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A Queuing Model for Determining the Optimal-Mix of ATMs AND POS terminals in a Commercial Bank: An Implication to the Cash-less Policy in Nigeria

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Abstract: In the bid to actualizing the cashless policy in Nigeria, this study criticized the multiplication of Automated Teller machines (ATM) in commercial banks as recommended by most authors and emphasized the role of the Point -of -Sale terminals (POS) in reducing queue lengths and long waiting times of customers in banks. The study succeeded in formulating easy to use models for converting the number of ATMs in a bank into Optimal-Mix of ATMs and POSs. This was done via the M/G/1 queue modeling methodology. The models were applied to three commercial banks in Kaduna State, Nigeria namely; Bank I, Bank II and Bank III with 4,3 and 2 ATMs respectively. The Optimal ATM-POS Mix determined for these banks are respectively ;(1 ATM, 2 POS), (0 ATM, 2 POS) and (2 ATM, 0 POS). The study recommends that in actualizing the cashless policy, the Central Bank of Nigeria should make frantic effort in directing commercial Banks to apply the methodology of this study, in converting their current ATMs into Optimal Mix of ATMs and POSs.

Keywords: Queue, Service, Cash-less

1. INTRODUCTION

[2] established that the Automatic Teller Machine (ATM) is indisputably an indispensable piece of machinery in actualizing the Cashless Policy in Nigeria but certainly not without challenges. They identified these challenges to include; congestion, long queue lengths and long waiting times as well as over utilizations that characterize ATM queues across commercial banks in Nigeria. [7] opined that these challenges are traceable to the recent upsurge in the customer base of most banks without equivalent increase in service capacity.

In an attempt to address these challenges, [2] began by criticizing the cost ineffective solution of multiplying the number of ATM machines proffered by most researchers. They recommended that it is cost effective to install fast ATMs or upgrade the speed of existing ones rather than multiplying the number of slow ATMs.

The current solution proffered by commercial banks across the country, is the introduction of a few number of Point -Of- Sale (POS) terminals inside the banking hall in order to handle large monetary transactions among others. This solution is not without the challenge of determining the optimal number of ATMs and POS terminals (Optimal-Mix), that will effectively reduce the waiting times and queue length of customers as well as reduce over utilization of the Machines. The reason has been that no scientific method is employed in doing this.

Preliminary investigation in this study revealed that while the POS transactions are free of customers' response to some initializing procedures before the actual business transaction, the ATMs are not free from these procedures. Rather, they request the customers to perform some activities like choice of language selection, do you want to continue? (yes or no), do you want a receipt?(yes or no), Saving or current account?, Select the bank (destination), Do you want to perform another transaction? etc. All these, increase the service time of customers which in turn translates into increase waiting times and establishes the possibility of the POS terminal been faster than the ATM machine.

Another reason for the anticipated fastness of the POS is the speed and the type of processor installed on it relative to that of the ATM machines. In addition, the expertise of the cashier involved in the POS transaction with the customers, obviously helps to eliminate the delay in the transaction that would have been posed by the ignorance of the

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customers. A strong supporter of this claim is [4] who affirmed the efficiency and superiority of the POS terminals over the ATM machines stating that the former can be used for both cash withdrawals and lodgments unlike most ATMs.

The questions that borders one's mind is; why multiplying ATM machines outside banking halls when POS terminals within the banking halls process more transaction loads and at faster rate too?

In the light of the aforementioned, the study seeks to formulate a queuing model for determining the Optimal-Mix of ATMs and POSs in a commercial bank that will effectively reduce the waiting times and queue length of customers as well as reduce over utilization of the Machines. This solution answers the question as to whether the services of these two machines should be retained at equal ratio or at different mix.

2. Review of Related Literature

The study employed the M/G/1 queuing model; this model specifies the Poisson arrival and General Service time distribution for a single server queue. [2] applied the M/G/1 queue model when they modeled ATM queuing systems across selected banks in Nassarawa state, Nigeria. Many other researchers have assumed the Poisson arrival distribution, the exponential service time distribution and automatically applied the M/M/1 or the M/M/c queue models for single server and multiple server queues respectively, in solving the problem of long waiting time of customers and server over utilization. The recommendation of these researchers is that the number of ATMs should be increased in the banks forgetting the consequence of incurring new cost of purchase, installation and maintenance. Among these researchers are [6], [3], [5], [9], and [1].

To the best of our knowledge, researches that consider the role of POS terminals as well as the determination of the Optimal -Mix of ATMs and POSs in commercial banks for actualizing the cash-less policy are rare. This work has been able to bridge this gap in knowledge via a queue modeling methodology. These we believe will assist in making the actualization of the cashless policy in Nigeria a reality. The rest of the paper is captured as follows; Methodology, Result, Discussion, Conclusion and Recommendation.

3. Methodology

In this section, we present the method of data collection, the distribution fit and the mathematical details of the M/G/1 queuing model. A simple formulation for determining the optimal-Mix of the number of ATMs and POS terminals is also presented.

3.1 Method of data collection

Data on arrival and service times where sourced from three Banks in Kaduna, Nigeria. The Banks are; Guarantee Trust Bank, kabala west bye pass, ECO Bank, Angwar-Muazu bye pass and Diamond Bank, Ahmadu Bello way. One ATM queue was randomly chosen alongside the only existing POS queue in each bank see table 1 for details. The arrival time of each customer to the ATM or POS terminal was recorded their respective service times were obtained via a stop watch. These times were collected during a peak day (Monday) and peak hours (8.00 a.m - 12.00 p.m) for the period of two (2) months in each Bank. There was no festive day during this period.

3.2 Distribution fit

Following the fact that if number of arrivals follow the Poisson distribution, then inter arrival times follow the exponential distribution, the data on arrival times of customers across the Banks, were converted into inter arrival times. The MINITAB and the SPSS statistical softwares were now used to ascertain whether the inter arrival times across the Banks fit the exponential distribution using the Anderson Darling and the Kolmogorov Smirnov Statistic respectively. Details of the fit are in table 1 below.

3.3 The Mathematical details of the M/G/1 queuing Model

The M/G/1 queuing model is a single channel Poisson arrival, General Service time distribution and First-Come-First-Serve queuing discipline model. Givenλ as the arrival rate at the service facility, mean ($E\{t\}$) and variance ($var\{t\}$) of the service time distribution, according to [8], it can be shown using sophisticated probability/Markov chain analysis that;

$$L_s = \lambda E(t) + \frac{\lambda^2 E^2(t) + var(t)}{2[1-\lambda E(t)]}\ ,\ \lambda E(t) < 1 \tag{1}$$

where L_s is the average number of customers in the system which is a queue performance measure. It is important to mention that other derived queue performance measures include; the average number of customers in queue (Lq),the average waiting time of customers in the system and queue, $Ws$and Wq, respectively and the server utilization (ρ) where;

$$\rho = \{t\} \tag{2}$$

The following relationships popularly known as the Littles' formula can be used in computing the aforementioned queue performance measures.

$$L_S = \lambda W_s \tag{3}$$

$$L_q = \lambda W_q \tag{4}$$

The model does not provide a closed form expression for probability (p_n) of having'n' number of customers in queue because of analytic intractability. The probability that the facility is empty (idle) is computed as

$$p_0 = 1 - \{t\} = 1 - \rho \tag{5}$$

3.4 Models for computing the Optimal-Mix of ATMs and POS terminals

Let the service rate of the ATM be $\mu_A$and that of the POSbe μ_B. Then the Mix-ratio (MR) can be defined as:

$$MR = \frac{\mu_A}{\mu_B} = \frac{xATM}{yPOS} \tag{6}$$

where, $xATM/yPOS$ is in its lowest term and $xATM$ and $yPOS$ are integers.

3.4.1 The Optimal-Mix

The Optimal Number of ATMs ($\mathrm{O}N_{ATM}$) is formulated as;

$$\mathrm{O}N_{ATM} = N_{ATM}\, Mod\; x_{ATM} \tag{7}$$

where, N_{ATM} is the number of ATMs in a Bank.

The Optimal Number of POS terminals ($\mathrm{O}N_{POS}$) is formulated as;

$$\mathrm{O}N_{POS} = Int\left(\frac{N_{ATM} * y_{POS}}{xATM}\right) \tag{8}$$

where, the Int function returns the integer part of $\frac{N_{ATM\,X} * y_{POS}}{xATM}$.

Note that if $N_{ATM} < xATM$ then $\mathrm{O}N_{POS} = 0$

The optimal ATM-POS Mix is obtained as: ($\mathrm{O}N_{ATM}$, $\mathrm{O}N_{POS}$).

When multiple ATMs and multiple POS terminals are considered in a Bank, with the coefficient of variations in the service rates of the ATMs as well as that of the POS terminals each less than 20%, then the Average service rate of the ATMs ($\mathrm{A}\mu_A$) and the Average service rate of the POS terminals (A μ_B) can be used to replace μ_A and μ_B in equation 6 above (Taha, 2003). Otherwise (Coefficient of variation > 20%) a Simulation approach to modeling the system should be employed.

4. RESULT

The data collected on arrival times at the ATM and POS terminal in each Bank, were used in fitting probability distributions of inter arrival times (table 1) as well as their arrival rates. This informed the selection of the M/G/1 queuing model. Their respective service times were used to compute their service rates. See tables 1 and2. The queue performance measures for the ATM and POS terminals across the Banks were also captured (table 2). The result of system performance measures across banks when the POS queues were theoretically replaced with the ATM queues is shown in table 3.

Furthermore, the Optimal-Mix of ATM and POS terminals across the Banks are displayed in table 4. Finally, the section captured associated graphs to some of these tables in figures 1-7.

Table 1. Goodness of fit summary of exponential inter arrival time distribution

	Test Statistics			
Bank/Machine	**Kolmogorov Smirnov (K- S)**	**P-value**	**Anderson Darling**	**P-value**
Bank I ATM	0.921	0.364	0.830	0.191
Bank I POS	0.718	0.681	0.511	0489
Bank II ATM	0.850	0.889	0.222	0.932
Bank II POS	0.803	0.540	0.772	0.225
Bank III ATM	0.721	0.675	0.730	0.254
Bank III POS	0.596	0.870	0.319	0.793

$\alpha = 0.01$

Table 2. System performance of ATM and POS terminals across the Banks

Bank/ Machine	$\boldsymbol{\lambda}$	**μ** $(E[t])$		$\overline{L_s}$	$\overline{L_q}$	$\overline{W_s}(mins.)$	$\overline{W_q}(mins.)$	ρ **(%)**
Bank I ATM	0.4290	1.8063	1.3200	5.0403	4.2654	11.7489	9.9427	77.4881
Bank I POS	0.2810	1.2438	0.8500	1.0967	0.7472	3.90292	2.6593	34.9494
Bank II ATM	0.4227	1.9467	1.5700	7.1660	6.3431	16.95292	15.0062	82.2870
Bank II POS	0.4100	1.2798	0.8300	1.6875	1.1628	4.1159	2.8361	52.4718
Bank III ATM	0.4285	1.9904	1.5100	8.4473	7.5945	19.7135	17.7235	85.2886
Bank III POS	0.3232	1.2895	0.8700	1.3115	0.8947	4.0579	2.7684	41.6766

*Arrival Rate ($\boldsymbol{\lambda}$) * Service Rate (μ) *Variance of service time (Var[t]) * Average number of customer in the system ($\overline{L_s}$) * Average Length of Queue ($\overline{L_q}$) *Average waiting time in queue ($\overline{W_q}$) * Average waiting time in the system ($\overline{W_s}$) * Utilization (ρ)

Table 3. The result of system performance measures when the POS queues are theoretically replaced with their respective ATM queues.

Bank	Measures of Performance				
	$\overline{L_s}$	$\overline{L_q}$	$\overline{W_s}(mins.)$	$\overline{W_q}(mins.)$	ρ **(%)**
Bank I	1.0967	0.7472	2.5564	1.7418	53.3569
Bank II	1.6875	1.1628	3.9922	2.7509	54.0972
Bank III	1.3115	0.8947	3.0607	2.0881	55.2551

* Average number of customer in the system ($\overline{L_s}$) * Average Length of Queue ($\overline{L_q}$) *Average waiting time in queue ($\overline{W_q}$) * Average waiting time in the system ($\overline{W_s}$) * Utilization (ρ)

Table 4. Optimal-Mix of ATM and POS machines across Banks

Bank	**CurrentNumber of ATMs**	**ATM Service Rate** (μ_A)	**POS Service Rate** (μ_B)	**Mix-Ratio** $(\frac{\mu_A}{\mu_B})$	**Optimal-Mix**
Bank I	4	1.8063	1.2438	1.5 $(\frac{3}{2})$	(1ATM, 2POS)
Bank II	3	1.9467	1.2798	1.5 $(\frac{3}{2})$	(0ATM, 2POS)
Bank III	2	1.9904	1.2895	1.5 $(\frac{3}{2})$	(2 ATM, 0POS)

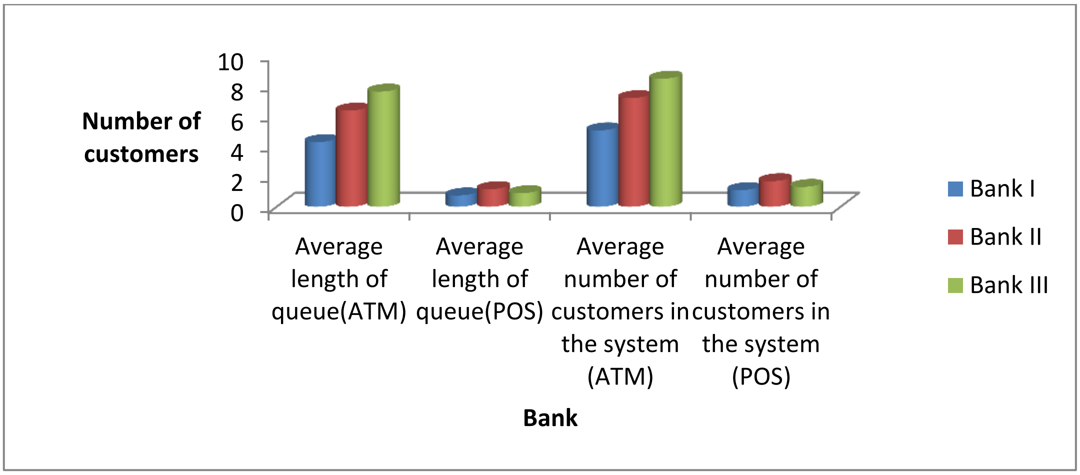

Figure1. Average number of customers waiting before the ATMs and POSs across Banks

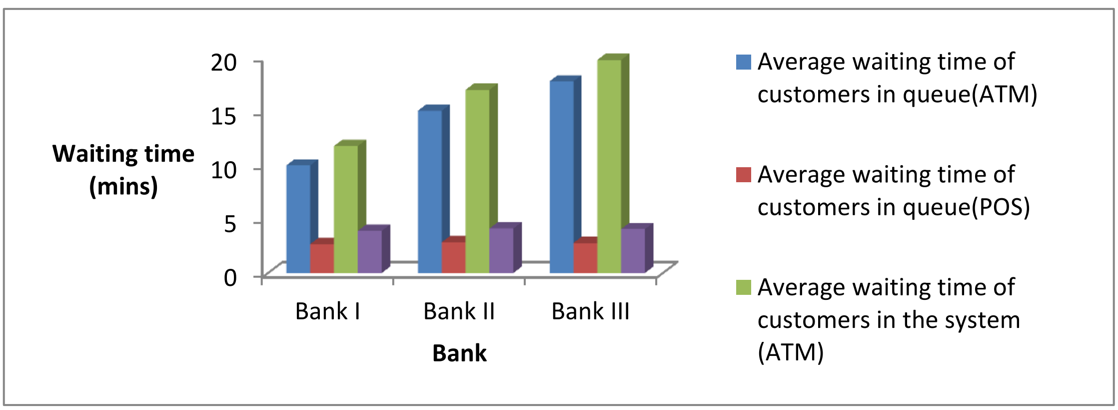

Figure 2. Average waiting time of customers before the ATMs and POSs across Banks

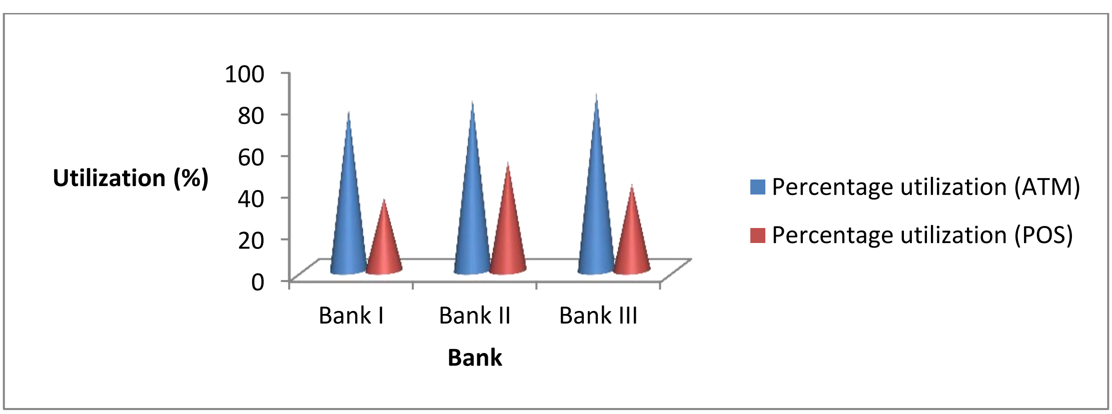

Figure 3. Utilization of ATMs and POSs across Banks

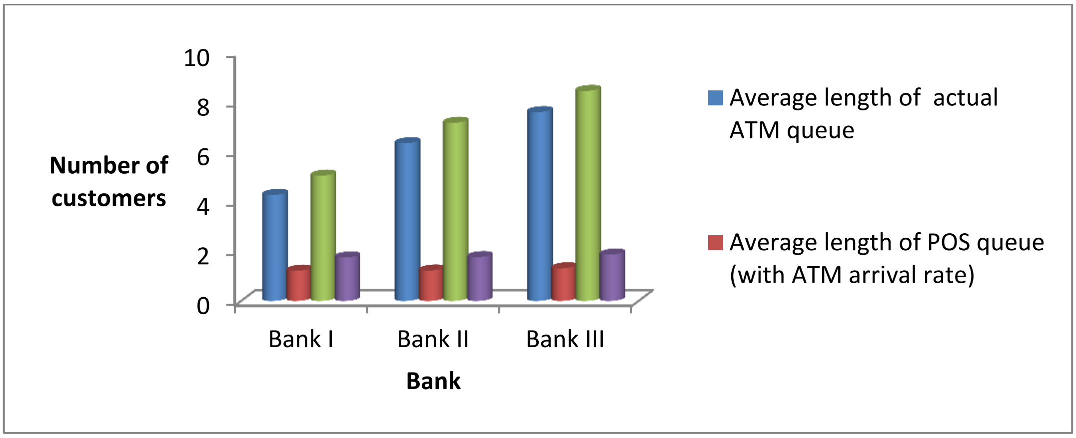

Figure 4. Average number of customers waiting before the POSs across Banks with the arrival rates of the ATMs

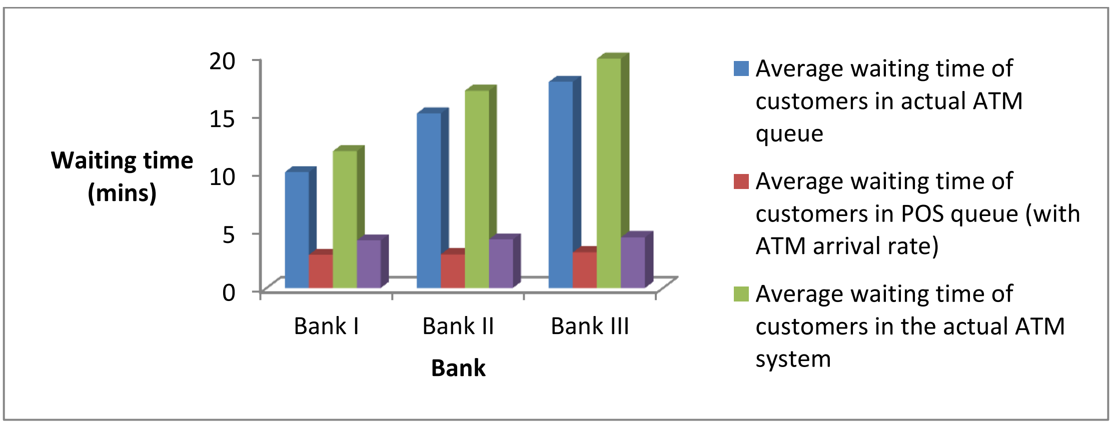

Figure 5. Average waiting time of customers before the POSs across Banks with the arrival rates of the ATMs

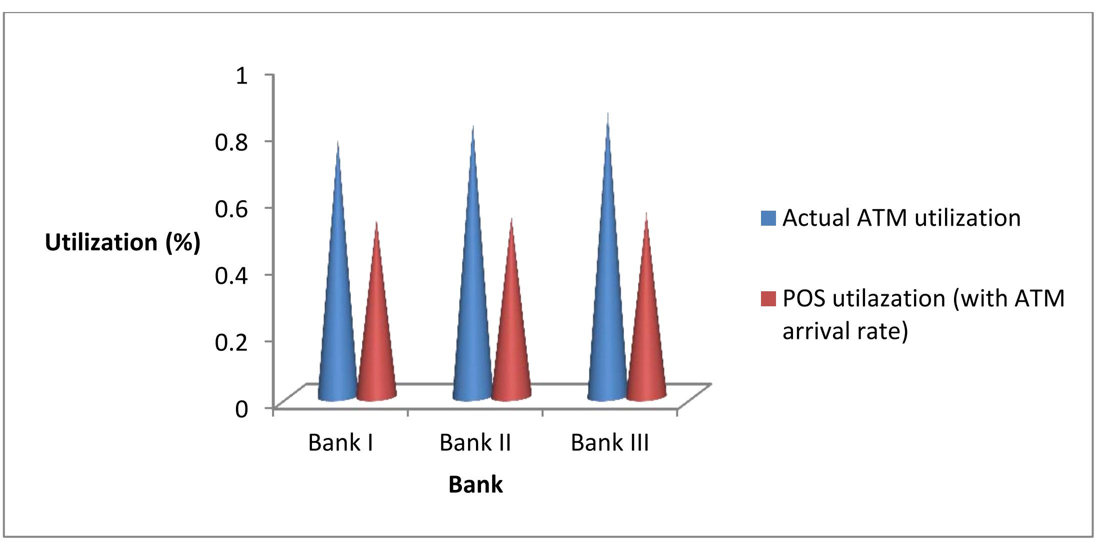

Figure 6. Utilization of the POSs across Banks with the arrival rates of the ATMs

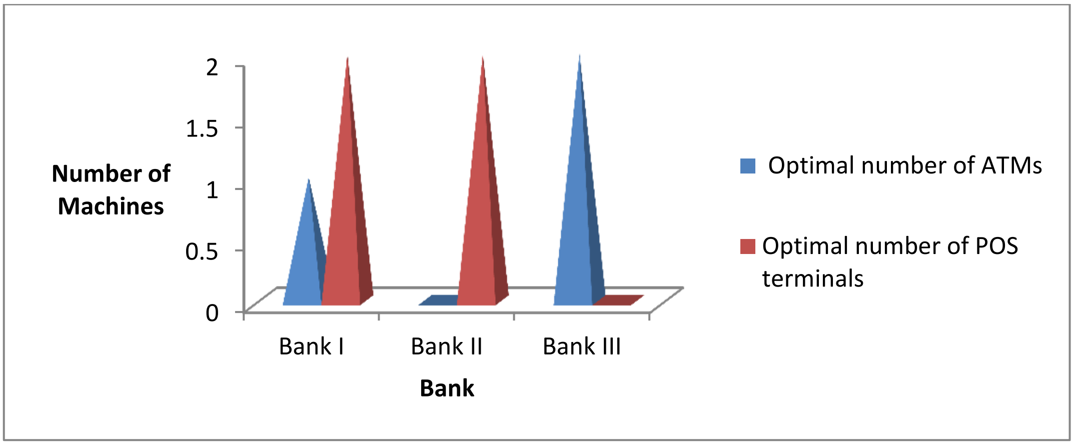

Figure 7. Optimal number of ATMs and POSs across Banks

5. DISCUSSION

This section presents discussion on the selection of the M/G/1 queuing model, performance evaluation of the ATMs and POS terminals across the Banks and the performance of the POS terminals across Banks when their queues are theoretically replaced with their respective ATM queues. It further discussed the optimal-Mix of the ATM and POS machines across Banks and finally, the study implication to the cash-less policy in Nigeria.

5.1 Selection of the M/G/1 model

The results of the goodness of fit of customer's inter arrival times to the exponential distribution (table 1), informed the selection of the M/G/1 model since the number of arrivals definitely follows the Poisson distribution and the distribution of service times is not required. All that is required of the service times is the mean and the variance. In addition, since the ATM and POS queues are each single channels with the First-In-First-Serve discipline, the M/G/1 queuing model is considered appropriate for this work.

5.2 Performance evaluation of the ATMs and POS terminals across Banks

The performance measures of the queuing system (table 2 and figures 1-3) were used in evaluating the current performance of the ATMs and POS terminals across the Banks. The table revealed that, the arrival rates of customers before the ATMs are higher than those of the POS terminals across the Banks, while customers spent less time in service before the POSs compared to the time they spent before the ATMs. These according to the table, translates into the POSs having lesser values of average length of queues and the average number of customers waiting in the system. It also translate into lesser values of average number of customers waiting in the queue and in the system compared to those of the ATMs across the Banks.

The table further revealed that the ATMs with respect to this study are considered over utilized (server utilization > 70%) while the POSs are considered underutilized (server utilization < 70%). This shows that the POSs are faster despite the fact that they handle more business transactions (as explained in section 1.0) than the ATMs and yet underutilized. One might argue that the fastness of the POSs and their underutilization might be due to the fact that they have lesser arrival rates, compared to the arrival rates of customers before the ATMs. An attempt to clarify this argument led to the replacement of each POS queue by that of the ATM in each Bank.

5.3 Performance evaluation of the POS terminals across Banks when their queues are theoretically replaced with their respective ATM queues.

In order to prove that the POSs outperform the ATMs across Banks despite the more number of transactions they handle and irrespective of the arrival rates of customers, the POS queues were replaced by those of their respective ATMs. Though it is practically impossible to achieve this, but it was theoretically achieved for the purpose of this evaluation by simply replacing the arrival rates of the POSs by those of their respective ATMs across Banks (table 3).

The table revealed that, the POSs still outperforms the ATMs as showed by the lesser values of average length of queues and the average number of customers waiting in the system compared to those of the ATMs across the Banks. This is also evident in the lesser values of average number of customers waiting in the queue and in the system compared to those of the ATMs across the Banks. See figures 1, 2, 4 and 5 for details. Another important measure of the performance of the POSs in respect of this replacement is the POSs utilization across Banks. Table 3 and figures 3 and 6 reveal that the POSs are underutilized (server utilization < 70%) despite this replacement. This shows that the POSs can accommodate more transactions or higher arrival rates of customers.

5.4 The Optimal Mix of ATMs and POS terminals across Banks

The bid to answer an earlier question posed in this work; that why multiplying the number ATMs when the POSs are faster and can handle more transactions? will no doubt lead to commercial Banks installing more number of POS terminals. This will definitely not be without the challenge of determining what number of POS is optimal? This work has successfully formulated easy to use models for determining the optimal number (Optimal-Mix) of ATMs and POS terminals across Banks. See equations 7 and 8 in section 2.3.1. The result of applying these equations in determining the Optimal-Mix of the ATMs and POSs across Banks using their current number of ATMs is captured in table 4 and in figure 7.According to this table, with the current number of ATMs (4, 3 and 2) in Bank I, Bank II and Bank III respectively, the optimal ATM-POS Mix is (1 ATM, 2POS), (0 ATM,2 POS) and (2 ATM, 0 POS) respectively.

5.5 Study implications to the success of the cashless policy in Nigeria.

[2] affirmed that the ATM machine is indisputably an indispensible piece of machinery in the successful implementation of the cash-less policy in Nigeria. Their work left out the tremendous role of the POS terminals as it affects this policy. This study has been able to show that the POS outperform the ATM in curbing the problems of long queue lengths, and long waiting times of customers in the system. These problems are indeed not desirable if the cash-less policy is to be successful. In actualizing this policy, this study stresses that there is no need multiplying the number of ATMs in Banks in order to solve these problems when the POSs handle more number of transactions at faster rates. To this end, the study formulated easy to use models for converting the current number of ATMs across Banks into Optimal-Mix of ATMs and POSs. This, the authors believe will enhance the actualization of the cash-less policy in Nigeria.

6. CONCLUSION AND RECOMMENDATION

6.1 Conclusion

The following conclusions were drawn from the study.

(i) There is no need multiplying the number of ATMs across Banks when the POSs can handle more transactions at faster rates.

(ii) The successful implementation and actualization of the cash-less policy in Nigeria, depends on the optimal ATM and POS Mix across Banks.

6.2 Recommendation

The following recommendations were made in this study.

(i) In the bid to actualizing cash-less economy in Nigeria, the Central Bank of Nigeria should make frantic effort in directing commercial Banks to apply

the methodology of this study, in converting their current ATMs into Optimal Mix of ATMs and POSs.

(ii) To further validate the result of this study, the extension of the study to other major cities in the country should be considered for further research.

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