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A Generalization of Generalized Gamma Distribution

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Abstract: In this paper, a generalization of generalized gamma distribution(GGGD) ,which includes the three-parameter generalized gamma distribution, two-parameter Weibull and gamma distributions, and exponential distribution as special cases, has been suggested and studied. The hazard rate function and the stochastic ordering of the distribution have been discussed. Maximum likelihood estimation has been discussed for estimation of parameters. Applications of the proposed distribution have been discussed with two real lifetime datasets and the goodness of fit shows quite satisfactory over generalized gamma, gamma, Weibull, and exponential distributions.

Keywords: Generalized gamma distribution, Weibull distribution, Hazard rate Function, Stochastic ordering, Goodness of fit

1. Introduction

A two parameter gamma distribution (G.D.) having parameter θ and α is defined by its probability density function (pdf) and cumulative distribution function (cdf)

$$f_1(y;\theta,\alpha)=\frac{\theta^{\alpha}}{\Gamma(\alpha)}e^{-\theta y}y^{\alpha-1};\quad y>0,\theta>0,\alpha>0$$

$$F_1(y;\theta,\alpha)=1-\frac{\Gamma(\alpha,\theta y)}{\Gamma(\alpha)}\;; y>0,\theta>0,\alpha>0$$

where θ is a scale parameter and α is a shape parameter and $\Gamma(\alpha,z)$ is the upper incomplete gamma function defined as

$$\Gamma(\alpha,z)=\int_z^{\infty}e^{-y}\,y^{\alpha-1}\,dy\,;\alpha>0,\; z\geq 0$$

A detailed study about properties, estimation of parameters and applications of a discrete analogue of two-parameter gamma distribution is available in [1]. The two-parameter gamma distribution reduces to exponential distribution at $\alpha=1$ having pdf and cdf

$$f_2(y;\theta)=\theta e^{-\theta y}; y>0,\theta>0$$

$$F_2(y;\theta)=1-e^{-\theta y}; y>0,\theta>0$$

A detailed study on exponential distribution and its role in life testing is available in [2]. Further, gamma distribution is the weighted exponential distribution. A discrete exponential distribution along with its properties and applications to meteorology has been discussed by [3]. [4] have obtained two-parameter generalized exponential distribution and discussed its statistical properties, estimation of parameter and applications. A discrete analogue of the generalized exponential distribution along with its properties, estimation of parameters and applications are available in [5]. The research works done by different researchers on exponential distributions are available in [6].

The Weibull distribution (WD), proposed by [7], having parameter θ and α is defined by its pdf and cdf

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$$f_3\left(x;\theta,\alpha\right)=\theta\,\alpha\,x^{\alpha-1}\,e^{-\theta x^{\alpha}}\quad ;x>0,\theta>0,\alpha>0$$

$$F_3\left(x;\theta,\alpha\right)=1-e^{-\theta x^{\alpha}}\quad ;x>0,\theta>0,\alpha>0$$

The Weibull distribution reduces to exponential distribution at $\alpha=1$. Further, the Weibull distribution is nothing but the power exponential distribution. Assuming $x=y^{1/\alpha}$ and $y=w\left(x\right)=x^{\alpha}$ in (1.4), we get

$$g\left(x;\theta,\alpha\right)=f\left[w\left(x\right)\right]w'\left(x\right)=\theta\,e^{-\theta x^{\alpha}}\,\alpha\,x^{\alpha-1}=\theta\,\alpha\,x^{\alpha-1}e^{-\theta x^{\alpha}}$$

which is the pdf of Weibull distribution defined in (1.6). [8] obtained discrete Weibull distribution. [9] introduced a new discrete Weibull distribution. Recently, [10] have comparative study on modeling of lifetime data using two-parameter gamma and Weibull distributions and it has been observed that in some datasets gamma gives better fit than Weibull whereas in some datasets Weibull gives better fit than gamma and hence these two distributions are competing each other. Most of the research works done on Weibull distributions are available in [11].

The generalized gamma distribution (GGD), suggested by [12], having parameters $\theta,\alpha,$ and β is defined by its pdf and the cdf

$$f_4\left(x;\theta,\alpha,\beta\right)=\frac{\beta\,\theta^{\alpha}}{\Gamma\left(\alpha\right)}x^{\beta\alpha-1}\,e^{-\theta x^{\beta}};\ x>0,\theta>0,\alpha>0,\beta>0$$

$$F_4\left(x;\theta,\alpha,\beta,\right)=1-\frac{\Gamma\left(\alpha,\theta\,x^{\beta}\right)}{\Gamma\left(\alpha\right)};\ \ x>0,\theta>0,\alpha>0,\beta>0$$

where α and β are the shape parameters and θ is the scale parameter, and $\Gamma\left(\alpha,z\right)$ is the upper incomplete gamma function defined in (1.3) . Note that Stacy (1962)[12]obtained GGD by taking $x=y^{1/\beta}$ and thus $y=w\left(x\right)=x^{\beta}$ in (1.1), and using the approach of obtaining the pdf of Weibull distribution in (1.6). Clearly the gamma distribution, the Weibull distribution and the exponential distribution are particular cases of (1.9) for $\left(\beta=1\right)$, $\left(\alpha=1\right)$ and $\left(\alpha=\beta=1\right)$, respectively. The parametric estimation for the GGD has been discussed by [13]. [14] has obtained a new discrete distribution related to generalized gamma distribution and discussed some of its properties. Note that the research works done on exponential distribution, Weibull distribution, gamma distribution, generalized gamma distributions, their extensions and applications for life testing and modeling are available in [15]. Recently, [16] have detailed critical and comparative study on modeling of lifetime data using three-parameter GGD and generalized Lindley distribution (GLD) introduced by [17], and observed that GGD gives much closer fit than GLD in real lifetime datasets.

Since GGD gives better fit than both Weibull and gamma distributions, it is expected and hoped that generalization of generalized gamma distribution (GGGD) will be a better model than GGD, Weibull and gamma. An attempt has been made to obtain GGGD which includes the three-parameter GGD, two-parameter Weibull and gamma distributions, and exponential distribution. Some of its properties including hazard rate function and stochastic ordering have been discussed. The estimation of its parameters has been discussed using maximum likelihood estimation. Two real lifetime data has been considered to test the goodness of fit of GGGD over GGD, Weibull, gamma and exponential distributions.

2. A GENERALIZATION OF GENERALIZED GAMMA DISTRIBUTION

Assuming $x=\frac{1}{\gamma}y^{1/\beta}$ and $y=w\left(x\right)=\left(\gamma\,x\right)^{\beta}$ in (1.1), and following the approach of obtaining the pdf of GGD by [12], the pdf of generalization of generalized gamma distribution (GGGD) can be expressed as

$$f_5(x;\theta,\alpha,\beta,\gamma)=\frac{\beta(\theta\gamma^\beta)^\alpha}{\Gamma(\alpha)}x^{\beta\alpha-1}e^{-\theta(\gamma x)^\beta};\ x>0,\theta>0,\alpha>0,\beta>0,\gamma>0 \qquad (2.1)$$

where (α,β,γ) are shape parameters and θ is the scale parameter. Further, GGD of [12], Weibull, gamma and exponential distributions are particular cases of GGGD for $(\gamma=1)$, $(\gamma=1,\alpha=1)$, $(\gamma=1,\beta=1)$ and $(\gamma=1,\alpha=1,\beta=1)$, respectively.

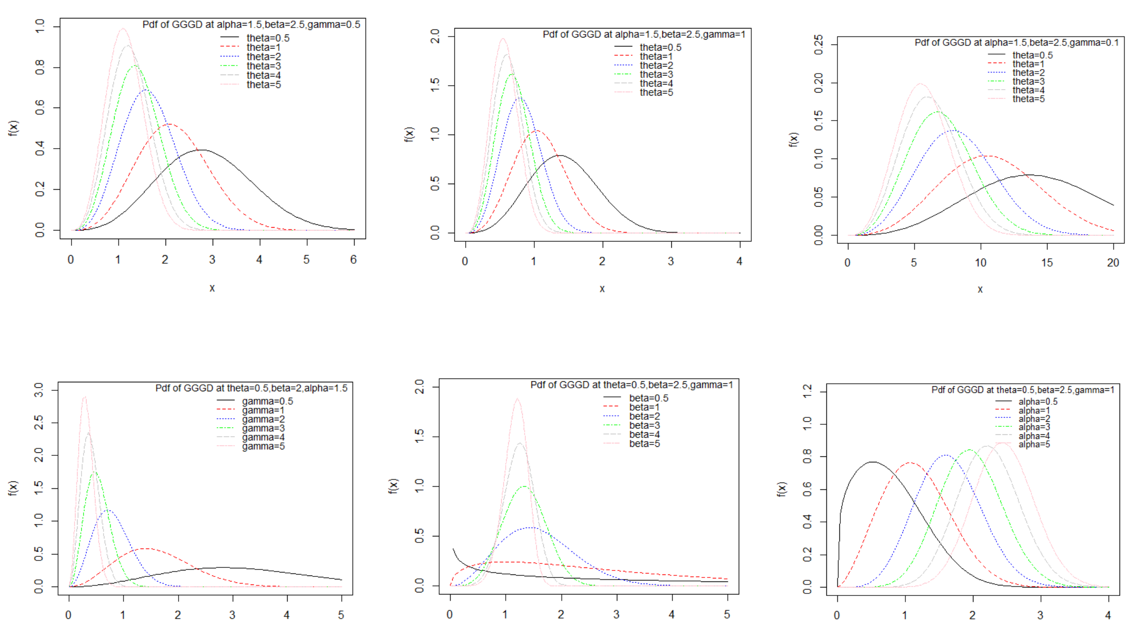
The cdf of GGGD can be expressed as

$$F_5(x;\theta,\alpha,\beta,\gamma)=1-\frac{\Gamma\left(\alpha,\theta(\gamma x)^\beta\right)}{\Gamma(\alpha)};\ x>0,\theta>0,\alpha>0,\beta>0,\gamma>0 \qquad (2.2)$$

where $\Gamma\left(\alpha,\theta(\gamma x)^\beta\right)$ is the upper incomplete gamma function defined as

$$\Gamma\left(\alpha,\theta(\gamma x)^\beta\right)=\int_{\theta(\gamma x)^\beta}^{\infty} e^{-y}\,y^{\alpha-1}\,dy \qquad (2.3)$$

Graphs of the pdf and the cdf of GGGD are shown in figures 1 and 2 for varying values of the parameters $\theta,\alpha,\beta,$ and γ.



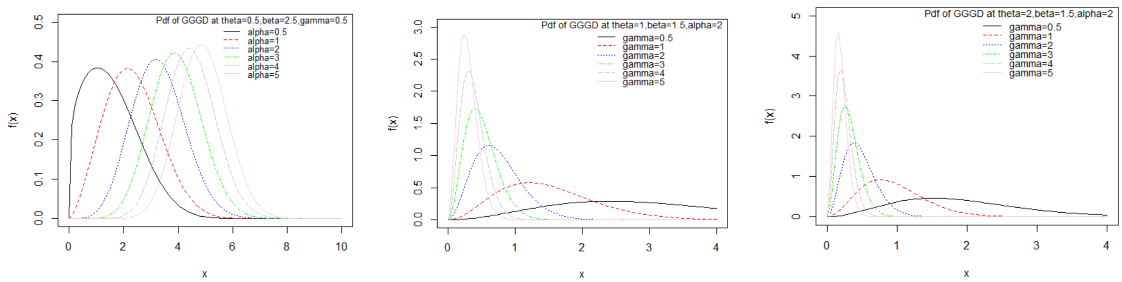


Figure 1. Behavior of the pdf of GGGD for parameters $\theta, \alpha, \beta,$ and γ .

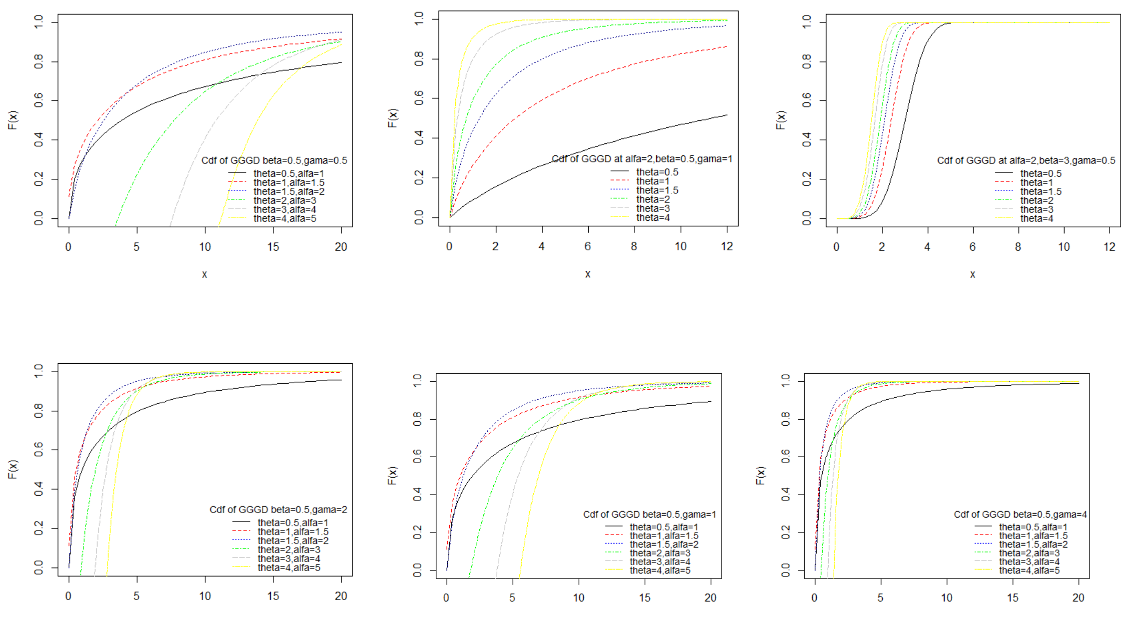


Figure 2. Behavior of the cdf of GGGD for parameters $\theta, \alpha, \beta,$ and γ .

3. HAZARD RATE FUNCTION

Let X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$. The hazard rate function (also known as the failure rate function) function of X is defined as

$$h(x) = \lim_{\Delta x \to 0} \frac{P\left(X < x + \Delta x \,\middle|\, X > x\right)}{\Delta x} = \frac{f(x)}{1 - F(x)}$$

Thus $h(x)$ of GGGD can be expressed as

$$h(x;\theta,\alpha,\beta,\gamma) = \frac{f_5(x;\theta,\alpha,\beta,\gamma)}{1 - F_5(x;\theta,\alpha,\beta,\gamma)} = \frac{\beta\left(\theta\gamma^{\beta}\right)^{\alpha} e^{-\theta(\gamma x)^{\beta}} x^{\beta\alpha-1}}{\Gamma\left(\alpha, \theta(\gamma x)^{\beta}\right)} , \tag{3.1}$$

where $\Gamma\left(\alpha,\theta\left(\gamma x\right)^{\beta}\right)$ is the upper incomplete gamma function defined in (2.3). Graphs of $h\left(x\right)$ of GGGD for parameters $\theta,\alpha,\beta,$ and γ are shown in figure 3.

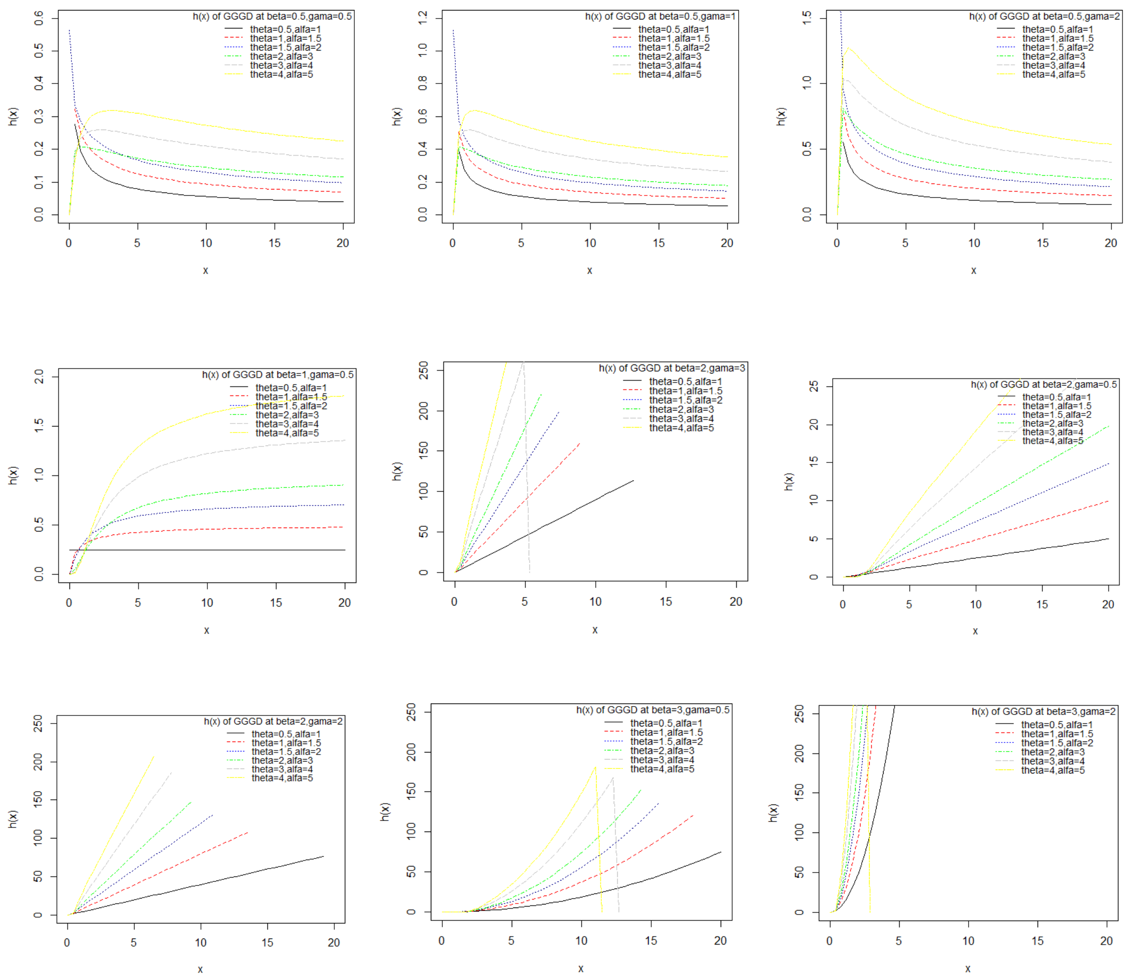


Figure3. Behavior of the $h\left(x\right)$ of GGGD for parameters θ,α,β and γ.

4. STOCHASTIC ORDERING

Stochastic ordering is an important tool for judging the comparative behavior of positive continuous random variables. Suppose X and Y are positive continuous random variables. Then Y is said to be greater than X in the

(a) stochastic order $\left(X\le_{st}Y\right)$ if $F_X\left(x\right)\ge F_Y\left(x\right)$ for every x

(b) hazard rate order $\left(X\le_{hr}Y\right)$ if $h_X\left(x\right)\ge h_Y\left(x\right)$ for every x

(c) mean residual life order $\left(X\le_{mrl}Y\right)$ if $m_X\left(x\right)\le m_Y\left(x\right)$ for every x

(d) likelihood ratio order $\left(X\le_{lr}Y\right)$ if $\dfrac{f_X\left(x\right)}{f_Y\left(x\right)}$ decreases in x.

The following interrelationship among various stochastic ordering of distributions given in [18]are useful

$$X\le_{lr}Y\Rightarrow X\le_{hr}Y\Rightarrow X\le_{mrl}Y \tag{5.1}$$
$$\Downarrow$$
$$X\le_{st}Y$$

In the following theorem an attempt has been made to show that GGGD is ordered with respect to the strongest 'likelihood ratio ordering'.

Theorem: Suppose $X \sim \text{GGGD}(\theta_1,\alpha_1,\gamma_1,\beta_1)$ and $Y \sim \text{GGGD}(\theta_2,\alpha_2,\gamma_2,\beta_2)$. If any one of the following conditions satisfied

(i) $\theta_1=\theta_2,\gamma_1=\gamma_2,\alpha_1=\alpha_2$ and $\beta_1<\beta_2$ (ii) $\theta_1=\theta_2,\gamma_1=\gamma_2,\beta_1=\beta_2$ and $\alpha_1<\alpha_2$

(iii) $\theta_1=\theta_2,\alpha_1=\alpha_2,\beta_1=\beta_2$ and $\gamma_1>\gamma_2$ (iii) $\alpha_1=\alpha_2,\beta_1=\beta_2,\gamma_1>\gamma_2$ and $\theta_1>\theta_2$

then $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

Proof: We have

$$\frac{f_X(x;\theta_1,\alpha_1,\gamma_1,\beta_1)}{f_Y(x;\theta_2,\alpha_2,\gamma_2,\beta_2)}=\frac{\beta_1\left(\theta_1\gamma_1^{\beta_1}\right)^{\alpha_1}\Gamma(\alpha_2)}{\beta_2\left(\theta_2\gamma_2^{\beta_2}\right)^{\alpha_2}\Gamma(\alpha_1)}e^{-\left\{\theta_1(\gamma_1 x)^{\beta_1}-\theta_2(\gamma_2 x)^{\beta_2}\right\}}x^{\beta_1\alpha_1-\beta_2\alpha_2}\quad ;\ x>0$$

Now

$$\ln\frac{f_X(x;\theta_1,\alpha_1,\gamma_1,\beta_1)}{f_Y(x;\theta_2,\alpha_2,\gamma_2,\beta_2)}=\ln\left[\frac{\beta_1\left(\theta_1\gamma_1^{\beta_1}\right)^{\alpha_1}\Gamma(\alpha_2)}{\beta_2\left(\theta_2\gamma_2^{\beta_2}\right)^{\alpha_2}\Gamma(\alpha_1)}\right]+\left(\beta_1\alpha_1-\beta_2\alpha_2\right)\ln x-\left\{\theta_1(\gamma_1 x)^{\beta_1}-\theta_2(\gamma_2 x)^{\beta_2}\right\}$$

This gives
$$\frac{d}{dx}\ln\frac{f_X(x;\theta_1,\alpha_1,\gamma_1,\beta_1)}{f_Y(x;\theta_2,\alpha_2,\gamma_2,\beta_2)}=\frac{\beta_1\alpha_1-\beta_2\alpha_2}{x}+\left\{\theta_2\beta_2\gamma_2(\gamma_2 x)^{\beta_2-1}-\theta_1\beta_1\gamma_1(\gamma_1 x)^{\beta_1-1}\right\}.$$

Thus under the given conditions in the theorem, $\frac{d}{dx}\ln\frac{f_X(x;\theta_1,\alpha_1,\gamma_1,\beta_1)}{f_Y(x;\theta_2,\alpha_2,\gamma_2,\beta_2)}<0$. This means that $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

5. ESTIMATION OF PRAMETERS USING MAXIMUM LIKELIHOOD

Suppose $(x_1,x_2,x_3,\ldots,x_n)$ be a random sample from $\text{GGGD}(\theta,\alpha,\gamma,\beta)$. Then, the natural log likelihood function can be expressed as

$$\ln L=\sum_{i=1}^{n}\ln f_5(x_i;\theta,\alpha,\beta,\gamma)$$

$$=n\left[\ln\beta+\alpha(\ln\theta+\beta\ln\gamma)-\ln\Gamma(\alpha)\right]+(\beta\alpha-1)\sum_{i=1}^{n}\ln x_i-\theta\gamma^{\beta}\sum_{i=1}^{n}(x_i)^{\beta}$$

The maximum likelihood estimates (MLEs) $(\hat{\theta},\hat{\alpha},\hat{\beta},\hat{\gamma})$ of parameters $(\theta,\alpha,\beta,\gamma)$ of GGGD (2.1) are the solutions of the log likelihood equations

$$\frac{\partial\ln L}{\partial\theta}=\frac{n\alpha}{\theta}-\gamma^{\beta}\sum_{i=1}^{n}x_i^{\beta}=0$$

$$\frac{\partial\ln L}{\partial\alpha}=n(\ln\theta+\beta\ln\gamma)-n\psi(\alpha)+\beta\sum_{i=1}^{n}\ln x_i=0$$

$$\frac{\partial\ln L}{\partial\beta}=\frac{n}{\beta}+n\alpha\ln\gamma+\alpha\sum_{i=1}^{n}\ln x_i-\theta\gamma^{\beta}\ln\gamma\sum_{i=1}^{n}x_i^{\beta}-\theta\gamma^{\beta}\sum_{i=1}^{n}x_i^{\beta}\ln(x_i)=0$$

$$\frac{\partial \ln L}{\partial \gamma} = \frac{n\alpha\beta}{\gamma} - \theta\beta\gamma^{\beta-1}\sum_{i=1}^{n} x_i^{\beta} = 0,$$

where $\psi(\alpha) = \frac{d}{d\alpha}\ln\Gamma(\alpha)$ is the digamma function.

These four log likelihood equations do not seem to be solved directly because these cannot be expressed in closed form. However, Fisher's scoring method can be applied to solve these equations iteratively. For, we have

$$\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{n\alpha}{\theta^2}$$

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -n\psi'(\alpha)$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{n}{\beta^2} - \theta\gamma^{\beta}(\ln\gamma)^2\sum_{i=1}^{n} x_i^{\beta} - 2\theta\gamma^{\beta}\ln\gamma\sum_{i=1}^{n} x_i^{\beta}\ln(x_i) - \theta\gamma^{\beta}\sum_{i=1}^{n} x_i^{\beta}\left(\ln(x_i)\right)^2$$

$$\frac{\partial^2 \ln L}{\partial \gamma^2} = -\frac{n\alpha\beta}{\gamma^2} - \theta\beta(\beta-1)\gamma^{\beta-2}\sum_{i=1}^{n} x_i^{\beta}$$

$$\frac{\partial^2 \ln L}{\partial \theta\,\partial \alpha} = \frac{n}{\theta} = \frac{\partial^2 \ln L}{\partial \alpha\,\partial \theta}$$

$$\frac{\partial^2 \ln L}{\partial \theta\,\partial \beta} = -\left[\gamma^{\beta}\ln\gamma\sum_{i=1}^{n} x_i^{\beta} + \gamma^{\beta}\sum_{i=1}^{n} x_i^{\beta}\ln(x_i)\right] = \frac{\partial^2 \ln L}{\partial \beta\,\partial \theta}$$

$$\frac{\partial^2 \ln L}{\partial \theta\,\partial \gamma} = -\beta\gamma^{\beta-1}\sum_{i=1}^{n} x_i^{\beta} = \frac{\partial^2 \ln L}{\partial \gamma\,\partial \theta}$$

$$\frac{\partial^2 \ln L}{\partial \alpha\,\partial \beta} = n\ln\gamma + \sum_{i=1}^{n}\ln x_i = \frac{\partial^2 \ln L}{\partial \beta\,\partial \alpha}$$

$$\frac{\partial^2 \ln L}{\partial \alpha\,\partial \gamma} = \frac{n\beta}{\gamma} = \frac{\partial^2 \ln L}{\partial \gamma\,\partial \alpha}$$

$$\frac{\partial^2 \ln L}{\partial \beta\,\partial \gamma} = \frac{n\alpha}{\gamma} - \theta\beta\gamma^{\beta-1}\ln\gamma\sum_{i=1}^{n} x_i^{\beta} - \theta\,\gamma^{\beta-1}\sum_{i=1}^{n} x_i^{\beta} - \theta\beta\gamma^{\beta-1}\sum_{i=1}^{n} x_i^{\beta}\ln(x_i) = \frac{\partial^2 \ln L}{\partial \gamma\,\partial \beta},$$

where $\psi'(\alpha) = \frac{d}{d\alpha}\psi(\alpha)$ is the trigamma function.

The MLE $\left(\hat{\theta},\hat{\alpha},\hat{\beta},\hat{\gamma}\right)$ of parameters $(\theta,\alpha,\beta,\gamma)$ of GGGD (2.1) are obtained by solving the following equations

$$\begin{bmatrix} \frac{\partial^2 \ln L}{\partial \theta^2} & \frac{\partial^2 \ln L}{\partial \theta \partial \alpha} & \frac{\partial^2 \ln L}{\partial \theta \partial \beta} & \frac{\partial^2 \ln L}{\partial \theta \partial \gamma} \\ \frac{\partial^2 \ln L}{\partial \alpha \partial \theta} & \frac{\partial^2 \ln L}{\partial \alpha^2} & \frac{\partial^2 \ln L}{\partial \alpha \partial \beta} & \frac{\partial^2 \ln L}{\partial \alpha \partial \gamma} \\ \frac{\partial^2 \ln L}{\partial \beta \partial \theta} & \frac{\partial^2 \ln L}{\partial \beta \partial \alpha} & \frac{\partial^2 \ln L}{\partial \beta^2} & \frac{\partial^2 \ln L}{\partial \beta \partial \gamma} \\ \frac{\partial^2 \ln L}{\partial \gamma \partial \theta} & \frac{\partial^2 \ln L}{\partial \gamma \partial \alpha} & \frac{\partial^2 \ln L}{\partial \gamma \partial \beta} & \frac{\partial^2 \ln L}{\partial \gamma^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0 \\ \hat{\beta}=\beta_0 \\ \hat{\gamma}=\gamma_0}} \begin{bmatrix} \hat{\theta}-\theta_0 \\ \hat{\alpha}-\alpha_0 \\ \hat{\beta}-\beta_0 \\ \hat{\gamma}-\gamma_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \ln L}{\partial \theta} \\ \frac{\partial \ln L}{\partial \alpha} \\ \frac{\partial \ln L}{\partial \beta} \\ \frac{\partial \ln L}{\partial \gamma} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0 \\ \hat{\beta}=\beta_0 \\ \hat{\gamma}=\gamma_0}}$$

where $\theta_0, \alpha_0, \beta_0$ and γ_0 are the initial values of parameters θ, α, β and γ. These equations are solved iteratively till sufficiently close estimates of $\hat{\theta}, \hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are obtained. In this paper, to estimate the parameters θ, α, β and γ for the considered datasets, R-software has been used.

6. GOODNESS OF FIT

The application of goodness of fit of GGGD has been discussed for with two real lifetime datasets. The first dataset is relating to fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second on 101 observations with maximum stress per cycle 31,000 psi available in [19]. The second dataset represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm, available in [20]. The fit has been compared with GGD, GD, Weibull and exponential distributions. The maximum likelihood estimates of parameters for the considered distributions for the datasets 1 and 2 have been presented in table 1. The values of $-2\ln L$, AIC (Akaike information criterion), K-S Statistic (Kolmogorov-Smirnov Statistic) and p-value for two datasets have been presented in table 2. The AIC and K-S Statistics are computed using the following formulae: $AIC = -2\log L + 2k$, and $K\text{-}S = \operatorname{Sup}_x \left| F_n(x) - F_0(x) \right|$, where k being the number of parameters involved in the respective distributions, n is the sample size and $F_n(x)$ is the empirical distribution function. The distribution corresponding to the lower values of $-2\ln L$, AIC and K-S statistic is the best fit distribution.

The goodness of fit in table 2 shows that that GGGD gives much better fit than GGD, GD, Weibull and exponential. The variance-covariance matrix of the parameters θ, α, β and γ of GGGD for datasets 1 and 2 are shown in tables 3 and 4, respectively.

TABLE 1. Summary of the ML estimates of parameters

Data set	Model	ML Estimates			
		$\hat{\theta}$	$\hat{\alpha}$	$\hat{\beta}$	γ
1	GGGD	0.414869	1.531572	2.540404	0.022695
	GGD	0.018820	4.829310	1.307120	-----
	GD	0.112483	7.685828	-----	------
	Weibull	0.002060	1.457312	-----	-----
	Exponential	0.014634	------	-----	------
2	GGGD	0.205558	2.435249	3.348127	0.816920
	GGD	0.304412	3.586110	2.648310	------
	GD	9.538435	23.381839	------	------
	Weibull	0.005589	5.335228	------	-------
	Exponential	0.407942	------	------	--------

TABLE 2. Summary of Goodness of fit by K-S Statistic

Data set	Model	$-2\log L$	AIC	K-S	p-value
1	GGGD	907.51	915.51	0.067	0.748
	GGD	912.44	916.44	0.087	0.429
	GD	915.76	919.76	0.099	0.271
	Weibull	982.66	986.66	0.280	0.000
	Exponential	1044.87	1046.87	0.366	0.000
2	GGGD	97.74	105.74	0.040	0.999
	GGD	100.58	106.58	0.044	0.987
	GD	100.07	104.07	0.058	0.973
	Weibull	99.31	103.31	0.060	0.963
	Exponential	261.73	263.73	0.448	0.000

TABLE 3. Variance-covariance matrix of the parameters θ, α, β and γ **of GGGD for dataset 1**

$$\begin{array}{c} \\ \hat{\theta} \\ \hat{\alpha} \\ \hat{\beta} \\ \hat{\gamma} \end{array} \begin{array}{c} \begin{array}{cccc} \hat{\theta} & \hat{\alpha} & \hat{\beta} & \hat{\gamma} \end{array} \\ \begin{bmatrix} 1.7410 & 0.0394 & -0.0295 & -2.9667 \\ 0.0394 & 4.8591 & -3.6715 & 1.3173 \\ -0.0295 & -3.6715 & 2.8634 & -1.0167 \\ -2.9667 & 1.3173 & -1.0166 & 5.4554 \end{bmatrix} \end{array}$$

TABLE 4. Variance-covariance matrix of the parameters θ, α, β and γ **of GGGD for dataset 2**

$$\begin{array}{c} \\ \hat{\theta} \\ \hat{\alpha} \\ \hat{\beta} \\ \hat{\gamma} \end{array} \begin{array}{c} \begin{array}{cccc} \hat{\theta} & \hat{\alpha} & \hat{\beta} & \hat{\gamma} \end{array} \\ \begin{bmatrix} 6.5169 & -0.0677 & 0.0904 & -0.0238 \\ -0.0677 & 0.5350 & -0.4904 & 0.0025 \\ 0.0904 & -0.4904 & 0.4852 & -0.0024 \\ -0.0238 & 0.0025 & -0.0024 & 0.0001 \end{bmatrix} \end{array}$$

7. CONCLUDING REMARKS

A generalization of generalized gamma distribution (GGGD) introduced by Stacy (1962)[12], which includes the three-parameter generalized gamma distribution (GGD), two-parameter Weibull and gamma distributions, and exponential distribution, has been suggested and investigated. The behavior of the hazard rate function and the stochastic ordering of the distribution have been discussed. The estimation of the parameters of the distribution has been explained using the method of maximum likelihood. The the goodness of fit of GGGD with two real lifetime datasets have been discussed and the fit by GGGD is quite satisfactory over generalized gamma, gamma, Weibull, and exponential distributions.

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