01}^*q_{10}^* \quad (2)$$

The steady state availability of the system will be given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to0} s\, A_0^*(s) = N_2(0)/D_2'(0)$$

On substituting values of $P'_{ij}s$ and $\Psi'_i s$, we get

$$N_2(0) = \lambda_3(\lambda_1 + \lambda_2)$$

$$D_2'(0) = (\lambda_3 + \lambda)(\lambda_1 + \lambda_2) + \lambda_3\lambda\left\{\lambda_2 \int_0^\infty \bar{\beta}(u)\,\mathrm{du} + \lambda_1 \int_0^\infty \bar{\mu}(u)\,\mathrm{du}\right\}$$

Now the mean up time during (0, t] is

$\mu_{up} = \int_0^t A_0(u)du$, So that

$$\mu_{up}^*(s) = A_0^*(s)/s$$

And the down time during (0,t] is

$$\mu_0(t) = 1 - \mu_{up}(t)$$

7. BUSY PERIOD ANALYSIS FOR REPAIRMAN:

We define $B_i(t)$ as the probability that the regular repairman is busy in the repair of the failed unit when the system initially starts from state $S_i \in E$. Using probabilistic arguments, taking the Laplace transform and solving the resultant set of equations for $B_0^*(s)$, we have

$$B_0^*(s) = N_3(s)/D_3(s)$$

Where

$$N_3(s) = q_{01}^* q_{13}^* \left(q_{37}^* Z_7^* + q_{38}^* Z_8^* + q_{39}^* Z_9^* \right) + q_{02}^* \{1 - q_{13}^* (q_{37}^* q_{71}^* + q_{38}^* q_{81}^* + q_{39}^* q_{91}^*)\} \left(q_{24}^* Z_4^* + q_{25}^* Z_5^* + q_{26}^* Z_6^* \right)$$

and $D_3(s) = D_2(s)$ is same as in availability analysis which is given by (2)
In the steady state, the probability that the regular repairman will be busy is given by

$$B_0^*(s) = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = \frac{N_3(0)}{D_2'(0)}$$

On substituting values of $p_{ij}'s$ and $\Psi_i's$ in above equation, we get,

$$N_3(0) = \lambda_3 \lambda \left\{ \lambda_2 \int_0^\infty \bar{\beta}(u)\,\mathrm{du} + \lambda_1 \int_0^\infty \bar{\mu}(u)\,\mathrm{du} \right\}$$

$$D_2'(0) = (\lambda_3 + \lambda)(\lambda_1 + \lambda_2) + \lambda_3 \lambda \left\{ \lambda_2 \int_0^\infty \bar{\beta}(u)\mathrm{du} + \lambda_1 \int_0^\infty \bar{\mu}(u)\mathrm{du} \right\}$$

Now the expected duration of the busy time of the repairman in (0,t]

$\mu_t(t) = \int_0^t B_0(u)du$, So that $\mu_t^* = B_0^*/s$

8. EXPECTED NUMBER OF VISITS BY REGULAR REPAIRMAN:

Let us define $V_i(t)$ as the expected number of visits by regular repairman during the time interval $(0, t]$ when the system initially starts from regenerative state S_i. Using probabilistic arguments, taking the Laplace transform and solving the resultant set of equations for $\tilde{V}_0(s)$, we get

$$\tilde{V}_0(s) = \frac{N_4(s)}{D_4(s)}$$

Where

$$N_4(s) = \tilde{q}_{02}\{1 - \tilde{q}_{13}(\tilde{q}_{37}\tilde{q}_{71} + \tilde{q}_{38}\tilde{q}_{81} + \tilde{q}_{39}\tilde{q}_{91})\} + \tilde{q}_{01}\tilde{q}_{13}$$

$D_4(s) = D_2(s)$ is same as in availability analysis given by (2)

In steady state, number of visits per unit time is given by

$$V_0(0) = \lim_{t\to\infty} \frac{V_0(t)}{t} = \frac{N_4(0)}{D_4'(0)}$$

On substituting values of $p_{ij}'s$ and $\Psi_i's$ in above equation, we get,

$$N_5(0) = \lambda_3 \lambda (\lambda_1 + \lambda_2)$$

$$D_2'(0) = (\lambda_3 + \lambda)(\lambda_1 + \lambda_2) + \lambda_3 \lambda \left\{ \lambda_2 \int_0^\infty \bar{\beta}(u)\mathrm{du} + \lambda_1 \int_0^\infty \bar{\mu}(u)\mathrm{du} \right\}$$

9. PROFIT ANALYSIS

The expected uptime, down time of the system and busy period of the repairman in (0, t] is given as:

$$\mu_{up}(t) = \int_0^1 A_0(u)du$$

$$\mu_{dn}(t) = 1 - \mu_{up}(t)$$

$$\mu_b = \int_0^1 B_0(u)du$$

So that

$$\mu_{up}^*(s) = A_0^*(s)/s$$

$\mu^*_{dn}(s) = 1/s^2 - \mu^*_{up}(s)$
$\mu^*_b(s) = B^*_0(s)/s$

The expected profits incurred in (0, t] = expected total revenue in (0, t] – expected total repair in (0, t] – expected cost of visits by repairman in(0, t]
Therefore, profit analysis of the system can be written as:
$P_1 = K_0A_0 - K_1B_0 - K_2V_0$
K_0 = Revenue per unit up time of the system,
K_1 = Cost per unit time for which the repair is busy,
K_2 = Cost per unit visits by the repairman.

10. GRAPHICAL STUDY OF SYSTEM BEHAVIOUR

The behavior of MTSF, Availability and Profit analysis of the system is studied graphically in this section. To plot their graphs, the repair time distributions of units are also assumed to be distributed exponentially. The graphs of MTSF, availability and that of profit are depicted with respect to the different parameters. It is observed that the MTSF, Availability and the profit analysis of the system decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF, Availability and the profit analysis increases with increasing repair rates. Hence, it can be concluded that the expected life of the system can be increased by decreasing failure rate and increasing repair rate of the unit which in turn will improve the reliability and hence the effectiveness of the system.

In fig 2, we plot MTSF w.r.t. λ and fixed values of parameter $\lambda_1, \lambda_2, \lambda_3, \gamma_1$ *and* γ_2 . It is observed that MTSF of the system decreases w.r.t. α_1 irrespective of the other parameters so that we conclude that expected life of the system increases with decreasing failure rate of unit in minor failure mode (λ).In fig 3, we plot MTSF γ_1 w.r.t. and fixed values of parameter $\lambda, \lambda_1, \lambda_2, \lambda_3, \gamma_2$. It is quiet clear that MTSF of the system increases w.r.t. γ_1 irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate of unit in repair mode (γ_1). In fig 4, we plot Availability w.r.t. λ and fixed values of parameter $\lambda_1, \lambda_2, \lambda_3, \gamma_1, \gamma_2$. It is observed that Availability of the system decreases w.r.t. λ irrespective of the other parameters. Therefore, we conclude that expected life of the system increases with decreasing failure rate of unit in failure mode (λ). In fig 5, we plot Availability γ_1 w.r.t. and fixed values of parameter $\lambda, \lambda_1, \lambda_2, \lambda_3, \gamma_2$. It is quiet clear that Availability of the system increases w.r.t. γ_1 irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate of unit in repair mode (γ_1). In fig 6, we plot profit w.r.t. λ and fixed values of parameter $\lambda_1, \lambda_2, \lambda_3, \gamma_1, \gamma_2$. It is observed that profit of the system decreases w.r.t. λ irrespective of the other parameters so that we conclude that expected life of the system increases with decreasing failure rate of unit in minor failure mode (λ). In fig 7, we plot profit w.r.t. γ_1 and fixed values of parameter $\lambda, \lambda_1, \lambda_2, \lambda_3, \gamma_2$. It is quiet clear that profit of the system increases w.r.t. γ_1 irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate of unit in repair mode (γ_1).

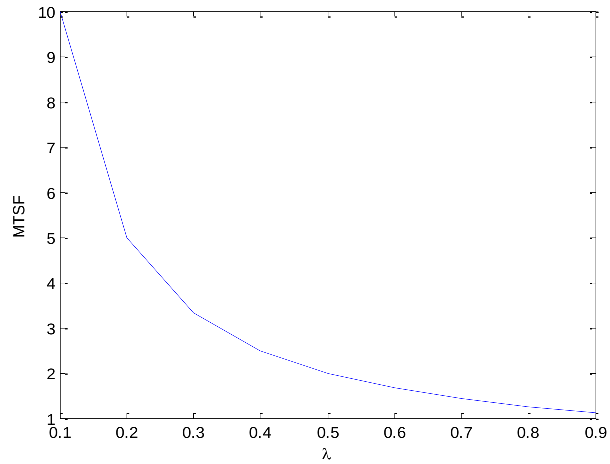

Figure.2 Behaviour of MTSF W.R.T. λ for different values of $\lambda_1, \lambda_2, \lambda_3, \gamma_1$ *and* γ_2

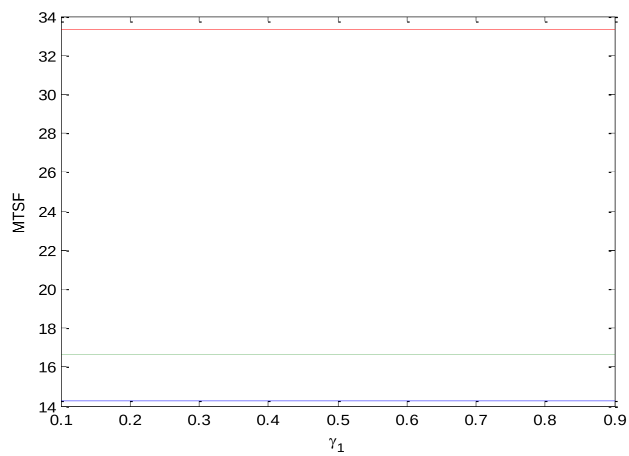

Figure.3 Behaviour of MTSF W.R.T. γ_1 for different values of $\lambda, \lambda_1, \lambda_2, \lambda_3$ *and* γ_2

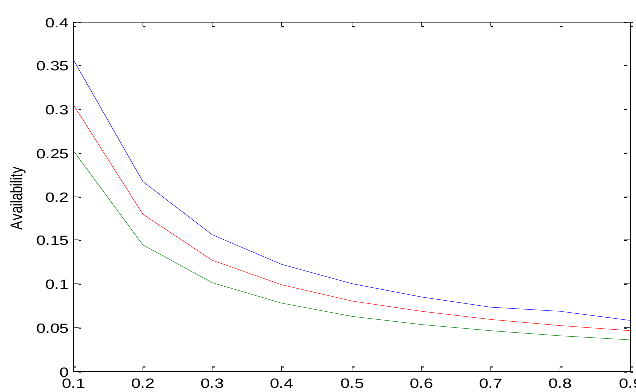

Figure.4 Behaviour of availability W.R.T λ for different values of $\lambda_1, \lambda_2, \lambda_3, \gamma_1, \gamma_2$

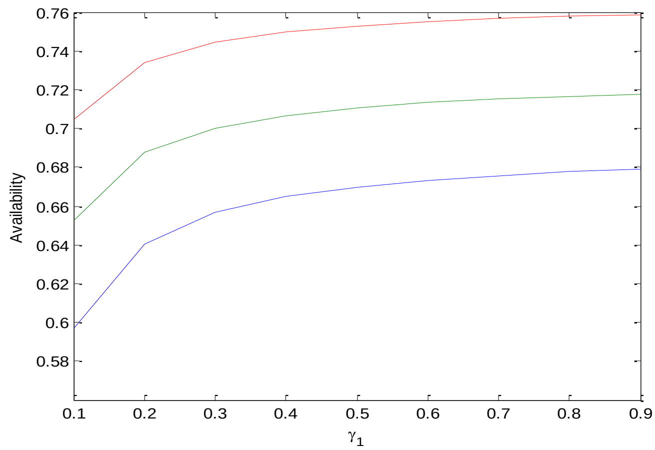

Figure.5 Behaviour of availability W.R.T γ_1 for different values of $\lambda, \lambda_1, \lambda_2, \lambda_3, \gamma_2$

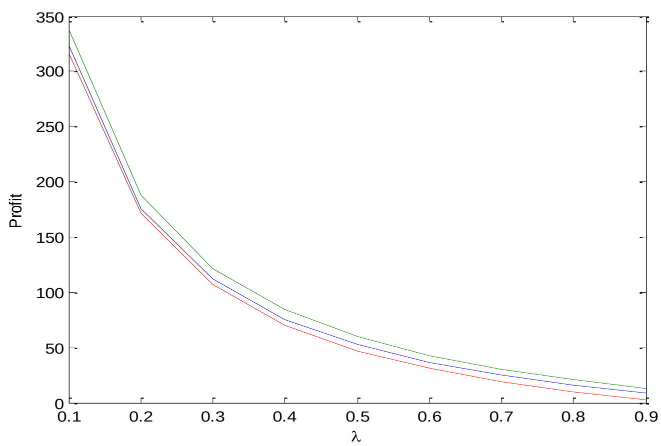

Figure.6 Behaviour of profit W.R.T. λ for different values of $\lambda_1, \lambda_2, \lambda_3, \gamma_1, \gamma_2, k_0, k_1 and\ k_2$

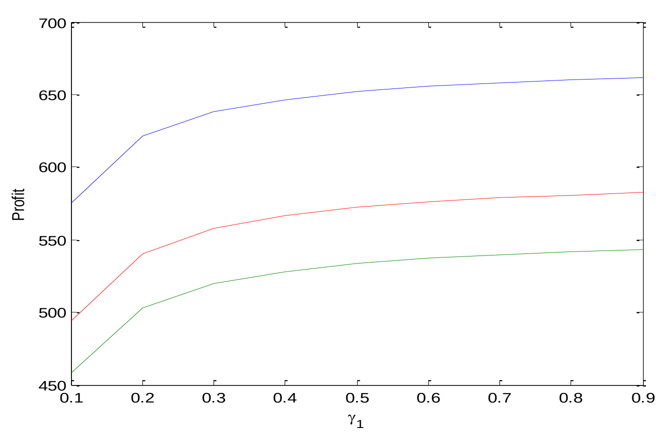

Figure.7 Behaviour of profit W.R.T γ_1 for different values of $\lambda, \lambda_1, \lambda_2, \lambda_3, \gamma_2, k_0, k_1 and\ k_2$

11. CONCLUDING REMARKS:

It is observed that the MTSF decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF increases with increasing repair rates. Thus, we can conclude that the expected life of the system can be increased by increasing repair rate of the unit. Further, it is observed that the availability of the system gradually decreases with increasing failure rates irrespective of type of failure and increases with increasing repair rate of the unit. Also, it is seen that profit analysis of the system decreases as failure rate increases irrespective of the other parameters and increases with increasing repair rate of the unit. Hence, it can be concluded that the expected life of the system can be increased by decreasing failure rate and increasing repair rate of the unit which in turn will improve the reliability and hence the effectiveness of the system.

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