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Indoor Path Planning using Harmonic Functions via Half-Sweep Arithmetic Mean Method

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Abstract: This paper presents the application of a two-stage Half-Sweep Arithmetic Mean (HSAM) iterative method to obtain the Harmonic functions to solve the path planning problem in a 2D indoor environment. Several path planning simulations in a known indoor environment were conducted to examine the effectiveness of the proposed method. It is shown that the proposed path planning algorithm is capable of generating smooth paths from various start and goal positions. Also, numerical results show that the proposed HSAM method converges much faster than the existing iterative methods, thus it drastically improves the overall performance of the path planning algorithm.

Keywords: Path Planning; Half-Sweep Arithmetic Mean method; Laplace's equation; Harmonic functions

1. Introduction

The aim of path planning algorithm is to construct a collision-free path from arbitrary position to a specified goal position within a workspace containing obstacle. In this study, we employ a mathematical model for the path planning problem that relies on the solutions of Laplace's equation i.e. harmonic functions, to provide virtual surface gradient that can be used for navigation purpose.

Harmonic functions are known to be very useful in robot navigation [1]. They offer a complete path planning algorithm and paths derived from them are generally smooth. When applied to the path planning problem, they have the advantage over simple potential field based approach, as they exhibit no spurious local minima. Global approach to path planning problem using the solutions of Laplace's equation was first introduced by [2].

After that, several studies were conducted using similar idea to solve various robot motion and navigation problems. Garrido, Moreno, Blanco and Martin [3] used harmonic functions obtained through finite elements method for robot motion. Pedersen and Fossen [4] employed harmonic functions via potential flow for marine vessel path planning. Harmonic functions were also successfully applied in behaviour-based robot [5, 6]. More recently, harmonic functions were applied for path planning of Unmanned Aerial Vehicles in 3D space [7].

2. Harmonic Functions In Path Planning

Global approach to path planning problem relies on the use of harmonic potential fields as guidance for robot navigation. The potential fields are computed in a global manner and the harmonic solutions to Laplace's equation are then used to find the path lines for a robot to move from start to the specified goal point. Obstacles are considered as current sources with relatively high potential values, and the goal is considered to be the sink with the lowest assigned potential value. This amounts to using Dirichlet boundary conditions. Then, by performing the standard Gradient Descent Search (GDS), it leads the path tracing process by following the negative gradient of the harmonic potentials to the lowest potential value i.e. goal point [2].

In the path planning literature, the solutions of Laplace's equation was computed using Gauss-Seidel (GS) [2] and Successive Over-relaxation (SOR) [8] iterative methods. In these studies, it was shown that SOR is superior to GS method. After that, Saudi and Sulaiman [9] discovered faster Laplace's solver using the combination of red-black strategy with half-sweep iteration and SOR method. The half-sweep approach was also successfully applied in behaviour-based robot [5, 6].

However, when the size of the environment increases, the computational time required for obtaining the harmonic potentials grows exponentially. Therefore in this study, we investigate the efficiency of two-stage Half-Sweep Arithmetic Mean (HSAM) iterative method in

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computing the harmonic potentials of the environment to solve path planning problem.

One major advantage of HSAM method is its suitability for parallel implementation, since the computation in the two stages can be executed independently. HSAM is actually a variant of the established Arithmetic Mean (AM) method [10], which is also known as Full-Sweep Arithmetic Mean (FSAM) method. HSAM was introduced by combining the concept of half-sweep iteration and AM method [11]. After that, HSAM method was applied to solve Poisson equation [12] and linear Fredholm integral equation [13] and Composite 6-Point Closed Newton-Cotes Quadrature Algebraic Equation [14]. These previous studies, however, were focusing on one-dimensional space only. In the case of path planning problem that was examined in this study, Laplace's equation in two-dimensional space was considered. In addition to the proposed HSAM method, the standard GS (also known as Full-Sweep Gauss-Seidel (FSGS)) and FSAM were also considered in this study for performance comparison purposes..

3. ARITHMETIC MEAN METHODS

Numerical solutions for Laplace's equation are readily obtained from finite difference method. Consider a 2D Laplace's equation defined as

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (1)$$

The application of Eq. (1) to model the potential values in path planning problem often results in large linear system with sparse coefficient matrix. Therefore, iterative method is often used to efficiently solve such large and sparse linear system. The standard five-point finite difference formula to approximate Eq. (1) is given as

$$u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j} = 0 \qquad (2)$$

Another type of finite difference approximation is obtained by rotating the *i-j* plane axis clockwise by 45^o [15]. Hence, the rotated five-point approximation formula is defined as

$$u_{i-1,j-1} + u_{i+1,j-1} + u_{i-1,j+1} + u_{i+1,j+1} - 4u_{i,j} = 0 \qquad (3)$$

Eq. (2) and (3) are used for the iterative scheme of full-sweep and half-sweep iteration cases, respectively. Figure 1 illustrates the portion of computational grid about point (i, j) for full-sweep and half-sweep cases.

The Gauss-Seidel (GS) iterative scheme for Eq. (2) can be written as [16]

$$u_{i,j}^{(k+1)} = \frac{1}{4}\left(u_{i-1,j}^{(k+1)} + u_{i+1,j}^{(k)} + u_{(i,j-1)}^{(k+1)} + u_{(i,j+1)}^{(k)}\right). \qquad (4)$$

By adding a weighted parameter, *w* the iterative schemes for full-sweep and half-sweep cases can be written as [17]

$$u_{i,j}^{(k+1)} = \frac{w}{4}\left(u_{i-1,j}^{(k+1)} + u_{i+1,j}^{(k)} + u_{(i,j-1)}^{(k+1)} + u_{(i,j+1)}^{(k)}\right) + (1-\omega)u_{i,j}^{(k)}, \qquad (5)$$

$$u_{i,j}^{(k+1)} = \frac{w}{4}\left(u_{i-1,j-1}^{(k+1)} + u_{i+1,j-1}^{(k+1)} + u_{(i-1,j+1)}^{(k)} + u_{(i+1,j+1)}^{(k)}\right) + (1-\omega)u_{i,j}^{(k)}. \qquad (6)$$

Eq. (4) is used for the implementation of FSGS iterative method. While the implementations of FSAM and HSAM methods employ Eq. (5) and (6), respectively.

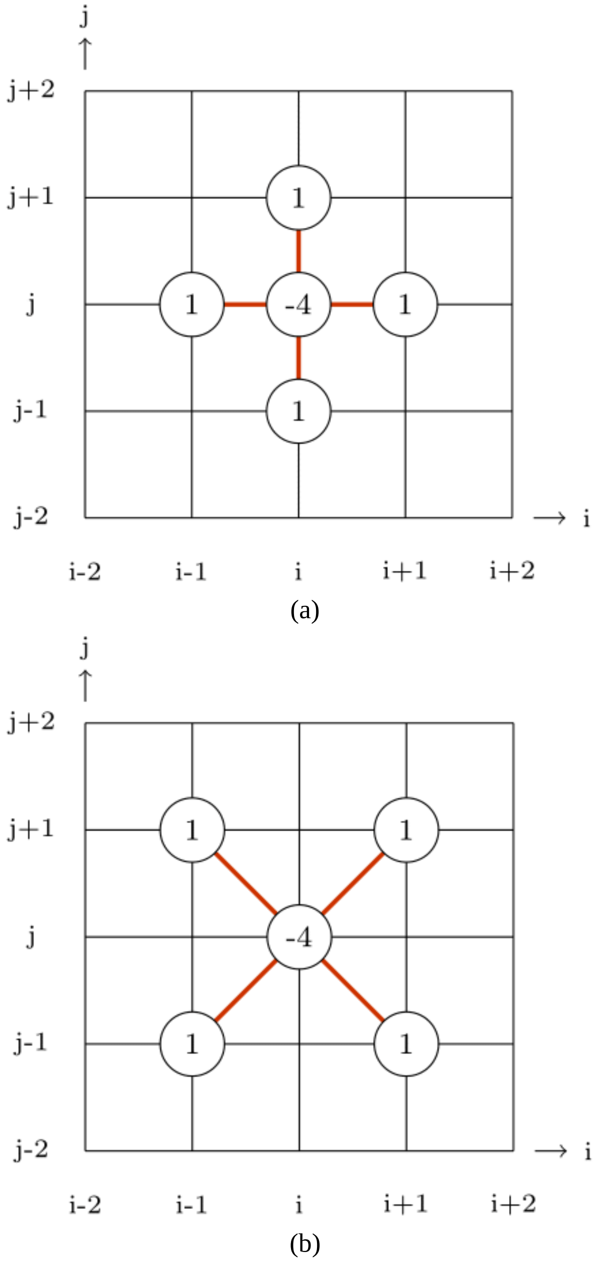

Figure1. Portion of the computational grid about point (i, j) for (a) full-sweep, and (b) half-sweep cases, respectively

Applying these finite difference approximations to Eq. (1) will result in a large and sparse linear system that can be stated in matrix form as

$$Au = b \qquad (7)$$

where A and b are known, and u is unknown. We can express the matrix A = (a_{ij}) as the matrix sum

$$A = L + D + U \tag{8}$$

where D = diag{a_{11}, a_{22}, ..., a_{nn}} and L and U are respectively, strictly lower and upper triangular matrices.

As stated in the previous section, AM is a two-stage iterative method and its iterative process involves of solving two independent systems such as $\hat{u}^{(1)}$ and $\hat{u}^{(2)}$. By adding an acceleration parameter, w the general iterative scheme for AM method can be defined as

$$\left.\begin{aligned}(D + wL)\hat{u}^{(1)} &= \big((1-w)D - wU\big)u^{(k)} + wb,\\ (D + wL)\hat{u}^{(2)} &= \big((1-w)D - wL\big)u^{(k)} + wb,\\ u^{(k+1)} &= \tfrac{1}{2}\big(\hat{u}^{(1)} + \hat{u}^{(2)}\big).\end{aligned}\right\} \tag{9}$$

Here $\hat{u}^{(0)}$is an initial vector approximation to u and w is a positive parameter. A proof identical to that given in [10] ensures that the iterative method (9) is convergent for 0 <w< 2. The optimal w can be obtained by conducting several experiments until it give the smallest number of iterations.

By determining the values of matrices D, L and U as stated in Eq. (8), the implementations of FSAM and HSAM methods are described in Algorithm 1 and 2, respectively. To obtain the harmonic solutions of Eq. (1), the two-step iteration procedures at Level 1 and 2 and matrix sum at Level 3 are repeatedly carried out until the specified convergence requirement $\|u^{(k+1)} - u^{(k)}\| < \epsilon$ is satisfied, where ϵ is the convergence criterion. In path planning problem, only non-occupied nodes are computed in the iteration process. All other nodes that are occupied either by obstacles or boundary walls are ignored, since their values are held fixed.

Algorithm 1: Full-Sweep Arithmetic Mean (FSAM)

Refer Figure 2(a)
Level 1 (Sweep forward)
for $i, j = 0,1,2, \ldots, N-1, N$ **do**
Compute

$$u_{i,j}^{(1)} = \frac{w}{4}\left(u_{i-1,j}^{(1)} + u_{i+1,j}^{(k)} + u_{i,j-1}^{(1)} + u_{i,j+1}^{(k)}\right) + (1-w)u_{i,j}^{(k)}$$

Level 2 (Sweep backward)
for $i, j = N, N-1, N-2, \ldots, 1, 0$ **do**
Compute

$$u_{i,j}^{(2)} = \frac{w}{4}\left(u_{i-1,j}^{(k)} + u_{i+1,j}^{(2)} + u_{i,j-1}^{(k)} + u_{i,j+1}^{(2)}\right) + (1-w)u_{i,j}^{(k)}$$

Level 3
for $i, j = 0,1,2, \ldots, N-1, N$ **do**
Compute

$$u_{i,j}^{(k+1)} = \frac{1}{2}\left(u_{i,j}^{(1)} + u_{i,j}^{(2)}\right)$$

Algorithm 2: Half-Sweep Arithmetic Mean (HSAM)

Refer Figure 2(b)
Level 1 (Sweep forward)
for first node **to** last node **do**
Compute

$$u_{i,j}^{(1)} = \frac{w}{4}\left(u_{i-1,j}^{(1)} + u_{i+1,j}^{(k)} + u_{i,j-1}^{(1)} + u_{i,j+1}^{(k)}\right) + (1-w)u_{i,j}^{(k)}$$

Level 2 (Sweep backward)
for last node **to** first node **do**
Compute

$$u_{i,j}^{(2)} = \frac{w}{4}\left(u_{i-1,j}^{(k)} + u_{i+1,j}^{(2)} + u_{i,j-1}^{(k)} + u_{i,j+1}^{(2)}\right) + (1-w)u_{i,j}^{(k)}$$

Level 3
for first node **to** last node **do**
Compute

$$u_{i,j}^{(k+1)} = \frac{1}{2}\left(u_{i,j}^{(1)} + u_{i,j}^{(2)}\right)$$

Figure 2 illustrates the computational grid for FSAM and HSAM methods. For FSAM method in Figure 2(a), all nodes in the problem domain will be considered, whereas for HSAM method in Figure 2(b) only black nodes are considered (i.e. node 1 to 41). Therefore, the computational complexity for HSAM method is approximately 50% less than FSAM method. In Level 1, the computation starts from the first node until the last node. Meanwhile in Level 2, the computation sweeps in reverse-order from the last node down to the first node. To obtain the harmonic functions, the algorithm is performed explicitly until the convergence criterion is satisfied. For HSAM method, after the convergence, the remaining white nodes are calculated using direct method [15].

4. THE PATH PLANNING SIMULATION

The simulation is designed using the proposed algorithm for path planning in static environment. Three map examples of indoor environment are used covering an area of 270 x 270, 290 x 290 and 414 x 120 for the respective Case 1, Case 2 and Case 3. The environment consists of various shapes of obstacles, inner walls, and outer boundary walls.

In the initial setup, nodes occupied by obstacles, inner and outer walls are fixed with high potential values, whilst the lowest potential value is assigned to the goal point. No initial values are assigned to all other non-occupied nodes. The computations are carried out on a PC equipped with an Intel i53570 CPU running at 3.40GHz speed with 8GB RAM.

The path planning simulation begins by computing the harmonic potentials of the environment using the considered iterative methods. Several values are tested to obtain the optimal weighted parameter, *w*. The iteration process for obtaining the harmonic potentials continues until the convergence criterion is satisfied. The convergence criterion must be set to a very small error tolerance, $\epsilon = 1.0^{-15}$, since lower precision is not sufficient to avoid flat areas in the resulting harmonic potential values. After the harmonic potentials of the environment are obtained, the required path from start to the specified goal point can be traced using the standard GDS. The GDS simply repeatedly moves to the next node with lower potential value until the goal point with the lowest potential value is found out. The implementation of path planning algorithm is described in Algorithm 3.

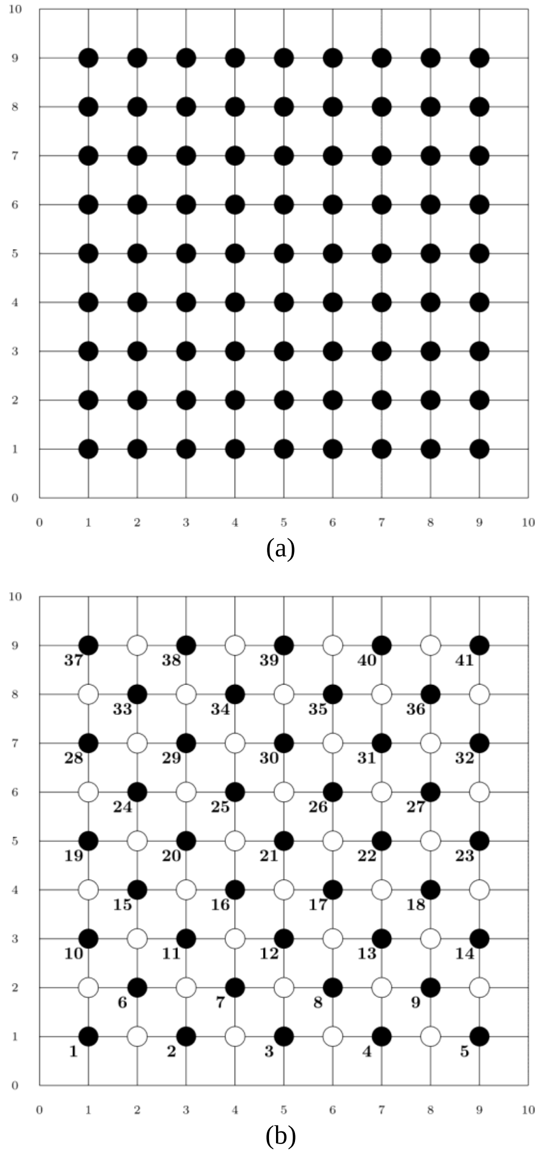

Figure 2. The computational grid for (a) full-sweep and (b) half-sweep cases, respectively

Algorithm 3: Path Planning Algorithm

(1) Load the map of the environment
(2) Setup matrices to store potential values of the environment
(3) Set potential values of all boundary nodes
(4) Set potential values of goal point
(5) Initialize potential values for other free spaces
(6) Compute the harmonic functions using the considered methods (e.g. Algorithm 1 and 2 for FSAM and HSAM methods, respectively).
(7) Perform GDS on the obtained harmonic functions to generate path from start to goal point
(8) Save the generated path

5. Results and Discussion

Table 1 provides a comparison of the performance of the considered iterative methods to solve Laplace's equation 1 in terms of iteration numbers and CPU time in seconds. The numerical results in this table show that the HSAM method is clearly superior to the existing methods. It can be clearly observed that the iterations and CPU time of both half-sweep methods are approximately 50% less than their corresponding full-sweep methods. FSAM performs much faster than the standard FSGS and the previous HSGS [5] methods. The AM variants of FSAM and HSAM methods are very much superior to their corresponding standard FSGS and HSGS methods. HSAM gives the best performance. Against FSAM, it reduces the number of iterations and CPU time approximately by 49% and 70%, respectively.

Table 1. Iteration, *k* and CPU time (in seconds), *t* of the considered methods

Methods	**CASE 1**	
FSGS	*k*	53697
	t	45.12
FSAM	*k*	6525
	t	10.67
HSAM	*k*	3361
	t	2.93
HSGS [5]	*k*	27760
	t	12.51
Methods	**CASE 2**	
FSGS	*k*	19805
	t	16.51
FSAM	*k*	2478
	t	3.50
HSAM	*k*	1311
	t	1.13
HSGS [5]	*k*	10202
	t	4.50
Methods	**CASE 3**	
FSGS	*k*	15009
	t	5.84
FSAM	*k*	1900
	t	0.77
HSAM	*k*	1039
	t	0.46
HSGS [5]	*k*	7752
	t	1.81

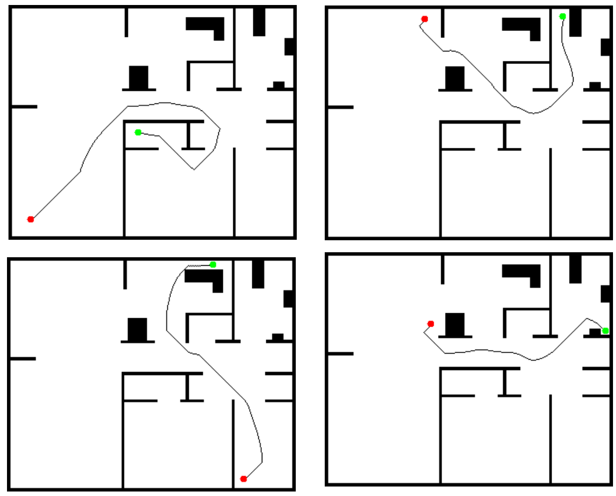

Figure 3. The generated paths for Case 1 covering an area of 330 x 270

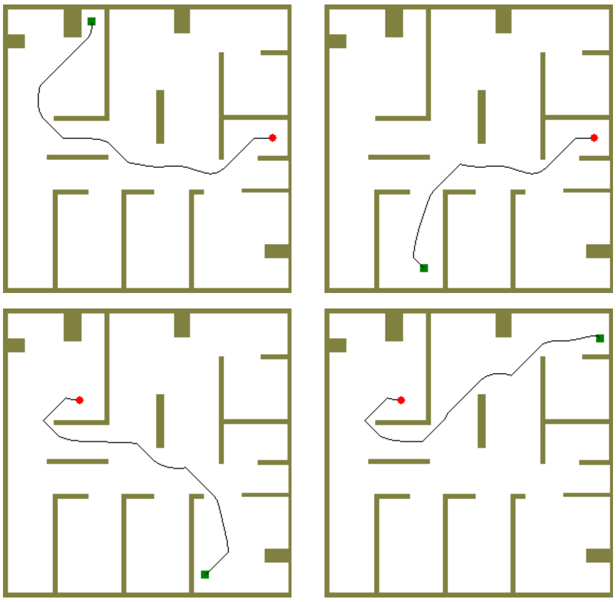

Figure 4. The generated paths for Case 2 covering an area of 290 x 290

Once the solutions of Laplace's equation (1) are obtained, they are then used to generate path from start to goal position. The generated paths for Case 1 – 3 obtained from the path planning simulation are shown in Figures 3 - 5, respectively. The solid square in green colour and circle in red colour denote start and goal point, respectively.

6. Conclusions

This study demonstrated the great potential of the two-stage HSAM iterative method in computing the harmonic functions for application in path planning problem. The HSAM method contributed significantly in improving the overall performance of the path planning algorithm. The path planning algorithm is very robust, where it capable of generating paths in the narrow space and difficult corners.

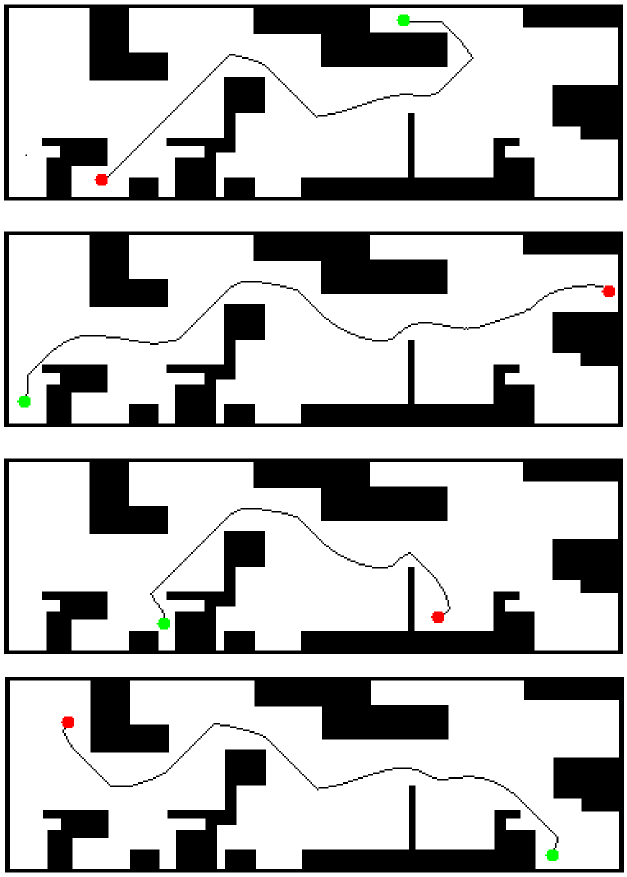

Figure 5. The generated paths for Case 3 covering an area of 414 x 120

Moreover, since AM methods possess separate and independent calculations at Level 1 and Level 2, parallel implementation is possible. In the future work, the path planning algorithm will be tested in more complex domains. Application in dynamic and unknown environment is an interesting idea to explore in the future. Furthermore, the HSAM method will be examined for path planning problem in space of higher dimensions. Faster iterative method by combining quarter-sweep iteration concept with AM method will also be considered.

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