



# A New Variable Length LMS Algorithm for Partial Update Adaptive Filtering Driven by Cyclostationary Signal

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**Abstract:** The problem with some methods of Partial Update (PU) is that when using a special kind of input (Cyclostationary Signal), the system fails to converge or becomes unstable. These problems have been solved in this paper by using a newly proposed algorithm with two concepts of dynamic length Least Mean Square (LMS). The first is the length variation of the total filter coefficients  $N$ , while the second variation applies to the number of coefficients to be updated at each iteration  $M$ . This algorithm is called New Variable Length LMS algorithm NVLLMS. In the NVLLMS algorithm, the value of the Mean Error Square (MSE) is used to control  $N$  and  $M$ . Moreover, the step size is time varying. The proposed algorithm shows better performance compared with PU LMS algorithms through simulation results of system identification.

**Keywords:** Partial Update (PU), Least Mean Square (LMS), New Variable Length LMS (NVLLMS), Cyclostationary Signal (CS).

## 1. INTRODUCTION

Cyclostationary Signals (CS), are random process signals with statistical parameters, that fluctuate periodically with time [1]. Some examples of these signals are produced by samplers, scanners, multiplexors, and modulators [1].

If an adaptive filter which uses a Least Mean Square (LMS) algorithm and is driven by correlated input signals (for example cyclostationary signal), then this algorithm suffers from some drawbacks like slow convergence or instability. One method presented overcomes these drawbacks is the selection of the adaptive filter length [2], or one can choose to update only part of the filter coefficient vector (Partial Update) at each time instant. Such algorithms can reduce computational complexity and on the other hand perform close to the full-update methods in term of convergence rate and Mean Error Square (MSE). There are several variants of the LMS algorithms with partial update methods [3-14]. In addition, partial-update adaptive filters may suffer from stability or convergence problems when the input signal is cyclostationary or periodic [15].

In this paper, a new algorithm is proposed which is called, New Variable Length LMS (NVLLMS) algorithm. Several variable lengths LMS is reported [16-22]. The NVLLMS algorithm represents a mix of PU

LMS and variable length filter (VL) to solve the stability and convergence problems, and to reduce the number of total coefficients required. This algorithm is applied to system identification for different cases, and compared with other PU LMS algorithms. The proposed algorithm shows better performance through the simulation and solves the problems of the PU LMS algorithms.

## 2. PARTIAL UPDATE ALGORITHMS

The Partial-Update (PU) adaptive filtering method is a straightforward approach to control the computational complexity because it only updates part of the coefficient vector instead of updating the full coefficient filter vector. This method plays an important role to improve the cost, performance, portability and size, since the computational complexity of an adaptive filter are an important consideration in many practical applications, especially the applications that require the adaptive filter to have a very large number of coefficients like the acoustic echo cancellation and channel equalization [15].

To explain the use of partial updates, the least-mean square (LMS) algorithm is presented [15]:

$$\begin{bmatrix} w_1(k+1) \\ \vdots \\ w_N(k+1) \end{bmatrix} = \begin{bmatrix} w_1(k) \\ \vdots \\ w_N(k) \end{bmatrix} + \mu e(k) \begin{bmatrix} x(k) \\ \vdots \\ x(k-N+1) \end{bmatrix}, \quad (1)$$

$k = 0, 1, 2, 3, \dots$

or

$$\begin{aligned} \mathbf{W}(k+1) \\ = \mathbf{w}(k) + \mu e(k)\mathbf{X}(k) \end{aligned}$$

Where  $\mathbf{w}(k)$  is tapping coefficient vector,  $\mu$  is the step size parameter,  $e(k)$  is the error signal, and  $\mathbf{X}(k)$  is an input signal vector.

Fig.1 shows the concept of the PU method, which indicates that only  $M$  out of  $N$  coefficients are updated at iteration  $k$ .

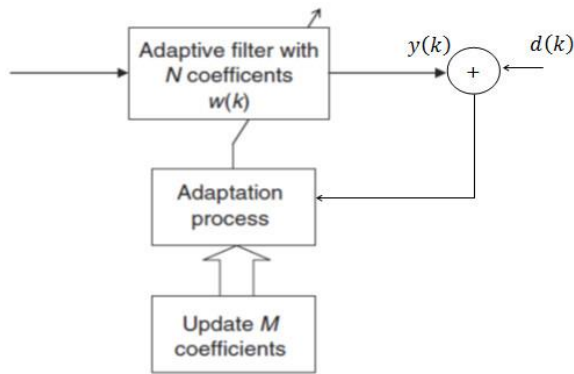


Figure1. The concept of PU method

This can be accommodated by modifying the adaptation algorithm in the equation (1) to

$$w(k+1) = w(k) + \mu e(k)I_M(k)x(k), \quad (2)$$

$k = 0, 1, 2, \dots$

Where  $I_M(k)$  is a diagonal matrix with  $M$  ones and  $N - M$  zeros on its diagonal indicating which  $M$  coefficients are to be updated at iteration  $M$  [15]:

$$I_M = \begin{bmatrix} i_1(k) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & i_N(k) \end{bmatrix}, \quad \sum_{j=1}^N i_j(k) = M, \quad (3)$$

$k = 0, 1, 2, 3, \dots$

If  $i_j(k) = 1, j = 1, \dots, N$ , then the coefficient  $w_j(k)$  gets an update at iteration  $k$ , otherwise it remains unchanged. In this paper, we consider some of the basic

partial update methods as follows:

#### A. Periodic Partial Update

In this method the update complexity is distributed over a number of iterations, in order to reduce the computational complexity [15].

$$\begin{aligned} w(k+1) &= w(k) + f(k), \quad k = 0, 1, 2, \dots \\ y(k) &= h(w(k), x(k)) \end{aligned} \quad (4)$$

Where  $y(k)$  is the output of adaptive filter, and

$$f(k) = [f_1(k), f_2(k), \dots, f_N(k)]^T \quad (5)$$

is the  $N \times 1$  update vector,

Using periodic partial updates for the adaptive filter coefficients in equation (4) results in [15]:

$$\begin{aligned} w((k+1)S) &= w(kS) + f(kS), \\ & \quad k = 0, 1, 2, \dots \\ w((kS+i)) &= w(kS), \\ & \quad i = 0, 1, 2, \dots, S-1 \\ y(k) &= h(w(\lfloor k/S \rfloor S), x(k)) \end{aligned} \quad (6)$$

Where  $S$  is the period of coefficient updates

The adaptive filter updates its coefficients every  $S^{\text{th}}$  iteration at  $k = 0, S, 2S, 3S, \dots$ ,

$$\begin{aligned} w((k+1)S) &= w(kS) + 2\mu e(kS)x(kS), \\ & \quad k = 0, 1, 2, 3, \dots \\ w((kS+i)) &= w(kS), \quad i = 0, 1, 2, \dots, S-1 \\ S &= \left\lceil \frac{N}{M} \right\rceil \end{aligned} \quad (7)$$

#### B. Sequential Partial Update

In order to reduce the computational complexity of updating a subset of the adaptive filter coefficients at each iteration, the sequential partial update method is used [3]. The sequential partial update method adapts a partition of the coefficient vector within the complexity constraints at each iteration [15]. The sequential PU method may choose the first element at iteration  $k$ , choose the second element at iteration  $k+1$ , and the third element at iteration  $k+2$ . Updating a subset of the weights can reduce computational complexity. The sequential-partial-update LMS algorithm is given by:

$$w(k+1) = w(kS) + \mu e I_M(k) x(k), \quad k = 0, 1, 2, \dots \quad (9)$$

$$I_M(k) = \begin{bmatrix} i_1(k) & 0 & \dots & 0 \\ 0 & i_2(k) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & \ddots & i_N(k) \end{bmatrix},$$

$$i_j(k) = \begin{cases} 1 & \text{if } j \in \zeta_{(k \bmod B)+1} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Where  $B = N/M$  is an integer equal to the number of subsets (each  $B$  includes  $M$  coefficient). The adaptation process is roughly proportional to  $B$  so the sequential partial update method designs the subset as [15]:

$$\begin{aligned} \zeta_1 &= \{1, 2, 3, \dots, M\}, \\ \zeta_2 &= \{M + 1, M + 2, \dots, 2M\}, \\ &\vdots \\ \zeta_B &= \{(B - 1)M + 1, (B - 1)M + 2, \dots, N\} \end{aligned} \quad (11)$$

C. Stochastic Partial Updates

The stochastic method can be implemented as a randomized version of the sequential method. The stochastic partial update has the same update equation as the sequential partial update. The method uses the following coefficient selection matrix [15]:

$$I_M(k) = \begin{bmatrix} i_1(k) & 0 & \dots & 0 \\ 0 & i_2(k) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & \ddots & i_N(k) \end{bmatrix},$$

$$i_j(k) = \begin{cases} 1 & \text{if } j \in \zeta_m(k) \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

where  $m(k)$  is an independent random process .

D. Updates M-Max

In the M-Max Update the weight vector is updated according to the position of the first M-max elements of the input  $x$ . This method aims to find the subset of the update vector that can make the biggest contribution to the convergence [15].The coefficient selection matrix  $I_M(k)$  is defined by:

$$I_M(k) = \begin{bmatrix} i_1(k) & 0 & \dots & 0 \\ 0 & i_2(k) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & \ddots & i_N(k) \end{bmatrix},$$

$$i_j(k) = \begin{cases} 1 & \text{if } |x(k - j + 1)| \in \max_{1 \leq l \leq N} (|x(k - l + 1)|, M) \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

3. VARIABLE LENGTH (VL) ADAPTIVE FILTER

The VLLMS algorithm is dynamically updated the tap-length of the LMS adaptive filter to optimize performance and to reduce the power in the adaptive filter core. This algorithm is also used as a PU when the number of tapes needed is greater than the existing. A variable length LMS adaptive filter will be introduced with the configuration shown in Fig. 2 [24].

Fig.2 shows three parallel filters. The coefficient vector of filter  $l$  is  $h_l(k)$  with length  $N_l (l = 1, 2, 3)$  . The three estimation errors  $e_l(k)$  are input into a length control unit which computes estimates of the MSE for each filter, and based on these estimates the updated filter lengths and step size are generated. The optimal filter length produced by filter number three.

The algorithm of operation can be summarized as follows [24]:

1) Initialization:

$$N_1(0) = N_0, N_2(0) = N_0 + 1, N_3(0) = N_0 + 2 \quad (14)$$

$$h_1(0) = 0_{N_1}, h_2(0) = 0_{N_2}, h_3(0) = 0_{N_3} \quad (15)$$

$$\mu_1(0) = \frac{\mu N_0}{N_1(0)}, \mu_2(0) = \frac{\mu N_0}{N_2(0)}, \mu_3(0) = \frac{\mu N_0}{N_3(0)}, \quad (16)$$

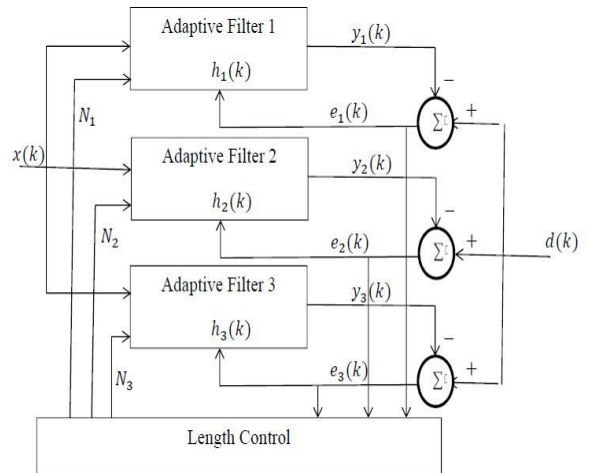


Figure 2. VLLMS filter configuration

Where  $N_0$  is the initial filter length,  $N_1, N_2$  and  $N_3$  are the initial filter length of filter 1, 2 and 3 respectively.  $\mu$  is the step size parameter of filter 1

$h_1(0), h_2(0)$  and  $h_3(0)$  are the initial filter coefficient of filter 1, 2 and 3 respectively.

$\mu_1(0), \mu_2(0)$  and  $\mu_3(0)$  are the initial step size of filter 1, 2 and 3 respectively



2) For each  $n = 1, 2, \dots$   
Compute the errors:

$$e_l(n) = d(n) - \sum_{i=0}^{M_l-1} h_{li}(n)x(n-i-l),$$

$$l = 1, 2, 3 \quad (17)$$

Where  $h_{li}(k)$  is the  $i^{\text{th}}$  coefficient of the adaptive filter  $l$ .

Update the coefficients:

$$h_l(n+1) = h_l(n) - \mu_{li}(n)x(n)e_l(n),$$

$$l = 1, 2, 3 \quad (18)$$

At each  $L^{\text{th}}$  iteration:

Compute the following average  $m_l$

$$m_l = \frac{1}{L} \sum_{j=n+L+1}^n e_l^2(j), l = 1, 2, 3 \quad (19)$$

Update the lengths:

$$N_1(n+1) = \begin{cases} N_1(n) + 1, & \text{if } m_1 > m_2 > m_3 \\ N_1(n), & \text{if } m_1 > m_2 \text{ or } m_{22} \leq m_{33} \\ N_1(n) - 1, & \text{otherwise} \end{cases} \quad (20)$$

$$N_2(n+1) = N_1(n+1) + 1, \quad N_3(n+1) = N_1(n+1) + 2 \quad (21)$$

Update step size

$$\mu_l(n+1) = \frac{\mu N_o}{N_l(n+1)}, l = 1, 2, \quad (22)$$

Where  $m_i$  ( $i = 1, 2, 3$ ) acts as the statistic square error in this algorithm. It is shown that for an uncorrelated input signal  $x(n)$  which satisfies:

$E[x(k)x(k-n)] = 0$  for  $n \neq 0$ , the following is valid:

$$\frac{J_{\infty}^{(l)}}{J_{\infty}^{(m)}} = \frac{J_{min}^{(l)}}{J_{min}^{(m)}} \begin{cases} \geq 1, & \text{if } N_1 \leq N_2 < N_{opt} \\ < 1, & \text{if } N_2 < N_1 < N_{opt} \\ 1, & \text{if } N_1 > N_{opt} \text{ or } N_2 > N_{opt} \end{cases}$$

$$l = 1, 2, 3 \quad m = 1, 2, 3 \quad l \neq m \quad (23)$$

Where  $J_{\infty}^{(l)} = E[e^2(\infty)^{(l)}]$ ,  $l = 1, 2, 3$  is the steady state MSE,  $J_{min}^{(l)}$ ,  $l = 1, 2, 3$  is the minimum steady state MSE and  $N_{opt}$  is the optimal filter length [24].

#### 4. CYCLOSTATIONARY PROCESS AND PU PROBLEMS

Cyclostationary processes are random data whose statistical characteristics vary periodically with time [25]. The cyclostationary input signal is defined as a signal of order  $n$  (in the wide sense) if and only if it is possible to find some  $n^{\text{th}}$ -order nonlinear transformation of the signal that will generate finite-strength additive sine-wave components, which result in spectral lines.

$$x(k) = a_i u(k),$$

$$i = \begin{cases} 1 & \text{if } k \bmod 8 = 1 \\ 2 & \text{if } k \bmod 8 = 2 \\ 3 & \text{if } k \bmod 8 = 3 \\ 4 & \text{if } k \bmod 8 = 4 \\ 5 & \text{if } k \bmod 8 = 5 \\ 6 & \text{if } k \bmod 8 = 6 \\ 7 & \text{if } k \bmod 8 = 7 \\ 8 & \text{if } k \bmod 8 = 0 \end{cases} \quad (24)$$

Where  $u(k)$  is an uncorrelated Binary-Phase-Shift-Keying (BPSK) signal taking on values  $\pm 1$  with equal probability, and the  $a_i$  are constant weights that multiply  $u(k)$  periodically, creating a cyclostationary signal with cyclic period 8.

In PU, there are some problems with the adaptive filter convergence and stability when cyclostationary inputs are applied as follows:

- 1) Periodic PU method: it has the problem of convergence difficulties.
- 2) Sequential PU method: it has the problem of convergence difficulties.
- 3) M-Max PU method: it has the problem of instability.

#### 5. A NEW PROPOSED ALGORITHM (NEW VARIABLE LENGTH LMS (NVLLMS))

The new proposed algorithm combines the benefits of the VLLMS and the PU-LMS algorithms in order to solve the convergence and stability problems of periodic, stochastic and M-Max PU. This algorithm is called NVLLMS. The concept of changing the number of coefficients to be updated at each iteration will be used.

##### 1) Design Aspects

The following design aspects are considered as follows:

- 1) Variable length of partial update coefficients (variable M).
- 2) Variable adaptive filter length (variable N).
- 3) The step size will be variable.
- 4) Modifying the length is depending on the value



of MSE at each iteration and the value of MSE for a group of iteration. The group of iteration can be defined by blocks, such that each block has the same size. The MSE is calculated for each block by the equation (Sum(e)/block size).

2) Proposed Algorithm

The proposed new algorithm introduces an adaptive filter with variable length N and also has the ability to vary the number of coefficients to be updated at each iteration M when using one of the PU methods. This can be introduced by the following configuration as shown in Fig. 3.

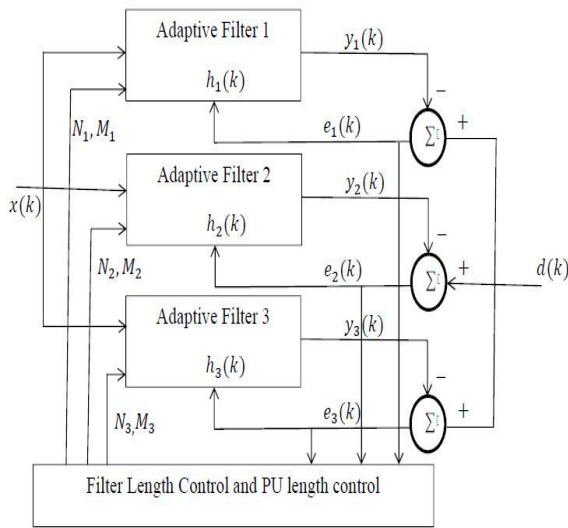


Figure 3. The NVLLMS filter configuration with variable N and M.

The algorithm of operation sequences can be summarized as follows:

1) Initialization:

$$N_1(0) = N_0, N_2(0) = N_0 + 1, N_3(0) = N_0 + 2 \tag{25}$$

$$h_1(0) = 0_{N_1}, h_2(0) = 0_{N_2}, h_3(0) = 0_{N_3} \tag{26}$$

$$\mu_1(0) = \frac{\mu_{N_1}(0)}{N_0}, \mu_2(0) = \frac{\mu_{N_2}(0)}{N_0}, \mu_3(0) = \frac{\mu_{N_3}(0)}{N_0} \tag{27}$$

The step size is increased instead of decreasing, in order to increase the speed of convergence [17]. Keeping that at all iterations the value of step size must be in the range of stability condition.

2) For each  $k = 1, 2, \dots$   
 Compute the errors:  $e_l(k)$

$$e_l(k) = d(k) - \sum_{i=0}^{N_l-1} h_{li}(k)x(k-i-l), l = 1,2,3 \tag{28}$$

Update the coefficients:

$$h_l(k+1) = h_l(k) - \mu_{li}(k)x(k)e_l(k), l = 1,2,3 \tag{29}$$

At each  $L^{th}$  iteration:

Compute the average of  $e_l(k)$  which denoted by  $m_l$ , where  $m_l$  is average error.

$$m_l = \frac{1}{L} \sum_{j=k+L+1}^n e_l^2(j), l = 1,2, \tag{30}$$

At each block of iteration with size D:

Compute the average of  $m_l$  for each block which denoted by  $m_b$ , where  $m_b$  is MSE.

$$m_b = \frac{1}{D} \sum_{j=k+L+1}^n e_l^2(j), l = 1,2,3 \tag{31}$$

Where D is the size's number of iterations that the block contained.

The number sample at each block can be determined by using the mod operation. If the size of the block is D then  $(k \text{ mod } D)$  must be zero to compute the average of each block. The value of M is updated when the system fails to converge or becomes unstable, and that happens when the value of average error for a block is greater than the previous block. The blocks are used to recognize the system output if it is converging and stable or not, and that is done by comparing the first item in the block with the last item from the same block, and also comparing both items with a reference value (this value is set according to the estimated output).

The system fails to converge

$$\text{if } m_b(i) \approx m_b(i+bs) \text{ and } m_b(i) \neq \text{output} \tag{32}$$

The system is unstable

$$\text{if } m_b(i) < m_b(i+bs) \tag{33}$$

Where the bs is the number of iterations per each block.

$$M(k+1) = \begin{cases} M(k) + \text{update size,} & \text{if system fail to converge} \\ & \text{or unstable} \\ M_0 & \text{if } M(k) = N - \text{update size} \\ M(k), & \text{otherwise} \end{cases} \tag{34}$$

The updated size differs according to the application and it may be positive or negative, the case of negative update is taking place when the value of M initially starts with high value so it has to be decreased to cover all values of M. The maximum value can be reached by M

must be less than  $N$  to keep the PU concept. If the variation of  $M$  has no effect on the output, then the system will reset the value of  $M$  to the initial value  $M_0$ .

Filter length variation is done according to the following procedure:

$$N_2(k+1) = N_1(k+1) + 1, \quad N_3(k+1) = N_1(k+1) + 2 \quad (36)$$

Update step size

$$\mu_l(k+1) = \frac{N_l(k+1)\mu_0}{N_0}, \quad l = 1, 2, \quad (37)$$

The equation of the step size update is changed to increase the step size in order to increase the convergence speed, but the boundary of stability is the same for both cases. The block size is another factor that can be set to increase the accuracy of the result. Decreasing the block size will increase the accuracy and vice versa.

## 6. SIMULATION RESULTS

The basic block diagram of system identification application is shown in Fig.4.

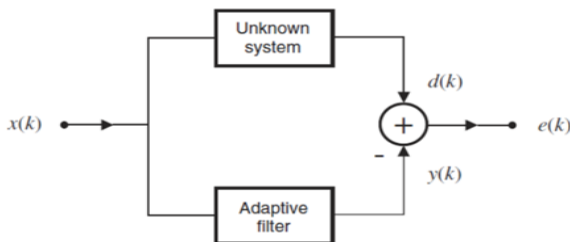


Figure 4. System Identification set up

The input signal  $x(k)$  is applied to the unknown system and to the adaptive filter [2, 17]. While the desired signal  $d(k)$  is the output of the unknown system.

Assume the unknown system has an impulse response given by [2]:

$h = [0.1 \ 0.3 \ 0.0 \ -0.2 \ -0.4 \ -0.7 \ -0.4 \ -0.2]^T$ . The input signal is white Gaussian with variance  $\sigma_n^2$  equals  $10^{-4}$ . The adaptive filter has 8 coefficients. The number of iterations  $k$  equals 10000 and  $\mu$  equals  $5 \times 10^{-4}$ . The error signal is then given by equation (38)

$$e(k) = d(k) - y(k) \quad (38)$$

$$y(k) = w^T(k)x(k) \quad (39)$$

At each iteration, the adaptive filter updates its coefficients in order to minimize an appropriate norm of

the error signal  $e(k)$ . When the error norm is minimized in a statistical sense, the corresponding  $w(k)$  gives an estimate of the unknown system parameters. The cyclostationary input given by equation (24) is used.

### A. Case(1): Traditional Full Update LMS and PU LMS with cyclostationary input :-

Fig. 5 shows the time-averaged learning curves for full update LMS and periodic, sequential, stochastic and M-Max PU LMS algorithms when cyclostationary input is applied

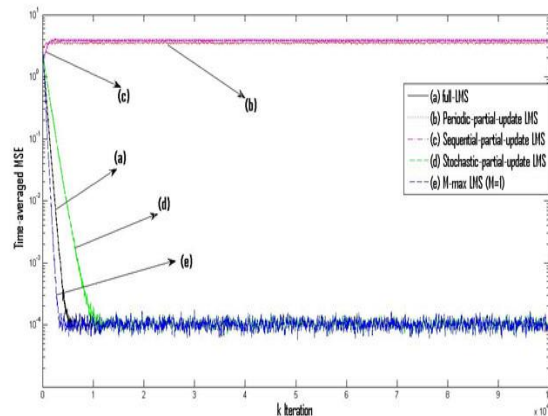


Figure 5. Time-averaged learning curves (MSE) for a full update, periodic, sequential, stochastic and M-Max PU LMS algorithms for a cyclostationary input.

It can be observed that the periodic and the sequential for  $S=2$  PU LMS fails to converge altogether, whereas the full-update and other PU LMS algorithms appear not to have been affected by the cyclostationary input signal. Moreover, also it can be seen that for  $M=1$ , the M-Max LMS is faster convergence than the full-update.

### B. Case(2): Traditional Full Update LMS and PU LMS with shift cyclostationary input :-

The shifted version of the cyclostationary input signal is considered and is given by:

$$x(k) = a_i u(k), \quad (40)$$

$$i = (k \bmod 8) + 1$$

The new signal has the same cyclic period as the previous one and it is obtained by advancing the signal shown in equation (24) by one sample [15]. Fig.6 shows the time-averaged learning curves for full update LMS and periodic  $S=2$ , sequential  $M=7$ , stochastic  $M=7$  and M-Max ( $M=1$ ) PU LMS algorithms.

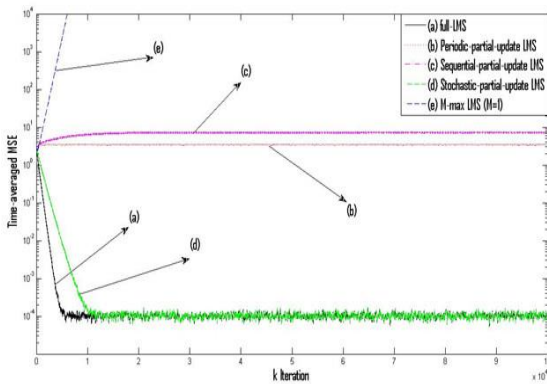


Figure 6. Time-averaged learning curves (MSE) for the full update, periodic, sequential, stochastic and M-Max M=1 PU LMS algorithms for a shifted cyclostationary input

Fig.6 shows that periodic and the sequential PU LMS fails to converge and the M-Max PU becomes unstable while the full-update and stochastic PU LMS algorithms appear not to have been affected by the new cyclostationary input signal and the stochastic is more stable compared to other methods of LMS PU.

C. Case(3): Applying a new proposed (NVLLMS) Algorithm

The NVLLMS algorithm will be used with the cyclostationary input, which is the same input that used before with periodic, sequential and M-max PU. NVLLMS will try to solve the problems of convergence and stability of PU algorithms.

1) The periodic PU.

Fig.7 shows the two learning curves of periodic PU, curve (a) when using periodic PU LMS and curve (b) when using the periodic PU NVLLMS algorithm. This figure shows that, when the NVLLMS is applied, the system successfully converged, but with a short delay, (about 500 iterations) This delay depends on the number of iterations at each block.

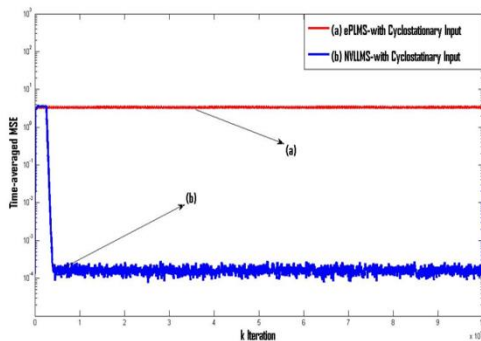


Figure 7. Time-averaged learning curves (MSE) for (a) a periodic PU LMS, (b) for a periodic PU NVLLMS algorithm for a cyclostationary input.

2) The sequential PU.

Fig. 8 shows the two learning curves of sequential PU, curve (a) when using the sequential PU LMS, and curve (b) for the sequential PU NVLLMS algorithm. It can be observed that when the NVLLMS is applied, the system also succeeded to converge with a small delay ( about 1500 iterations) similar to the previous case.

3) The M-max PU.

Fig. 9 shows the learning curve of M-max PU, curve (a) when using M-max PU LMS, and curve (b) for the M-max PU NVLLMS algorithm. As shown, the proposed algorithm produces a stable system ( after 2000 iterations) instead of the old situation which showing system instability.

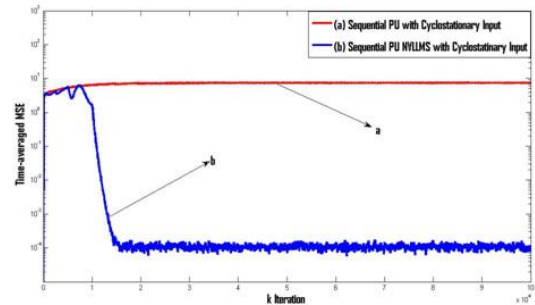


Figure 8 . Time-averaged learning curves (MSE) for (a) a sequential PU LMS, (b) a sequential PU NVLLMS algorithm for a cyclostationary input.

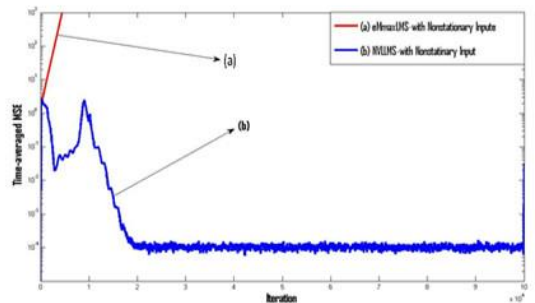


Figure 9. Time-averaged learning curves (MSE) for (a) an M-Max PU LMS, (b) an M-Max PU NVLLMS algorithm for a cyclostationary input

Using NVLLMS with periodic and sequential PU makes the system succeeded to converge, and in the case of M-Max it produces a stable system .

4) Case(4):System Identification with Nonstationary Impulse Response

The cyclostationary signal is a special class of nonstationary signals which are random in nature, but exhibit periodicity in their statistics [26]. The nonstationary impulse response is the case when the impulse response of the system is not constant, but it is at



some point will be changed, causing the system to start learning again to follow the change. In this section, the impulse response will be changed by multiplying the original one by -2 at iteration 3000. This can be used as a performance test for the NVLLMS algorithm. Fig.10 shows the learning curve for both full update LMS and NVLLMS algorithms. It is evident that the NVLLMS has faster convergence rate and better tracking capability compared with a full update LMS algorithm.

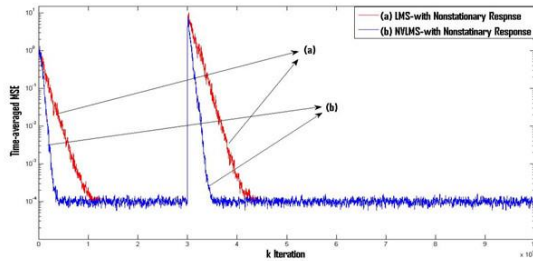


Figure 10. Time-averaged learning curves (MSE) with a nonstationary impulse response for full update (a) LMS, (b) NVLLMS algorithm

## 7. CONCLUSIONS

In this paper a new proposed algorithm called NVLLMS is proposed in order to overcome some of problems of periodic, stochastic PU (fail to converge) and M-Max PU (becomes unstable) algorithms that are driven by cyclostationary signal. The proposed algorithm used three design aspects which are variable length of partial update coefficients (variable M), variable adaptive filter length (variable N), and variable step size. Both N and M parameters are controlled by MSE. Simulation results using system identification set up shows that, using NVLLMS with periodic and sequential PU makes the system succeeded to converge after 500 and 1500 iterations respectively, and in the case of M-Max it produces a stable system after 2000 iterations. This delay (500-2000 iterations) depends on the number of iterations at each block (D).

Moreover, the proposed algorithm shows the fast convergence rate, and better tracking compared to a full band LMS algorithm for nonstationary environments.

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