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Integrated Inventory Policy for Deteriorating Items with Time Dependent Demand when Trade Credit is Offered

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Abstract: Now-a-days to attract buyers, vendor uses promotional tool viz trade credit period which is considered to be the most beneficial policy. In this article an attempt is made to maximize the joint total profit of the supply chain with respect to buyer's order quantity during a cycle time. A mathematical model for integrated inventory system is developed when demand rate linearly depends on time. By analyzing the total channel profit function, we developed some useful results to characterize the optimal solution and provided buyer's order quantity, optimal special cycle time. The units in inventory are subject to constant deterioration. A numerical example is given to support the proposed model. The sensitivity analysis of model parameter is carried out. Managerial insights are also obtained.

Keywords: Integrated Inventory Model, Time Dependent Demand, Deterioration, Trade Credit.

1. INTRODUCTION

As per the existing EOQ Model, the general assumption is that buyer should pay instantly after receiving the goods, which is not always practical. Now a days, supplier is ready to offer some time limit for payment to survive in the competition. During this credit time allowed by the supplier, the retailer can sell the goods, generate cash and earn interest on it. But the supplier can charge heavy interest, if payment is not made before the credit period. This model was introduced by Haley and Higgins [17]. Goyal [13] developed economic order quantity model under conditions of permissible delay in payments. Aggarwal and Jaggi [1] presented the economic ordering policies for deteriorating items in the presence of permissible delay payments. Hwang and Shinn [22] developed the joint price and lot size determination problem for an exponentially deteriorating product when the supplier offers a certain fixed credit period. Jamal et al. [24] developed a model for an optimal ordering policy for deteriorating items with allowable shortage and permissible delay in payment. Sarkar et al. [32] developed Supply chain models for perishable products under inflation and permissible delay in payment. Further Liao et al. [27] presented an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible under inflation. Chung et al. [8] developed the cycle time determination problem for an exponentially deteriorating product when the supplier offers a certain fixed credit period. Chang et al. [4], Chung and Liao [6] presented an EOQ model for deteriorating items when supplier credits linked to order quantity. Ouyang et al. [30] provided the optimal policy for the customer to obtain its minimum cost when the supplier offers not only a permissible delay but also a cash discount. Teng et al. [36] developed an algorithm for a retailer to determine its optimal price and lot size simultaneously when the supplier offers a permissible delay in payments by considering the difference between the selling price and the purchase cost. Chung and Liao [7] presented an inventory system for deteriorating items under the conditions of using the discounted cash-flows (DCF) approach to the permissible delay payment related to order quantity to generalize Jaggi and Aggarwal [23]. Chen and Ouyang [5] extended Jamal et al. [24] model by fuzzifying the carrying cost rate, interest paid rate and interest earned rate simultaneously, based on the interval- valued fuzzy numbers and triangular fuzzy number to fit the real world. Huang and Liao [21] explored an economic lot sizing model that incorporates a realistic feature such as the deterioration rate following an exponential distribution, making a broader application scope of Chang and Teng [3]. Tsao and Sheen [38] adopted a price- and time-dependent demand function to model the finite time horizon inventory for deteriorating items.

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De and Goswami [9] developed a probabilistic inventory model for items that deteriorate at a constant rate and the demand is a random variable under trade credit financing. Thangman and Uthayakumar [37] characterized a profitable decision policy between a supplier and the retailers by an agreement on the trade credit scenario such as permissible delay in payments (two echelon trade credit financing) for perishable items in a supply chain when demand depends on selling price and credit period.

Dye and Ouyang [11] have done a particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing for deteriorating items. Dye [10] considered a finite horizon deteriorating inventory model with two-phase pricing and time-varying demand and cost under trade credit financing using particle swarm optimization. Hou and Lin [20] extended economic order quantity model for deteriorating items under inflation and permissible delay in payments where demand rate is a linear function of price and decreases negative exponentially with time. Singh and Pattanayak [34] designed an EOQ model for a deteriorating item with time dependent exponentially declining demand under permissible delay in payment. Guchhait et al. [16] developed inventory policy of a deteriorating item with variable demand under trade credit period. Shastri et al. [33] developed a supply chain inventory model in inflationary environment by incorporating some realistic features such as ramp type demand, deterioration, partial backlogging, inflation and trade credit.

The above stated models are discussed either from buyer's or vendor's point of view. However, these one-sided optimal inventory models neglected interaction and cooperation opportunity between the buyer and the vendor. Therefore, to improve the collaboration of supply chain partners, determining the optimal policies based on the integrated total profit function is more reasonable which realizes the need of developing a win-win strategy for the buyer and vendor. Goyal [12] developed a single-vendor single-buyer integrated inventory model. Banerjee [2] assumed a lot-for-lot shipment policy for vendor in Goyal [12]. Goyal [14] relaxed the lot-for-lot policy and established that the inventory cost reduces significantly if vendor's economic production quantity is an integral multiple of the buyer's purchase quantity. Many Researchers Lu [28], Goyal [15], Viswanathan [39], Hill [18,19], Kelle *et al.* [26], Yang and Wee [40] established that more batching and frequent shipment policies are advantageous for the integrated inventory models. Mahata [29] developed an integrated production- inventory model with back order and lot for lot policy in fuzzy sense. Tayal et al. [35] developed an integrated production- distribution model for deteriorating items in a two echelon supply chain with allowable shortages and investment in preservation technology.

In this article, we develop an integrated vendor-buyer inventory model when demand rate linearly depend on time, the units are subject to deterioration at a constant rate and trade credit is allowed only by the vendor to the buyer. For this purpose, we, Kawale and Sanas [25], first took detailed review on inventory models under trade credit. The joint total profit per unit time is maximized with respect to order quantity and optimal special cycle time. A computational procedure is derived to find the best optimal decision. The numerical example and sensitivity analysis are given to validate the developed model.

2. NOTATIONS

The following notations are used in the proposed article:

S_v: Vendor's set up cost per set up.

S_b: Buyer's ordering cost per order.

C_v: Production cost per unit.

C_b: Buyer's purchase cost per unit.

C_c: The unit retail price to customers where $C_c > C_b > C_v$.

I_v: Vendor's inventory holding cost rate per unit per annum, excluding interest charges.

I_b: Buyer's inventory holding cost rate per unit per annum, excluding interest charges.

I_{v0}:Vendor's opportunity cost/$/unit time.

I_{b0}: Buyer's opportunity cost/$/unit time.

I_{be}: Buyer's interest earned/$/unit time.

ϱ: Capacity utilization which is ratio of demand to the production rate, $\varrho < 1$ and known constant.

M: Allowable credit period for the buyer offered by the vendor.

Q: Buyer's order quantity.

T: cycle time (decision variable).

n : Number of shipments from vendor to the buyer.

ɵ: constant rate of deterioration.

TVP: Vendor's total profit per unit time.

TBP: Buyer's total profit per unit time.

п : TVP + TBP Joint total profit per unit time.

3. ASSUMPTIONS

In addition, the following assumptions are made in derivation of the model:

- The supply chain under consideration comprise of single vendor and single buyer for a single product.
- Shortages are not allowed.
- The demand rate considered is time dependent, increasing demand rate. The constant part of linear demand pattern changes with each cycle.
- Replenishment rate is instantaneous for buyer
- The units in inventory are subject to deteriorate at a constant rate of ɵ, 0 < ɵ < 1. The deteriorated units can neither be repaired nor replaced during the cycle time.
- Finite production rate.
- Vendor produces the nQ items and then fulfils the buyer's demand, so at the beginning of production item, there is small possibility of deterioration in general. Moreover vendor is a big merchant who can have capacity to prevent deterioration. So in this model, deterioration cost is considered for buyer only at the rate ɵ is assumed to be constant.
- Vendor offers the buyer a permissible delay period M. During this permissible delay period, the buyer sells the items and uses the sales revenue to earn interest at a rate of I_{be} /unit/annum. At the end of this time period buyer settles the payments due against the purchase made and incurs opportunity cost at a rate of I_{b0} /unit/annum for unsold items in stock.

4. MATHEMATICAL FORMULATION

Throughout each production run, vendor manufactures, at a rate R, in batches of size nQ and incurs batch set up cost Sv. The expected cycle length for the vendor is nQ/D where D is the market demand rate. When total required amount nQ is accomplished, vendor stops production. The accumulated inventory for the vendor can be obtained as

$$nQ * \left[\frac{Q}{R} + (n-1) * \frac{Q}{D}\right] - \frac{1}{2} * nQ * \frac{nQ}{R}$$

The vendor distributes first Q units to the buyer as soon as it has been produced, that means vendor will make the delivery on average every Q/D units of time. Hence, the accumulated inventory for the buyer, receiving n shipments, each equal to Q, in a production cycle is given by

$$Q * \frac{Q}{D} * [1 + 2 + ------ + (n-1)]$$

The vendor's inventory per cycle = vendor's accumulated inventory level - the buyer's accumulated inventory level
The vendor's average inventory per unit time can be calculated as follows

$$\frac{\left\{\left\{nQ * \left[\frac{Q}{R} + (n-1) * \frac{Q}{D}\right] - \frac{n^2Q^2}{2R}\right\} - \frac{Q^2}{D}[1 + 2 + ---- + (n-1)]\right\}}{\frac{nQ}{D}}$$

$$= \frac{Q}{2}[(n-1)(1-\varrho) + \varrho] \quad \text{where } \varrho = \frac{R}{D}$$

The above derivation is similar to the Ouyang et al. (2005).

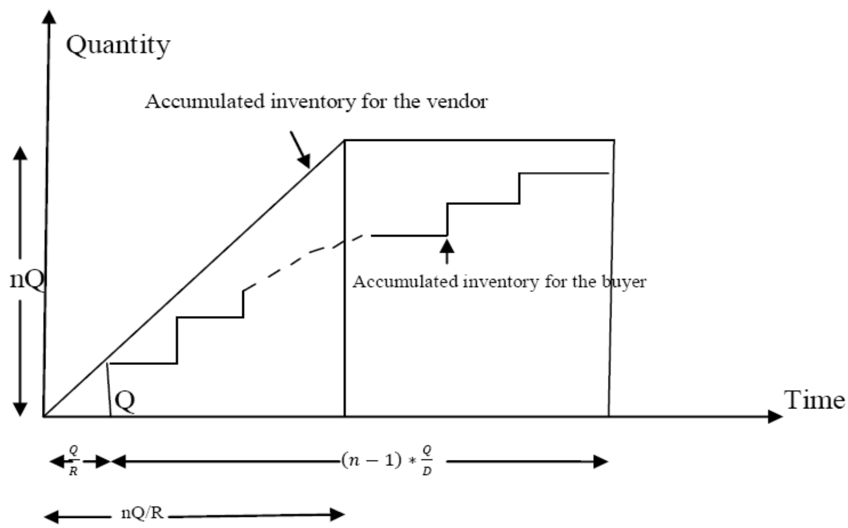

Figure 1. Integrated Vendor- Buyer inventory system.

4.1 Net profit function for vendor consists of following elements:-

1. Sales revenue: the total sales revenue per unit time is $(C_b - C_V)\frac{Q}{T}$.

$$= \frac{(C_b - C_v)}{T}\left\{(\frac{a}{\theta} - \frac{b}{\theta^2})(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

See Appendix A for computation of Q

2. Set-up cost : nQ units are manufactured in one production run by the vendor. Therefore the setup cost per unit time is $\frac{S_v}{nT}$

3. Holding cost : using vendor's average inventory per unit time

$$\frac{C_v(I_v+I_{v0})}{T}[(n-1)(1-\varrho)+\varrho]\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}(T-Te^{\theta T}+\frac{\theta T^2}{2})\}$$

4. Opportunity cost : opportunity cost per unit time because of offering permissible delay period is $\frac{C_b I_{v0} MQ}{T}$

$$= \frac{C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

Hence the total profit per unit time for vendor is = Sales revenue – Set up cost – Holding cost – Opportunity cost.

$$\text{TVP} = \frac{(C_b-C_v)}{T}\left\{(\frac{a}{\theta}-\frac{b}{\theta^2})(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\} - \frac{S_v}{nT} - \frac{C_v(I_v+I_{v0})}{T}[(n-1)(1-\varrho)+\varrho]\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}(T-Te^{\theta T}+\frac{\theta T^2}{2})\} - \frac{C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\} \quad (1)$$

4.2 Net profit function for the buyer consists of following elements:-

1. Sales revenue: The total sales revenue per unit time is $\frac{(C_c-C_b)Q}{T}$

$$= \frac{(C_c - C_b)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

2. Ordering cost : Ordering cost per unit time is $\frac{S_b}{T}$

3. Holding cost: The buyer's holding cost (excluding interest charges) per unit time is

$\frac{C_b I_b}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}(T-Te^{\theta T}+\frac{\theta T^2}{2})\}$

4. Deteriorating cost : Deteriorating cost per unit time is $\frac{C_b}{T}[Q-\int_0^T(a+bt)dt]$

$$=\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\}$$

Based on the length of the credit period offered by the vendor, two cases arise namely $M<T$ and $M \geq T$.

Case 1] When $M<T$

In this case buyer starts getting the sales revenue and earns interest on average sales revenue for the time period till M, at M accounts are settled, if the stock still remains, finances are to be arranged to make payments to the vendor.

5. Interest earned per unit time during the credit period [0, M] is $\frac{I_{be}C_c}{T}\int_0^M(a+bt)tdt$

$$=\frac{I_{be}C_c}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}\right]$$

6. Interest payable per unit time during time span [M, T] is $\frac{C_b I_{b0}}{T}\int_M^T I(t)dt$

$$=\frac{C_b I_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)\left(1+\theta(T-M)-e^{\theta(T-M)}\right)-\frac{b}{\theta^2}(T-Te^{\theta(T-M)}+\theta(\frac{T^2}{2}-\frac{M^2}{2})\}$$

Therefore profit of the buyer in this case can be expressed as :-

TBP1= Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned – Interest paid.

$$=\frac{(C_C-C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}-\frac{S_b}{T}-\frac{C_b I_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)\right\}-\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\}+\frac{I_{be}C_c}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}\right]-\frac{C_b I_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)\left(1+\theta(T-M)-e^{\theta(T-M)}\right)-\frac{b}{\theta^2}(T-Te^{\theta(T-M)}+\theta(\frac{T^2}{2}-\frac{M^2}{2})\} \quad (2)$$

Case2] When $M \geq T$.

The first 4 components of the profit function remain same. The sixth cost component does not exist for $M \geq T$. The interest earned per unit time during time span [0, M] is

$$\frac{(I_{be}\ C_c)}{T}\{\int_0^T(a+bt)tdt+Q(M-T)\}$$
$$=\frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}\right]+\frac{I_{be}C}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}(M-T)$$

In this case profit for the buyer is given by

TBP2 = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned.

$$\text{TBP2}=\frac{(C_c-C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}-\frac{S_b}{T}-\frac{C_b I_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)\right\}-\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\}+\frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}\right]+\frac{I_{be}C_c}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}(M-T) \quad (3)$$

4.3 Joint total profit per unit time

In integrated system, the vendor and the buyer to take joint decision which maximizes the profit of the supply chain, the joint total profit per unit time for integrated system is

$$\pi = \begin{cases} \pi 1 = TVP + TBP1 & M < T \\ \pi 2 = TVP + TBP2 & M \geq T \end{cases}$$

Considering $e^{\theta T} = 1 + \theta T + \frac{\theta^2 T^2}{2}$

$$TVP = (C_b - C_v - C_b I_{v0} M)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v\left(I_v + I_{v0}\right)[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) \quad (4)$$

$$TBP1 = (C_c - C_b)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be} C_c}{T}\left[\frac{aM^2}{2} + \frac{bM^3}{3}\right] - C_b I_{b0}\left\{a\frac{(T-M)^2}{2T} - b\frac{(T-M)^2}{2\theta T} - b\left(-\frac{(T-M)}{\theta} - \frac{(T-M)^2}{2} + \frac{T}{2\theta} - \frac{M^2}{2\theta T}\right)\right\} \quad (5)$$

$$TBP2 = (C_c - C_b)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be} C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + I_{be} C_c (M - T)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) \quad (6)$$

$$\pi 1 = (C_b - C_v - C_b I_{v0} M)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v\left(I_v + I_{v0}\right)[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) + (C_c - C_b)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be} C_c}{T}\left[\frac{aM^2}{2} + \frac{bM^3}{3}\right] - C_b I_{b0}\left\{a\frac{(T-M)^2}{2T} - b\frac{(T-M)^2}{2\theta T} - b\left(-\frac{(T-M)}{\theta} - \frac{(T-M)^2}{2} + \frac{T}{2\theta} - \frac{M^2}{2\theta T}\right)\right\} \quad (7)$$

$$\pi 2 = (C_b - C_v - C_b I_{v0} M)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v\left(I_v + I_{v0}\right)[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) + (C_c - C_b)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be} C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + I_{be} C_c (M - T)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) \quad (8)$$

The optimum value of cycle time can be obtained by setting $\frac{d\pi}{dT} = 0$ for fixed n. The necessary condition for maximizing total profit is $\frac{d^2\pi}{dT^2} < 0$.

5. Numerical examples

To illustrate the above developed model, an inventory system with the following data is considered a=1000, b= 50 , θ=0.1, ϱ=0.7, C_v= \$5/unit, C_b= \$25/ unit, C_c = \$ 55 / unit, S_v =\$1500/setup, S_b= \$100/order, I_v= 1%/unit/annum, I_b=1%/unit/annum, I_{v0}= 2%/unit/annum, I_{b0}= 5%/unit/annum, I_{be} = 8%/unit/annum and M = 30days

Using computational procedure optimum cycle time T* for above data is 21 days for n = 5. The buyer's order quantity Q* are 1,21,580 units/order. Vendor's total profit TVP is \$9246.1 and buyer's total profit TBP is \$ 2,98,920. The maximum total joint profit of the integrated system π is \$3,08,170.

5.1 Sensitivity analysis

Sensitivity analysis of the integrated system with respect to parameters: demand scale parameter, demand rate parameter, deterioration rate and capacity utilization is presented in table 1, table 2 , table 3, and table 4. In each analysis the base parameter values are as assumed in Example1 and only the parameter of interest is varied holding all other parameter constant.

Table 1. Sensitive analysis for the demand scale parameter

Parameter a	T(days)	Q	Vendor	Buyer	Joint Profit
1,500	19	1,20,370	12,156	3,81,480	3,93,640
2,000	18	1,30,190	15,280	4,67,030	4,82,310
3,000	16	1,38,450	21,237	6,43,670	6,64,910

Table 2. Sensitive analysis for the demand rate parameter

Parameter b	T(days)	Q	Vendor	Buyer	Joint Profit
60	22	1,51,230	10,011	3,26,780	3,36,790
75	23	1,94,490	11,121	3,69,510	3,80,630
100	23	2,29,410	12,341	4,42,780	4,55,120

Table 3. Sensitive analysis for the deterioration rate

Parameter θ	T(days)	Q	Vendor	Buyer	Joint Profit
0.15	21	2,62,620	15,625	3,60,920	3,76,550
0.2	21	5,96,430	21,505	4,22,490	4,43,990
0.3	21	34,15,900	32,766	5,45,180	5,77,940

Table 4. Sensitive analysis for the capacity utilization

Parameter ϱ	T(days)	Q	Vendor	Buyer	Joint Profit
0.6	21	1,21,580	7,805	2,98,920	3,06,730
0.8	21	1,21,580	10,687	2,98,920	3,09,610
0.9	21	1,21,580	12,128	2,98,920	3,11,050

From table 1 it is observed that as demand scale parameter increases vendor's profit, buyer's profit and joint total profit of the supply chain also increases. Similarly it is shown from table 2,3,4 that as demand rate parameter, deterioration rate and ratio between production rate and the market demand rate increases joint total profit of integrated inventory system is also increases. From table 4 we can conclude that if there is a change in capacity utilization parameter then only vendor's total profit changes, buyer's total profit and order quantity remain same. Above table shows that Profit gains in percentage are positive for the entire supply chain. Therefore permissible delay period is beneficial to the supply chain as a whole.

6 Conclusion

In this paper, we formulate an integrated vendor-buyer inventory system with assumption that market demand is linearly dependent on time and vendor offers a permissible delay period to buyer for the payments of the procured items. Units in the inventory system are subject to constant deterioration. By analyzing the total channel profit function, we develop a solution to determine optimum order quantity and optimum special cycle time. Numerical example is presented to validate the proposed model and sensitivity analysis of the optimal solution is also indicated. Based on the results, it is observed that joint profit for the supply chain increases in joint decision. To attract the buyer for the joint decision, vendor should offer credit period. In future, one can study optimum threshold for the vendor to offer credit period to study inventory polices.

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Appendix A

$$\frac{dI(t)}{dt} + \theta\, I(t) = -(a + bt) \qquad 0 \le t \le T$$

With boundary condition I(0) =Q and I(T)=0 , We get:

$$Q = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bT}{\theta} e^{\theta T}, \qquad 0 \le t \le T.$$

$$I(t) = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta(T-t)} - 1) + \frac{b}{\theta}(Te^{\theta(T-t)} - t), \qquad 0 \le t \le T.$$