A Note on Some Measures of Income Distribution

F. M. Ahmed[1]

[1] *Department of Statistics, Mathematics and Insurance, Faculty of Commerce, University of Benha, Egypt*

Abstract: Methods to measure income indexes have for quite some time now been an important subject in statistics and econometric research. Various measures were proposed and studied. These measures are based on incomplete moments or incomplete conditional moments and they take into consideration the shape of the income distribution but suffer sometimes from low efficiency and or lack or robustness. In this note, some measures of income are reviewed such as Lorenz curve, Gini' index, entropy index and Schutz's index.

Keywords: Lorenz curve, Gini's coefficient, Schutz's coefficient.

1. INTRODUCTION

Equality of income distribution is found when every income unit receives its proportional share of the total income. The income units may be further defined as individual, head of family, and a consumption unit; see for example, Cowell (1980), Lambert and Lanza (2003). Inequality may be defined as any deviation from equality. Thus, if any person received less than his proportionate share of the aggregate income, the distribution would be unequal, see for example Schutz (1951) and Zheng and Formby (2000). we review the Axioms of income inequality measures. Also, we review some of income inequality measures and the advantages and disadvantages of these measures.

2. INCOME MEASURES

To begin with, we want to review the desirable properties of inequality measure to know which measure satisfies these properties and know the best measure. Furthermore, we review several famous inequality measures such as Lorenz curve, Gini coefficient, Entropy indexes, and Schutz coefficient.

A. Axioms for inequality measures

If $x_1 \leq x_2 \leq \cdots \leq x_n$ is an ordered income distribution among n individuals denoted by a nonnegative vector

$$X = (x_1, x_2, \ldots x_n)$$

The inequality measure $\theta(X)$ is defined as a unique function of $x_1, x_2, \ldots x_n$ satisfying certain desirable properties. These axioms will be presented below,
however, all axioms may not be satisfied in a single measure.

Axiom 1. Income scale independence.

If $Y = \alpha X (\alpha > 0), then\ \theta(X) = \theta(Y)$. This axiom requires the inequality measure to be invariant to income changes by the same proportion. In other words, this axiom implies that the inequality measure should remain unaffected if each income is altered by the same proportion, therefore, the inequality measure should be independent of the scale of measurement. Most standard measures pass this test except the variance since var $(\lambda Y) = \lambda^2$ var (Y) where λ is scalar and Y is a vector of incomes; see Revallion (2004).

Axiom 2. Principle of population.

The population principle requires inequality measures to be invariant to replications of the population. That means if a proportionate number of persons are added at all income levels.

Axiom 3. Transfer Principle.

This axiom requires the inequality measure to rise (or at least not fall), when an income is transferred from a poorer to a richer person. On the other hand, when an income is transferred from a richer person to a poorer person should register a fall (or at least not increase); see Castagnoli and Muliere (1990) and Chateauneuf and Moyes (2006).

Axiom 4. Symmetry.

$\theta(X) = \theta\big(\pi(X)\big)$ where π is any permutation of X. This axiom implies if two individuals interchange their

E-mail address: noufel.ahmed@gmail.com

income positions, inequality remains unchanged. Moreover, the inequality depends only on the frequency distribution of incomes and not on the order in which individuals are ranked within the distribution. In addition, This axiom requires that the inequality measure be independent of any characteristic of individuals other than their income; Gaertner and Namezie (2003).

Axiom 5. Decomposability.

This axiom requires that the inequality is seen to rise (decrease) amongst each sub-group of the population when we would expect inequality overall to increase (decrease). In some measures when we sum the within and between-group inequality, they don't sum to the total inequality.

Axiom 6. The inequality measure lies in the range of zero to one.

The inequality measure takes zero, when all individuals have equal income. Also, inequality measure takes the value unity, when one individual gets all the income.

3. LORENZ CURVE

The Lorenz curve is a tool used to represent income distributions: it tells us which proportion of total income is in the hands of a given percentage of population. The Lorenz curve can be defined as the relationship between the cumulative proportion of income units and the cumulative proportion of income received when units are arranged in ascending order of their income; see for example   Cowell   (1995),   Sen(1997),   and Shahateet(2006). Also, The Lorenz curve has been used as a graphical device to represent size distribution of income and wealth. In addition, The Lorenz Curve is not limited to a specific type of population or variable that is distributed among that population; see Aaberge (2000) and Bellú and Liberati (2005).

The Lorenz curve can be represented by a function L(F) where F is the horizontal axis and L is the vertical axis. The points in the Lorenz curve are indexed in non-decreasing order that mean Lorenz curve is linear function; see Daniel, J. (2009) and Frosini (2005).

The Lorenz curve can measure the inequality in any society by the distance between the equality line and Lorenz curve. If the Lorenz curve is farther away from the line of perfect equality, then that case is considered to have less equality than a case with a curve nearer to the equality line; see Gastwirth (1972) and Zenga (2007).

Let a vector of income X (a positive random variable) from a continuous distribution with cumulative distribution function (cdf) F(x) =F, density function $f(x)$, $x(F)=F^{-1}(x)$ quantile function and let $X_{i:n}$ denote the corresponding order statistics for a general distribution   function F(x).

$$F(x) = \int_0^x f(x)\, dx$$

Where F(x) can be interpreted as the proportion of units having an income less than or equal to x; F(x) varies from 0 to 1. Furthermore, if the mean μ of the distribution exists , the first-moment distribution function of X is defined as

$$F_1(x) = \frac{1}{\mu} \int_0^x x f(x)\, dx$$

Also, $F_1(x)$ varies form 0 to 1 and $F_1(x)$ can be interpreted as the proportional share of total income of the units having income less than or equal of x.

If $f(x)$ is continuous , the derivate of $F_1(x)$ exists and is given by

$$\frac{dF_1(x)}{d\,x} = \frac{x\, f(x)}{\mu}$$

Which implies that $F_1(x)$ is a monotonically nondecreasing function of x.

The Lorenz curve $\big(L(p)\big)$ is the relationship between the variables F(x) and $F_1(x)$ and is obtained by inverting functions   F(x) and $F_1(x)$, and eliminating x if the functions are invertible. Alternatively, the curve can be plotted by generating the values of F(x) and $F_1(x)$ by considering the arbitrary values of x.

In addition, the slope of the Lorenz curve is obtained as

$$\frac{dF_1}{dF} = \frac{x}{\mu}$$

This is always positive for positive income. Similarly, the second derivative of the curve is

$$\frac{d^2 F_1}{dF^2} = \frac{1}{\mu f(x)} > 0,$$

These two derivatives imply that the slope of the Lorenz curve is positive and increases monotonically. Form this it follows that $F_1 \leq F$. The straight line $F_1 = F$ is called the egalitarian line (equal line).
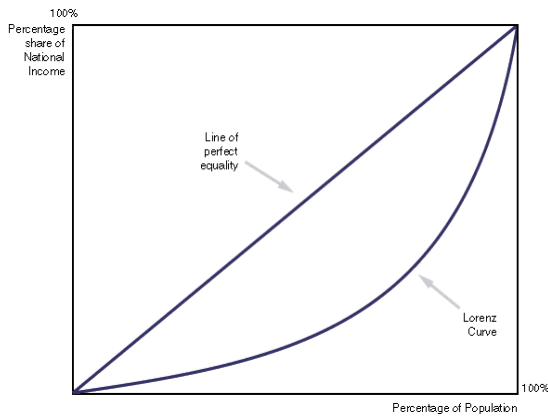
Figure 1. Lorenz curve

If the curve coincides with the equal line, it is implied that each unit receives the same income; this is the case of perfect equality of incomes. In the case of perfect inequality of incomes, it implies that all the income is received by only one unit in the population Kakwani and podder (1973) represented the relation as

$$p = F(x) \qquad and \qquad L(p) = F_1(x)$$

And $0 \leq p \leq 1$. Also, $L(p)$ is interpreted as the fraction of total income received by the lowest p^{th} fraction of the families. In addition, it satisfies the following conditions

a) If $p = 0$, $\quad L(p) = 0$;
b) If $p = 1$, $\quad L(p) = 1$;
c) $L'(p) = \frac{x}{\mu} \geq 0$ and $L''(p) = \frac{x}{\mu f(x)} > 0$;
d) $L(p) < p$.

Lorenz curve for several well-known income distribution functions

Pareto distribution

Before taking about pareto distribution, we have to introduce the pareto law. The pareto law of income distribution is given by the formula

$$R(x) = \left(\frac{x}{k}\right)^{-a} \qquad \text{when } x > k$$

$$=1 \qquad \text{when } x < k$$

Where $R(x) = 1 - F(x)$ is the proportion of income units having income x or greater, and a is found to be approximately 1.5; see kakwani (1980).

The density function of the pareto distribution is obtained by differentiating $R(x)$ with respect to x

$$f(x) = a\, k^a\, x^{-(a+1)} \quad \text{when } x > k$$

$$=0 \qquad\qquad \text{when } x < k$$

Where k is the scale factor, and a the pareto parameter. The curve $R(x)$ can be transformed to the logarithmic form

$$\log R(x) = a \log k - a \log x,$$

And, therefore, the graph of this curve on the double logarithmic scale will be a straight line with the slope $-a$.

According to $R(x)$ if income x increases by 1 percent, the proportion of units having income greater than or equal to x declines by a percent. The parameter a can be interpreted as the elasticity of decrease in the number of units when passing to a higher income class.

The mean and variance of the Pareto distribution are given by

$$E(x) = a\, k^a \int_k^\infty x^{-a}\, dx = \frac{k\, a}{(a-1)}$$

$$V(x) = \frac{k^2\, a}{(a-2)(a-1)^2}$$

Therefore the mean of the pareto distribution is proportional to the initial income k and the variance exists only if a > 2. Pareto observed that the value of a is approximately 1.5, which means that the variance of the estimated Pareto distribution will not be finite.

So that we can derive Lorenz curve for pareto distribution as follows. If the mean of the pareto distribution exists and is given by μ

Where

$$\mu = E(x) = \int_0^\infty x\, ak^a\, x^{-(a+1)}$$

$$\mu = \frac{ak}{(a-1)}$$

the first-moment distribution function(Lorenz curve) can be expressed as

$$F_1(x) = L(x) = \frac{1}{\mu} \int_k^x x\, f(x)\, dx = \frac{1}{\mu} \int_k^x x\, a\, k^a\, x^{-(a+1)}\, dx$$

$$F_1(x) = \frac{a\, k}{\mu(a-1)}\left[1 - \left(\frac{k}{x}\right)^{a-1}\right]$$

Substituting μ into $F_1(x)$ gives the equation of the Lorenz curve as

$$L(x) = 1 - (1-p)^{\frac{(a-1)}{a}}$$

And when $\frac{(a-1)}{a} = 1$, the Lorenz curve coincides with the equality line.

Lognormal distribution

The probability distribution function is denoted by

$$F(x) = \int_0^x \frac{1}{X\, \sigma\sqrt{2\pi}}\, exp^{\left(\frac{-1}{2\sigma^2}(\log X - \mu)^2\right)}\, dX$$
$$= \Lambda(x|\mu, \sigma^2)$$

The first-moment distribution function can then be written as

$$F_1(x) = \frac{\int_0^x Xd\ \Lambda(x|\mu, \sigma^2)}{\int_0^1 Xd\ \Lambda(x|\mu, \sigma^2)}$$

$$= \frac{\int_0^x \frac{1}{X\sqrt{2\pi}\sigma}\, exp^{\left(\frac{-1}{2\sigma^2}(\log X - \mu)^2\right)}dX}{e^{\mu + \frac{\sigma^2}{2}}}$$

$$F_1(x)\ = \Lambda(x|\mu + \sigma^2, \sigma^2)$$

Note that $e^{\mu + \frac{\sigma^2}{2}}$ is the mean of the lognormal distribution with parameters $\mu\ and\ \sigma^2$. Define the relation

$$x = \emptyset(t)$$

So that

$$t = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{1}{2X^2}}\ dX$$

Then, if $p = \Lambda(x|\mu, \sigma^2)$, it follows that

$$\frac{\log x - \mu}{\sigma} = \emptyset(p)$$

Similarly, if $L(p) = F_1(x)$ equation $F_1(x)$ becomes

$$\frac{\log x - \mu - \sigma^2}{\sigma} = \emptyset(L(p))$$

Eliminating   log x from $\emptyset(p)\ and\ \emptyset(L(p))$ gives the equation of the Lorenz curve as

$$\emptyset(L(p)) = \emptyset(p) - \sigma$$

Which depends only on the parameter σ.

The advantages of the Lorenz curve; see Bellù (2005)

- Lorenz curve suitable for any population and variable.
- Lorenz curve is easy to analyses.
- Lorenz curve always starts at (0.0) and ends at (1,1).
- Lorenz curves are scale invariant. They are based on how the variable values are distributed.
- Lorenz curve can be used to look at a distribution of single year or the changes in the Lorenz curve over time.

The disadvantage of the Lorenz curve:

- The Lorenz curve can't rise above the line of perfect equality and can't sink below the line of perfect equality.
- The Lorenz curve is not defined if the mean of probability distribution is zero or infinite.
-

4.   GINI INDEX

Gini coefficient is the most used single measure of inequality. It depends on Lorenz curve where it is a ratio of the areas on a Lorenz curve. The Gini coefficient is defined as A/(A+B), where A is the area between the line of perfect equality and Lorenz curve and the area under Lorenz curve is B. since A+B = $\frac{1}{2}$, the Gini coefficient, G= A/$\frac{1}{2}$ = 2A= 1-2B; see Deutsch and Silber(1997) and Gastwirth (1972).
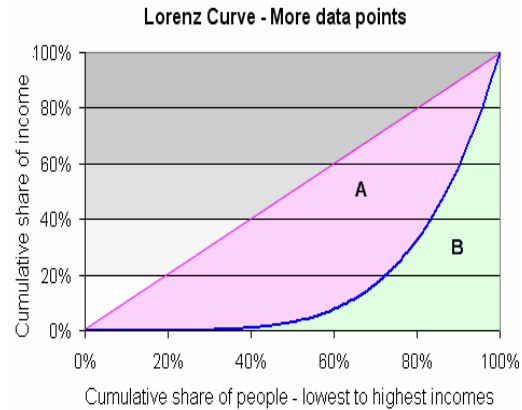

Figure 2 Lorenz curve and Gini index

Gini proposed two forms to measure income inequality. In 1912, he presented the first one which is defined as

$$G = \frac{\Delta}{2\mu}$$

Where

$$\Delta = \frac{1}{n(n-1)} \sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|,$$

x_i being the income of the ith individual, and n the total number of individuals.

In the continuous case, Δ is

$$\Delta = \int_0^\infty \int_0^\infty |x - y| f(x)f(y)dx\, dy.$$

Note that Δ is the arithmetic average of the $n\,(n-1)$ differences taken as absolute value. The 2μ is the maximum value of Δ, which is obtained when one individual got the all income, so the Gini index will be one. On the other hand, the minimum value of Δ is zero when all individuals have the same amount of income, so the Gini index will be zero.

In 1914, Gini presented the second form of the Gini measure depend on the Lorenz curve.

If the Lorenz curve is presented by the function y=L(x), the value of B can be found with integration and

$$G = 1 - 2 \int L(x)\,dx$$

If A = 0 the Gin coefficient becomes zero which means perfect equality, whereas if B= 0 the Gini coefficient becomes one which means complete inequality.

Note that both of these forms are equal

$$G = \frac{1}{2\mu} \int_0^\infty \int_0^\infty |x - y| f(x) f(y) dx\,dy.$$

$$G = \frac{1}{2\mu} \int_0^\infty \left[\int_0^x (x - y) f(y)\,dy \right. \\ + \int_x^\infty (y \\ \left. - x) f(y)\,dy \right] f(x)\,dx$$

$$G = \frac{1}{\mu} \int_0^\infty [xF(x) - xF_1(x)]\, f(x)\,dx$$

Note that $F(x)$ is the probability distribution function, and that $F_1(x)$ is the first-moment distribution function. Integrating the first term by parts

$$G = \frac{1}{\mu} \int_0^\infty xF(x)\,f(x)\,dx = 1 - \int_0^\infty F_1(x)\,f(x)\,dx$$

When , on substituting into G, yields

$$G = 1 - 2 \int_0^\infty F_1(x)\,f(x)\,dx \quad (2.8)$$

Which equal to one minus twice the area under the Lorenz curve.

Kakwani (1980) showed two lemmas

Lemma 1. If the distribution is Lorenz superior(inferior) to the distribution X , $G(Y)$ is less (greater) than $G(X)$ wher $G(X)$ stands for Gini index of the distribution X.

Lemma 2. The Gini index attaches more weight to transfers of income near the mode of the distribution than at the tails.

The advantages of Gini coefficient; see Zagier (1983) and Zoli (2002).

- If all incomes were doubled, the measure wouldn't change.
- If the population were to change, the measure of inequality should not change.
- Symmetry. The measure concentrate on the measured variable and if you and I swap incomes, there should be no change in the measure of inequality.
- Gini coefficient satisfies the principle of transfer sensitivity. Under this criterion, the transfer of income from rich to poor redues measured inequality.
- Gini coefficient is often used as a metric of inequality.
- Gini coefficient can be used to indicate how a distribution changes over time and if this change shows that equality is increasing or decreasing.

The disadvantages of Gini coefficient:

- Gini is not affected by the shape of Lorenz curve.
- Gini doesn't indicate how the inequality is distributed, only the total amount of inequality.
- Gini index isn't easily decomposable across groups. Gini is only decomposable if the partions are non-over lopping. That is, the total Gini of society is not equal to the sum of the Gini coefficient of its subgroups.
- When use Gini, we can't able to test for the significance of changes is the index over time.

5. GENERALIZED ENTROPY MEASURES

Entropy class could be the most complex inequality measures. However, the use of complex inequality measures will not give any information about the characteristics of the distribution like location and shape.

Entropy has the meaning of deviations from perfect equality; see Theil (1967) and Yitzhaki and Lerman1991.

The definition of a generalized inequality measure is the following:

$$G\,E\,(\propto) = \frac{1}{n(\propto^2 - \propto)} \sum_i \left[\left(\frac{y_i}{\bar{y}} \right)^{\alpha} - 1 \right]$$

Where $\bar{y}$ is the mean income (or expenditure per capita) and the parameter $\propto$ in the G E represents the weight given to distances between incomes at different parts of the income distribution in addition, $\propto$, can take

any real value. For lower values of $\propto$, GE is more sensitive to changes in lower tail of the distribution, and for higher values GE is more sensitive to changes that affect the upper tail. $\propto$ may range from minus infinity to infinity. However, $\propto$ is usually chosen to be non-negative, as for $\propto < 0$ this case of indexes is undefined if there are zero incomes, but the commonest values of $\propto$ used are 0,1 and 2. GE (1) is theil's T index

$$\text{GE (1)} = \frac{1}{n} \sum \left(\frac{y_i}{\bar{y}}\right) \ln \left(\frac{y_i}{\bar{y}}\right)$$

and GE (0) is theil's L of the mean log deviation measure

$$\text{GR (0)} = \frac{1}{n} \sum_{i=1}^{n} \ln \left(\frac{y_i}{\bar{y}}\right)$$

The values of GE measures vary between 0 and ∞ , with zero representing and equal distribution and higher value representing a higher level of inequality.

The upper value of GE $(\propto) = \frac{n^\propto - n}{n \, (\propto^2 - \propto)}$ And $\quad$ GE $(1) = \ln n$ But $\quad$ GE (0) is not bound

It is worth defining a class of relative entropy indexes RGE, defined as the ratio between the value of the original entropy index and the maximum value for each member of that class.

$$\text{RGE} (\propto) = \frac{GE\ (1)}{\max\ GE\ (1)} = \frac{\frac{1}{n} \sum \left(\frac{y_i}{\bar{y}}\right) \ln\left(\frac{y_i}{\bar{y}}\right)}{\ln(n)}$$

and

$$\text{RGE (1)} = \frac{GE\ (\propto)}{\max\ GE\ (\propto)} = \frac{\frac{1}{n(\propto^2 - \propto)} \sum \left(\left(\frac{y_i}{\bar{y}}\right)^\propto - 1\right)}{\frac{n^\propto - n}{n\ (\propto^2 - \propto)}}$$

But RGE (0) = 0 because the upper limit is not bound.
The advantage of entropy class:

- All members of GE class are scale invariant. If all incomes are multiplied by same factor, the ratio between each income and mean income remains the same.
- All members of GE class satisfy the principle of transfer. If income is redistributed from richer individuals to poorer individuals, the measure decreases. The opposite holds true if income is redistributed from poorer individuals to richer individuals.
- GE satisfy symmetry principles, these, measures are independent from any characteristic of individual other than their income.

The disadvantages of entropy class

- All members of generalized entropy are not translation invariant. By adding (subtracting) the same amount of money to all incomes, GE would decrease (increase).

- All members of entropy class don't define if there are zero incomes. In this case entrap class can only be calculated by replacing zero incomes with arbitrary (very small incomes).

6. SCHUTZ COEFFICIENT

Let a vector of income X (a positive random variable) from a continuous distribution with cumulative distribution function (cdf) F(x)=F, density function $f(x)$, $x(F)= F^{-1}(x)$ quantile function and let $X_{i:n}$ denote the corresponding order statistics for a general distribution function F(x).

The S-coefficient in terms of Lorenz curve is the maximum vertical distance between the Lorenz curve or the cumulative portion of the total income held below a certain income percentile, and the perfect equality line, that is the 45° degree line of equal incomes. So S-coefficient can be derived as

$$S = DP = Max\big[F(x) - L\big(F(x)\big)\big]$$
$$= F(x) - L\big(F(\mu)\big)$$
$$= \int_0^\mu x\, f(x) \left(1 - \frac{1}{\mu}\right) dx = \frac{MAD}{2\mu}$$

The mean absolute deviation (MAD) about population mean μ is defined as

$$MAD = E|X - \mu| = \int |x - \mu|\, f(x) dx$$

This equation expresses the S-coefficient as a ratio between the mean absolute deviation and the twice of the population mean; see for example, Schutz (1951) and Elamir (2012)
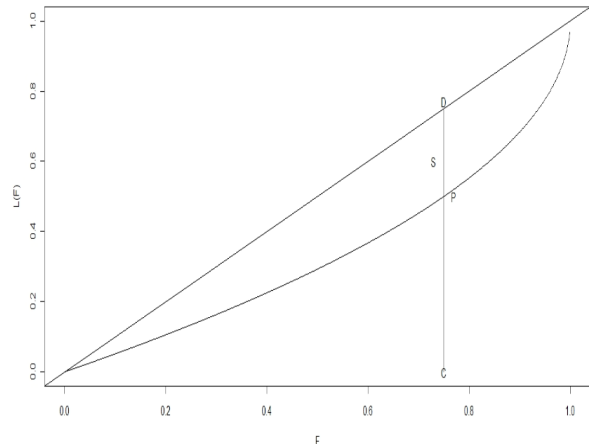

Figure 3. S-coefficient and Lorenz curve

S-coefficient in terms of "overs" and "unders"

Elamir (2012) presented population S-coefficient as the gap between the individual's income x_i and the population mean income μ. The MAD can be rewritten as:

$$MAD = E|x - \mu| = 2 \int_{\mu}^{\infty} (x - \mu)f(x)\, dx$$

$$= -2 \int_{-\infty}^{\mu} (\mu - x)f(x)\, dx$$

Therefore, the S-coefficient is

$$S = \frac{E(D_i)}{\mu}$$

Where

$$D_i = \begin{cases} (X_i - \mu), & X_i > \mu \\ 0 & X_i \le \mu \end{cases}$$

Note that the values of D_i represent a person's income to be more than the population mean μ ("*overs*") and $E(D_i)$ represents the expected amount of money which have to be transferred from households above the mean to those below the mean to achieve equality. Moreover, in the same manner we may write S-coefficient as:

$$S = \frac{E(\bar{D}_i)}{\mu}$$

Where

$$\bar{D}_i = \begin{cases} (X_i - \mu), & X_i > \mu \\ 0 & X_i \le \mu \end{cases}$$

The values of $\bar{D}_i$ represent a person's need of money to achieve equality ("*unders*") and $E(\bar{D}_i)$ represent the expected need of money to achieve equality.

Example

If $X_1, X_2, \ldots \ldots, X_n$ from pareto distribution with density function

$$pdf = ak^a x^{-(a+1)}, \quad a \le x < \infty,$$

with shape and scale parameters; a and k, cumulative distribution function

$$F = 1 - \left(\frac{k}{x}\right)^a,$$

quantile function

X(F) = k $(1 - F)^{\frac{-1}{a}}$, and

mean = $\frac{ak}{(a-1)}$. The MAD can be derived as:

$$MAD = 2 \int_{\mu}^{\infty} (x - \mu)ak^a x^{-(a+1)}\, dx$$

Hence,

$$MAD = \frac{2k}{a-1} \left(\frac{a}{a-1}\right)^{-(a-1)}$$

Therefore

$$S = \frac{MAD}{2\mu} = \frac{1}{a}\left(\frac{a}{a-1}\right)^{-(a-1)} = \frac{1}{a-1}\left(\frac{a-1}{a}\right)^{a}$$

This depends on the scale parameter a.

For a sample or population of size, n, an income distribution is $x_1, x_2, \ldots, x_n$ of nonnegative values and their order statistics are $x_{1:n}, x_{2:n}, \ldots, x_{n:n}$. We define the following nonparametric estimators for S-coefficient using data.

1. Based on the mean absolute deviation

$$\hat{S} = \frac{mad}{2\bar{x}} = \frac{\sum_{i=1}^{n}|x_i - \bar{x}|}{2n\bar{x}}$$

Where $\bar{x}$ is the mean an *n* the number of observations.

2. Based on the ranking of the income in ascending order

$$\hat{S} = \frac{\sum_{i=v}^{n}(x_{i:n} - \bar{x})}{n\bar{x}} = \frac{-\sum_{i=1}^{n}(x_{i:n} - \bar{x})}{n\bar{x}}$$

Where $x_{i:n}$ the order income, and $v = [\sum_{i=1}^{n} I(x_i < \bar{x})]$ is the number of values less than mean and I indicator function , 1 if true and 0 false.

3. Based on the expected money transferred from rich to poor

$$\hat{S} = \frac{\sum_{i=1}^{n} d_{i:n}}{n\bar{x}} = \frac{\bar{d}}{\bar{x}}$$

Where

$$d_i = \begin{cases} x_i - \bar{x}, & x_i > \bar{x} \\ 0 & x_i \le \bar{x} \end{cases}$$

The advantage of S-coefficient

*   Income scale invariant(mean independence). This means that the S-coefficient remains unchanged, when all incomes were multiplied by a factor, because the numerator and the denominator increases with the same factor.
*   Symmetry. When two persons swap their income, S-coefficient will not change as the value of d_i will not change and we sum for all individuals.
*   Pigou-Dalton transfer sensitivity. When income is redistributed from richer to poorer, S-coefficient decreases as numerator $d_i = 0$ (decreases). The opposite holds true for redistributions from poorer to richer individuals. It is worth noting that S-

coefficient react to redistribution only for transfers across the mean. In other words, if the change in the same side, S-coefficient will not change.

- Population size independence: requires inequality measures to be invariant to replications of the population; merging two identical distributions should not alter inequality.

- The Range S-coefficient is between zero and unity. When all incomes are equal, the numerator of S is equal to zero, as any difference between any income and the mean is zero. When all incomes are zero but the last one is not, S-coefficient has a maximum value at 1, because

$$m = \frac{(x_n - \bar{x})/n}{x_n/n} = 1 - \frac{\bar{x}}{x_n} \approx 1 \quad \text{where } \bar{x} < x_n$$

The disadvantage of S-coefficient

- S-coefficient is not translation invariant: when all incomes of the original income distribution are added (subtracted) the same amount, the numerator of S-coefficient remains unchanged, while the denominator increases (decreases) by an amount which is equal to original addition (subtraction) times the number of observations. Therefore S-coefficient decreases (increases).

- Decomposability. It was difficult to break down the inequality by population groups or income sources or in other dimension. However, Elamir (2012, 2013 and 2015) decomposed S-coefficient by sub-groups.

REFERENCES

Aaberge, R. (2000). Characterizations of Lorenz Curves and Income distributions . Soc Choice Welfare.

Bellu, L.G.and Liberati, P. (2005). Charting Income Inequality the Lorenz Curve. EASYPOL

Bellu,L.G.(2005). The Lorenz curve. EASYPOL

Castagnoli, E. and Muliere, P.(1990). A note on inequality measures and the Pigou-Dalton principle of transfers. In: Dagum C. and Zenga, M(eds) Income and wealth distribution, Inequality and poverty. Springer, Berlin, pp171-182.

Chateauneuf, A. and Moyes, P. (2006). Measuring Inequality Without the Pigou-Dalton Condition. In: McGillivray M (ed) Inequality, Poverty and Well-Being-Palgrave MaCmillan, Basingtoke, Hampshire\ New-York, PP. 22 – 65.

Cowell, F.A.(1980). On the structure of additive inequality measures. Rev Econ Stud 47, pp 521-531

Cowell,F.A.(1995). Measuring Inequality. (2nd edition). Harvester Wheatsheaf, Hamel Hempstead.

Daniel, J. (2009). How to Generate a Lorenz Curve and Gini Coefficient. Sealtle Washington.

Deutsch, J. and Silber, J.(1997). Gini's " transvariazione" and the measurement of distance between distributions. Empirical Economics 22, 547-554.

Elamir, E.A.H.(2012). On the decomposition of the Schutz coefficient: and exact approach with and application. Electronic Journal of Applied Statistical Analysis. Vol.5,pp187-198.

Elamir, E.A.H (2013). On estimation of some abbreviated social welfare measures. Quality & Quantity 47:1561–1576

Elamir, E. A. H. (2015) "Analysis of mean absolute deviation for randomized block design under Laplace distribution" American Journal of Theoretical and Applied Statistics, 43, 138-150.

Frosini B, (2005). Inequality measures for histograms. Statistica, 65, pp 27-40.

Gaertner, W. and Namezie, C. (2003). Income Inequality, Risk, and Transfare Principle: a Questionnaire Experimental Investigation. Math Soc Sci. 45: 229 – 245.

Gastwirth, J.L.(1971). A general definition of the Lorenz curve . Econometrica,vol.39,pp.1037-1039

Gastwirth, J. (1972). The estimation of the Lorenz curve and Gini index. Rev Econ Stat 54,pp 306-316.

Kakwani, N.C.and Podder (1973). On the estimation of Lorenz curves from grouped observations. International Economic Review, vol.14, pp.278-291.

Kakwani, N.(1980). Income inequality and poverty. Oxford University Press, London

Lambert, P. J. and Lanza, G (2003). The Effect on Inequality of Changing One or Two Incomes. Economics Department Working Paper, No. 2003, 15, University of Oregon.

Revallion, M. (2004). Competing Concepts of Inequality in The Globalization Debate. Policy Research Working Paper, No. 3243, World Bank.

Schutz, R.R.(1951). on the measurement of income inequality. American Economic Review, vol.41, pp.107-122.

Sen, A. (1997). On Economic Inequality. Clarendon, Oxford, UK.

Shahateet, M. I(2006). How serious regional economic inequality in Jordan? Evidence from two national household surveys. American Journal of Applied sciences, 3, 1735-1744.

Theil, H. (1967).Economics and information theory. North Holland, Amsterdam.

Yitzhaki, S. and Lerman, R.(1991). Income stratification and income inequality. Rev Income Wealth 37, pp 313-329.

Zagier, D. (1983). Inequality for the Gini coefficient of composite populations. J Math Econ 12, pp 103-118

Zenga, M.(2007). Inequality curve and inequality index based on the ratios between lower and upper arithmetic means. Stat Appl 6,pp 137-151.

Zheng, B. and Formby, J. (2000). Inequality Orderings, Stochastic Dominance and Statistical Inference. Journal Bus Econ Stat 18: 490 – 488.

Zoli, C. (2002). Inverse Stochastic Dominance, Inequality Measurement and Gini Indices. In: Moyes P. Seidl C, Shorrocks A. (eds) Inequalities: Theory Experiments and Applications. Journal Econ Supply: 119 – 161.