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Performance Comparison of High Order Moments Blind Channel Identification

Rabah Maoudj [1], Juan Medeiros [2] and Michel Terré[3]

[1,3] *CEDRIC/LAETITIA Laboratory, Le Conservatoire National des Arts et Métiers, Paris, France*
[2] *Electrical Engineering laboratory, Universidade Federal do Ceará, Sobral, Brazil*

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Abstract: In this paper, we compare different order of statistical moments to blindly estimate the propagation channel in an OFDM based wireless network. We first derive the theoretical expression of the channel estimation error and we compare it to simulation results in different scenarios. These simulation results show a good agreement with the theoretical expression. Furthermore, we show that blind channel identification algorithms, based on different moment's order, exhibit similar performance in terms of the Normalized Mean Square Error (NMSE) and in terms of the Bit Error Rate (BER). We conclude that the choice of the moment's order is uniquely based on the ambiguity solving stage in the algorithm. The proposed study shows that the choice of the 4th moment is a trade-off between channel estimation performance and ambiguity solving complexity.

Keywords: Blind channel, High order moment.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) [1], [2] has generated considerable interest as a highly suitable technique for high-bit-rate transmission. It has been adopted in modern wireless access networks such as LTE [3], [4], TEDS [5], WiFi [6]. In an OFDM waveform based transmission, a great number of subcarriers are generally devoted to pilot symbols which reduces the spectral efficiency. To overcome this problem, blind channel identification and equalization have attracted considerable interests during the past few years. Thus, many algorithms have initially been proposed using the subspace approach [7], the ML estimation [8], the adaptive tracking [9], and a constant modulus algorithm [10]. Furthermore, several works have also focused on using second-order statistics techniques [11] based on over-sampling or on spatial diversity. Higher order statistics were also investigated in blind channel estimation and shown good performances [12]–[15]. Thus, a relevant question arises in the case of algorithms investigated in [14], [15]: Is there an optimum moment's order for blind channel estimation? According to the author's best knowledge; there is no study about comparison of the impact of this moment's order on the blind channel estimation. To this end, in this paper, we compare both estimation errors and BER results between the 4th, 8th and 12th moment's order in OFDM transmissions using respectively 4-QAM, 16-QAM and 64-QAM modulation schemes; we derive analytical expressions of an approximation of the power of the estimation error and the BER for different moment's order. The obtained theoretical results are compared to the simulation results. The paper is organized as follows. Section II is dedicated to system model and some recalls about p^{th} order statistics. In section III, estimation channel algorithm is presented and derivation of the estimation error and the BER are detailed. Numerical results are presented and discussed in section IV. Finally, section V concludes this paper.

2. System Model

A. Data and Channel Model

Consider a *SISO* (Simple input Simple Output) transmission system with one transmit antenna and one receive antenna. Let's $s_{i,k}$ be the *M*-QAM symbols transmitted over the i^{th} OFDM symbol and the k^{th} subcarrier of an OFDM frame. Consider that the frequency response of the propagation channel at the k^{th} subcarrier h_k is constant over $N > M$ OFDM consecutive symbols. Assume also well time frequency synchronization at the receiver and cyclic prefix correctly sized to contain the channel impulse response. Thus, the

E-mail: rabah.maoudj@univ-paris13.fr, juca@ufc.br, terre@cnam.fr

transmission can be modeled according to the following equation,

$$r_{i,k} = h_k s_{i,k} + n_{i,k}. \tag{1}$$

Where $n_{i,k}$ is the circular AWGN (Additive White Gaussian Noise) at the i^{th} OFDM symbol and k^{th} subcarrier.

B. p^{th} order statistics

In this section we present some statistics concepts used in our blind channel estimation algorithm. This latter is based on high order statistic of the M-QAM received signal.

We define the (*p*, *q*) order moment of an complex variable *x* as follows,

$$\mu_{x,(p)}^{(q)} = E[x^p x^{*q}]. \tag{2}$$

$E[-]$ denotes the mathematical expectation.

If x is a circular complex variable, like it is the case for the AWGN in our system model, then $\forall\, p \neq q\ ,\ \mu_{x,(p)}^{(q)} = 0$.

If we assume that the distribution of the complex *M*-QAM symbols x is uniform, we can define (p, q) order moment as:

$$\forall\, p \neq q\ \ et\ p+q \leq M,\ \ \mu_{x,(p)}^{(q)} = \begin{cases} E[x^M] & ,p = M\ or\ q = M \\ 0 & ,\ otherwise. \end{cases} \tag{3}$$

In the sequel of this paper we use (p, 0) order moment of the received *M*-QAM signal. For 4-QAM symbols uniformly distributed, (*p*, 0) order moment is given by:

$$\mu_{x,(p)}^{(0)} = \begin{cases} a^p e^{jp\varphi},\ p = 4l, \forall l \in \mathbb{N}^* \\ 0,\ otherwise\,. \end{cases} \tag{4}$$

Where a and φ are respectively magnitude and initial phase of the constellation.

We can easily observe that each *M*_QAM constellation can be seen as a set of *4-QAM* constellations weighted by a magnitude a_i and shifted by a phase φ_i. For example, if x is a *16*_QAM symbol therefore $\in \left\{4 - QAM_{\sqrt{2},\frac{\pi}{4}},\ 4 - QAM_{\sqrt{10},\frac{\pi}{9.764}},\ 4 - QAM_{\sqrt{10},\frac{\pi}{2.515}},\ 4 - QAM_{\sqrt{18},\frac{\pi}{4}}\right\}$, as shown in figure 1.

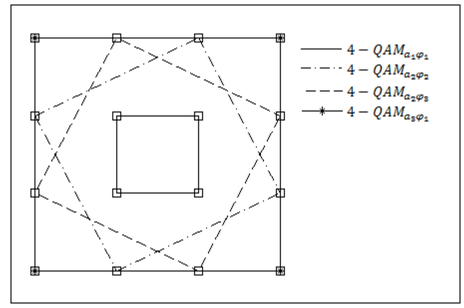

Figure 1. 16-QAM constellation

The p[th] order moment of x is given by the expression hereafter,

$$\mu_{x,(p)}^{(0)} = \begin{cases} \mu_{x_0,(p)}^{(\mathbf{0})} \sum_{i=1}^{\frac{M}{4}} a_i^p e^{jp\varphi_i},\ p = 4l, \forall l \in \mathbb{N}^* \\ 0,\ otherwise. \end{cases} \tag{5}$$

Where $\mu_{x_0,(p)}^{(0)}$ is the moment of $x_0 \in 4\_QAM_{1,0}$

We note that the unbiased estimator of $\mu_{x,(p)}^{(0)}$ over N samples is given by the following equation,

$$\hat{\mu}_{x,(p)}^{(0)} = \frac{1}{N}\sum_{i=0}^{N-1} x_i^P\,. \tag{6}$$

TABLE I. P^{TH} ORDER MOMENT WITH P = 14, 8 AND 12

	$\boldsymbol{\mu_{x,(4)}^{(0)}}$	$\boldsymbol{\mu_{x,(8)}^{(0)}}$	$\boldsymbol{\mu_{x,(12)}^{(0)}}$
4-QAM	-1	1	-1
16-QAM	-0.68	2.203	-8.879
64-QAM	-0.619	1.911	-8.662
128-QAM	-0.181	-0.653	4.488

C. Blind Channel Identification Algorithm

The present blind channel identification method exploits the fact that the p^{th} order moment of the *M*-QAM symbols is no-null however the first p^{th} moment of the AWGN is null. Therefore from equations (1 to 5), the frequency channel response at the k^{th} subcarrier is estimated as follows,

$$\tilde{h}_k e^{j\frac{2l\pi}{p}} = \hat{h}_k = \left(\frac{1}{N}\frac{\sum_{i=0}^{N-1} r_{i,k}^p}{\mu_{s,(p)}^{(0)}}\right)^{\frac{1}{p}},\quad l \in \mathbb{N}. \tag{7}$$

With, $\mu_{s,(p)}^{(0)}$ is given by equation (5) and $e^{j\frac{2l\pi}{p}}$ is the ambiguity resulting from the p^{th} root of 1, like $e^{j\frac{2l\pi}{p}} = \sqrt[p]{1}$. This ambiguity term is assumed well corrected with dedicated algorithm [13].

D. Estimation error derivation

Merging equation (1) into (7), we obtain,

$$\hat{h}_k = \left(h_k^p + \frac{1}{\mathrm{N}\mu_{s,(p)}^{(0)}} \sum_{i=0}^{\mathrm{N}-1} \sum_{m=1}^{p} \binom{p}{m} h_k^{p-m} s_{i,k}^{p-m} \mathrm{n}_{i,k}^m \right)^{\frac{1}{p}}. \quad (8)$$

By setting $\varepsilon_{h_k}^{(p)} = \frac{1}{\mathrm{N}\mu_{s,(p)}^{(0)}} \sum_{i=0}^{\mathrm{N}-1} \sum_{m=1}^{p} \binom{p}{m} h_k^{p-m} s_{i,k}^{p-m} \mathrm{n}_{i,k}^m$ equation (8) becomes,

$$\hat{h}_k = \left(h_k^p + \varepsilon_{h_k}^{(p)} \right)^{\frac{1}{p}} = h_k \left(1 + \frac{\varepsilon_{h_k}^{(p)}}{h_k^p} \right)^{\frac{1}{p}}. \quad (9)$$

Consider that $\left| \frac{\varepsilon_{h_k}^{(p)}}{h_k^p} \right| < 1$, this assumption may be widely admitted since we consider that the element of noise $\mathrm{n}_{\mathrm{i,k}}^{\mathrm{m}}$ is AWGN and this leads to consider $\sum_{\mathrm{m}=1}^{\mathrm{p}} \binom{\mathrm{p}}{\mathrm{m}} \mathrm{h}_{\mathrm{k}}^{\mathrm{p-m}} \mathrm{s}_{\mathrm{i,k}}^{\mathrm{p-m}} \mathrm{n}_{\mathrm{i,k}}^{\mathrm{m}}$ also as AWGN. We know in this case that the first moment of an AWGN is null, therefore $\mathrm{E}\left[\sum_{\mathrm{m}=1}^{\mathrm{p}} \binom{\mathrm{p}}{\mathrm{m}} \mathrm{h}_{\mathrm{k}}^{\mathrm{p-m}} \mathrm{s}_{\mathrm{i,k}}^{\mathrm{p-m}} \mathrm{n}_{\mathrm{i,k}}^{\mathrm{m}} \right] = 0$. In conclusion, $\left| \sum_{i=0}^{\mathrm{N}-1} \sum_{m=1}^{p} \binom{p}{m} h_k^{p-m} s_{i,k}^{p-m} \mathrm{n}_{i,k}^m \right| \ll 1$, when $\mathrm{N} \gg 1$.

Therefore equation (9) can be approximated by the first order Taylor series [14], as given by the following equation,

$$\hat{h}_k \cong h_k + \frac{1}{p h_k^{p-1}} \varepsilon_{h_k}^{(p)}. \quad (10)$$

By setting, $\varepsilon_{h_k} = \frac{1}{p h_h^{p-1}} \varepsilon_{h_k}^{(p)}$, the last equation becomes,

$$\hat{h}_k \cong h_k + \varepsilon_{h_k}. \quad (11)$$

With ε_{h_k} is the channel estimation error. Using equation (8) we can write:

$$\varepsilon_{h_k} h_k^{p-1} = \frac{1}{p\mathrm{N}\mu_{s,(p)}^{(0)}} \sum_{i=0}^{\mathrm{N}-1} \sum_{m=1}^{p} \binom{p}{m} h_k^{p-m} s_{i,k}^{p-m} n_{i,k}^m. \quad (12)$$

To evaluate the channel estimation accuracy we have to calculate error power defined as $\mathrm{E}\left[\left| \varepsilon_{h_k} \right|^2 \right]$. For this aim and to reduce calculation complexity, we admit some assumptions:

First we consider that $\sum_{i=0}^{\mathrm{N}-1} h_k^{1-j_1} s_{i,k}^{p-m_1} n_{i,k}^{m_1}$ and $\sum_{i=0}^{\mathrm{N}-1} h_k^{1-m_2} s_{i,k}^{p-m} n_{i,k}^{m_2}$ are independent for $m_1 \neq m_2$. We also assume that h_k^{p-m} and $n_{i,k}^m$ are not correlated and ε_{h_k} and h_k^{p-1} are independent. Therefore,

$$\mathrm{E}\left[\left| \varepsilon_{h_k} \right|^2 \right] = \frac{1}{\left(p\mathrm{N}\mu_{s,(p)}^{(0)} \right)^2} \frac{\sum_{m=1}^{p} \binom{p}{m}^2 \mathrm{E}[|s^{p-m}|^2] \mathrm{E}\left[\left| h_k^{p-m} \right|^2 \right] \mathrm{E}\left[\left| \sum_{i=0}^{\mathrm{N}-1} n_{i,k}^m \right|^2 \right]}{\mathrm{E}\left[\left| h_h^{p-1} \right|^2 \right]}. \quad (13)$$

In the sequel, we focus on the derivation of this estimation error in the case of Rayleigh propagation channel. This latter can be written as $h_k = h_{Rk} + j h_{Ik}$ with, $(h_{Rk}, h_{Ik}) \sim \mathcal{N}\left(0, \frac{\sigma_h^2}{2} \right)$. Following the system model defined by equation (2), the noise is also written as $n_{i,k} = n_{Ri,k} + j n_{Ii,k}$ and $(n_{Ri,k}, n_{Ii,k}) \sim \mathcal{N}\left(0, \frac{\sigma_n^2}{2} \right)$. Since $n_{i,k}^m$ is an uncorrelated centered complex Gaussian variable of variance σ_n^2 then,

$$\mathrm{E}\left[\left| \sum_{i=0}^{\mathrm{N}-1} n_{i,k}^m \right|^2 \right] = \sum_{i=0}^{\mathrm{N}-1} \mathrm{E}\left[\left| n_{i,k}^m \right|^2 \right]. \quad (14)$$

Based on [15] the expectation $\mathrm{E}\left[\left| n_{i,k}^m \right|^2 \right]$ is expressed according to the following equations,

$$E\left[\left| n_{i,k}^m \right|^2 \right] = \sum_{l=0}^{m} \binom{m}{l} E\left[n_{Ri,k}^{2l} \right] E\left[n_{Ii,k}^{2(m-l)} \right]. \quad (15)$$

This equation can be rewritten also as,

$$E\left[\left| n_{i,k}^m \right|^2 \right] = \sum_{l=0}^{m} \binom{m}{l} \mu_{|2l|} \mu_{|2(m-l)|}. \quad (16)$$

With, $\mu_{|q|}$ is the central absolute q^{th} moment of a real random Gaussian variable $n_{Ri,k}$. $\mu_{|q|}$ is given as follows [16, 17],

$$\mu_{|q|} = \sigma_n^q \frac{\Gamma\left(\frac{q+1}{2} \right)}{\sqrt{\pi}}. \quad (17)$$

Thus,

$$\mathrm{E}\left[\left| \sum_{i=0}^{N-1} n_{i,k}^m \right|^2 \right] = \mathrm{N} \sum_{l=0}^{m} \binom{m}{l} \sigma_n^{2m} \frac{\Gamma\left(\frac{2l+1}{2} \right) \Gamma\left(\frac{2(m-l)+1}{2} \right)}{\pi}. \quad (18)$$

Similarly, $\mathrm{E}[|h_k^m|^2]$ is obtained,

$$\mathrm{E}\left[\left| h_k^{p-m} \right|^2 \right] = \sum_{l=0}^{p-m} \binom{p-m}{l} \frac{\Gamma\left(\frac{2l+1}{2} \right) \Gamma\left(\frac{2(p-m-l)+1}{2} \right)}{\pi}. \quad (19)$$

Finally and using equation (18), (19) and Table I, estimation error can be written as equation (20) (at the end of the sixth page of this article).

3. Numerical Analysis

For simulations, we consider an OFDM transmission. The studied *M*-QAM constellations are *16*-QAM and *64*-QAM. For each constellation, we investigated the encoded Bit Error Rate BER and Normalized Mean

Square Error $NMSE = \frac{\|h-\hat{h}\|^2}{\|h\|^2}$. We evaluated the performance on blind channel estimation as function of *p*, the moment order and *N*, the number of OFDM symbol used for channel estimation. The theoretical result obtained in equation (20) is compared to Monte Carlo simulation. The channel modulus is chosen as a Rayleigh random complex value drawn each iteration.

Figure 2 shows the NMSE respect to Eb/N0 for 16-QAM constellation and using N = 64 OFDM symbols for channel estimation. NMSE is estimated theoretically and with simulations for 4^{th}, 8^{th} and 12^{th} order moment channel estimation algorithm.

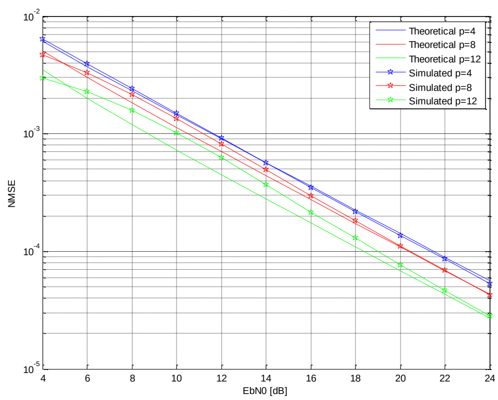

Figure 2. NMSE vs Eb/N0 for 16-QAM and 64 OFDM symbols.

As illustrated in figure. 2 the channel estimation error obtained in equation (20) is in good agreement with the simulated result whatever the moment order. Furthermore, the estimation error does not significantly decrease when the moment order increases.

Regarding BER, Figure 3 shows it as a function of Eb/N0. The simulation results are obtained for a 16-QAM constellation, and using N = 64 OFDM symbols for channel estimation. The BER is steel following the estimated error power given by the equation (20) and illustrated by the figure 2 at high Eb/N0. This joined the hypothesis token in section III of the high number of the averaged OFDM symbols to justify the validity of the Taylor linearization.

However, when the number of averaged OFDM symbols is not sufficient to guarantee this hypothesis therefore we observe a divergence in results with the predicted estimation error power.

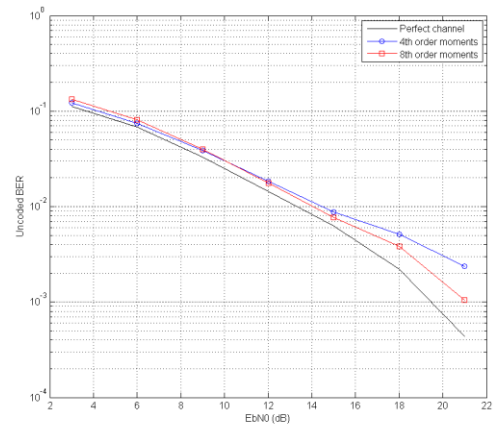

Figure 3. BER vs Eb/N0 for 16-QAM and 64 OFDM symbols.

We investigated also in this paper 4-QAM transmission. Figures 4 and 5 shown respectively NMSE and BER with 16 OFDM symbols used for the channel estimation.

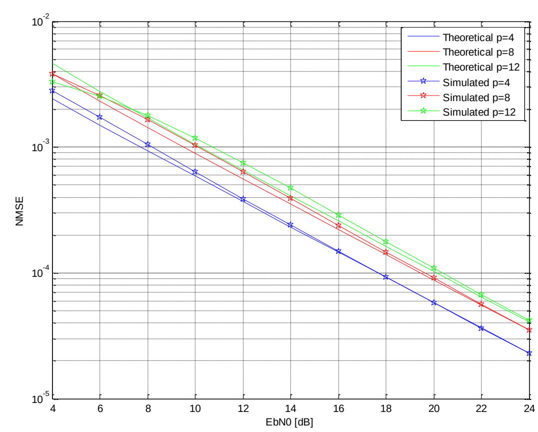

Figure 4. NMSE vs Eb/N0 for 64-QAM and 128 OFDM symbols.

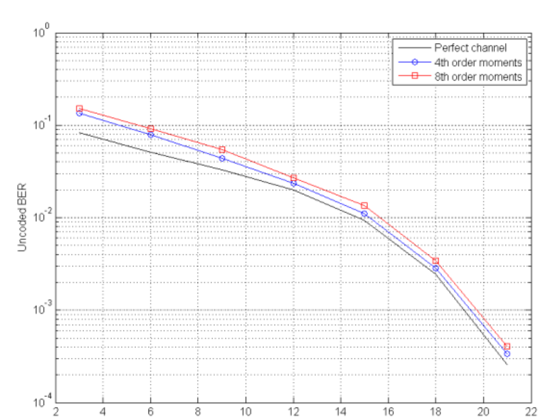

Figure 5. BER vs Eb/N0 for 4-QAM and 16 OFDM symbols.

From figure 4, we can easily see that a significant degradation is bringing by increasing the moment order in channel estimation. The fourth order moment algorithm performs better than the others in term of theoretical NMSE illustrated by figure 4 or uncoded BER given in figure 5.

Finally, the estimation error is widely acceptable in both studied scenarios with small number of OFDM symbols witch is suitable for fast varying channel estimation in 4-QAM.

In light of these results concerning NMSE and BER, it is difficult to make a choice to use a moment order despite other for a given QAM modulation order. For 16-QAM a high order seems more appropriate while for 4-QAM it is the turn of a weak order to give a better result. However, in all cases, the different moment orders provide nearly similar performance.

In section III.A, we introduced the ambiguity term resulting from the p^{th} root of 1, like $e^{j\frac{2l\pi}{p}} = \sqrt[p]{1}$.

A. Removing the $\frac{2l\pi}{p}$ Ambiguity

An overview of the inter subcarrier ambiguity removing which is included [13] in given below.

Consider an OFDM waveform transmission with an OFDM symbol length equal to K and a guard interval length of L then, we can write:

$$\boldsymbol{h} = \boldsymbol{F}_{K,L}\boldsymbol{a}. \quad (21)$$

Where:

$\boldsymbol{F}_{K,L}$ is the discrete Fourier transform matrix with K lines and L first columns. The elements k, l of the matrix $\boldsymbol{F}_{K,L}$ are given by the following formula:

$$\mathrm{F}_{\mathrm{K,L}}(\mathrm{k},\mathrm{l}) = \frac{1}{\sqrt{\mathrm{K}}}\exp\left(-\mathrm{j}\frac{2\pi\mathrm{kl}}{\mathrm{K}}\right), 0 \le \mathrm{k} < K \text{ and } 0 \le l < L$$

$\boldsymbol{a}$ is the finite impulse response of the propagation channel represented by the $(L \times 1)$ vector:

$$\boldsymbol{a} = \begin{pmatrix} a_0 \\ \vdots \\ a_{L-1} \end{pmatrix}.$$

Consider the vector $\boldsymbol{v}$ of the discrete Fourier transform of the propagation channel h:

$$\boldsymbol{F}_{K,K}^{H}\boldsymbol{h} = \boldsymbol{v}. \quad (22)$$

Where $\boldsymbol{F}_{\boldsymbol{K},\boldsymbol{K}}$ is the discrete Fourier transform matrix and $\boldsymbol{v}$ is a $(K \times 1)$ vector obtained by the concatenation of the $\boldsymbol{a}$ vector and a $(K-L \times 1)$ null vector:

$$\boldsymbol{v} = \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{0} \end{pmatrix}. \quad (23)$$

We introduce the $(K \times K)$ diagonal $\boldsymbol{I}_{\hat{h}}$ matrix defined as $diag(\boldsymbol{I}_{\hat{h}}) = \widehat{\boldsymbol{h}}$. The ambiguity correction can then be obtained by the identification of the vector $\boldsymbol{c}$:

$$\boldsymbol{c} = \begin{pmatrix} c_0 \\ \vdots \\ c_{K-1} \end{pmatrix}, \forall i \in [0 \cdots K-1], c_i \in e^{j\frac{2l\pi}{p}}, l = 1 \cdots p. \quad (24)$$

Such that:

$$\boldsymbol{I}_{\hat{h}}\boldsymbol{c} = \boldsymbol{h} + \boldsymbol{\varepsilon}_h. \quad (25)$$

The $\boldsymbol{c}$ vector has to represent the conjugates of the ambiguity terms on each subcarrier and $\boldsymbol{\varepsilon_h}$ stands for an estimation noise.

Left multiplying equation (25) by $\boldsymbol{F}_{K,K}^{H}$ and taking into account equation (22) we arrive to:

$$\boldsymbol{F}_{K,K}^{H}\boldsymbol{I}_{\hat{h}}\boldsymbol{c} = \boldsymbol{v} + \boldsymbol{\varepsilon}_v. \quad (26)$$

Notice that: $\boldsymbol{\varepsilon}_v = \boldsymbol{F}_{K,K}^{H}\boldsymbol{\varepsilon}_h$

Equation (26) can be written as following:

$$\boldsymbol{Bc} = \boldsymbol{v} + \boldsymbol{\varepsilon}_v. \quad (27)$$

$\boldsymbol{B}$ is given by: $\boldsymbol{B} = \boldsymbol{F}_{K,K}^{H}\boldsymbol{I}_{\hat{h}}$. This matrix can itself be split in two matrices namely a $(L \times K)$ matrix $\boldsymbol{B}_1$ and in a $(K-L \times K)$ matrix $\boldsymbol{B}_2$ defined as follows:

$$\begin{bmatrix} \boldsymbol{B_1} \\ \boldsymbol{B_2} \end{bmatrix}\boldsymbol{c} = \begin{bmatrix} \boldsymbol{a} + \boldsymbol{\varepsilon_{v1}} \\ \boldsymbol{\varepsilon_{v2}} \end{bmatrix}. \quad (28)$$

Like the matrix $\boldsymbol{B}$, the error vector $\boldsymbol{\varepsilon}_v$is split as following:

$\boldsymbol{\varepsilon}_v = \begin{bmatrix} \boldsymbol{\varepsilon_{v1}} \\ \boldsymbol{\varepsilon_{v2}} \end{bmatrix}$, the dimensions of $\boldsymbol{\varepsilon_{v1}}$ and $\boldsymbol{\varepsilon_{v2}}$ are $L \times 1$ and $K-L \times 1$ respectively.

Assuming the hypothesis of low rank channel, the length of the channel impulse response is at most equal to L so as not to exceed the guard interval of the OFDM symbol. With this hypothesis, we can consider that the $\boldsymbol{c}$ vector has to minimize the quadratic 2-norm of the $(K-L \times 1)$ $\boldsymbol{\varepsilon}_{v2} = \boldsymbol{B}_2\boldsymbol{c}$ vector. We will then take in consideration the following criterion:

$$\boldsymbol{c} = \arg_{\boldsymbol{c}} \min \left(\|\boldsymbol{\varepsilon_{v2}}\|^2\right). \quad (29)$$

In order to avoid the trivial null solution for $\boldsymbol{c}$ and to be compliant with equation (24), we introduce the constraint $\|\boldsymbol{c}\|^2 = K$. The $\boldsymbol{c}$ vector is then proportional to the eigenvector associated to the minimal eigenvalue of the $(K \times K)$ $\boldsymbol{B}_2^H\boldsymbol{B}_2$ matrix.

Nevertheless, by construction we have $rank(\boldsymbol{B}_2^H\boldsymbol{B}_2) = K-L$, hence, we have $\boldsymbol{c} \in$ ker $(\boldsymbol{B}_2)$ meaning that this vector belongs to the space spanned by eigenvectors linked to null eigenvalues of $\boldsymbol{B_2^H}\boldsymbol{B}_2$.

In order to find a solution, we propose to reduce the dimension of the $\boldsymbol{B}_2^H\boldsymbol{B}_2$ matrix. For that purpose, we propose to discard the L smallest (modulus) values of the

$\widehat{\boldsymbol{h}}$ vector. Discarding these L smallest components, we obtain a new $(K-L\times 1)$ vector $\widehat{\boldsymbol{h}}'$.

Following then same steps as previously, we introduce the $(K-L\times K-L)$ diagonal $\boldsymbol{I}_{\widehat{h}'}$ matrix defined as: $diag(\boldsymbol{I}_{\widehat{h}'}) = \widehat{\boldsymbol{h}}'$ and the $(K\times K-L)$ $\boldsymbol{B}'$ matrix defined as: $\boldsymbol{B}' = \boldsymbol{F}^{H}_{K,K-L}\boldsymbol{I}_{\widehat{h}'}$. The $\boldsymbol{B}'$ matrix can itself be split in a $(L\times K-L)$ $\boldsymbol{B}'_1$ matrix and in a $(K-L\times K-L)$ $\boldsymbol{B}'_2$ matrix. Equation (18) becomes then:

$$\begin{bmatrix}\boldsymbol{B}'_1\\ \boldsymbol{B}'_2\end{bmatrix}\boldsymbol{c}' = \begin{bmatrix}\boldsymbol{a}' + \boldsymbol{\varepsilon}'_{v1}\\ \boldsymbol{\varepsilon}'_{v2}\end{bmatrix}. \quad (30)$$

And the $\boldsymbol{c}'$ vector is now defined as the minimum eigenvector of the $\boldsymbol{B}'^{H}_2\boldsymbol{B}'_2$ matrix multiplied by $\sqrt{K-L}$. $\boldsymbol{c}'$ is then viewed as an ambiguity plus noise multiplicative correction vector, but we have to constraint it to be in the set $\left\{c_i \in e^{j\frac{2l\pi}{p}}, l = 1 \cdots p\right\}$. It can be done by a Maximum Likelihood (ML) demodulation. In the proposed algorithm each components of $\boldsymbol{c}'$ are just rounded to the closest neighbor among: $\left\{c_i \in e^{j\frac{2l\pi}{p}}, l = 1 \cdots p\right\}$. Once $\boldsymbol{c}'$ obtained, the final step of the algorithm leads to estimate the channel impulse response. From equation (30) we write:

$$\widehat{\boldsymbol{a}} = \boldsymbol{B}'_1\boldsymbol{c}'. \quad (31)$$

Once $\widehat{\boldsymbol{a}}$ estimated, the channel frequency estimation, including intrinsically the smoothing stage is finally given by:

$$\widehat{\widehat{\boldsymbol{h}}} = \boldsymbol{F}_{K,L}\widehat{\boldsymbol{a}}. \quad (32)$$

We have to notice that the main numerical complexity resides in the minimal eigenvector estimation, other steps being relatively easy to implement on hardware targets.

In term of overall error probability of the blind estimation of the propagation channel performed with the presented algorithm, can be given by the product of the error probability estimated from $\left|\boldsymbol{\varepsilon}_{h_k}\right|^2$ obtained in the equation (20) and the power probability of the inter subcarrier ambiguity resolving derivated from the quantity $|\boldsymbol{c}-\boldsymbol{c}'|^2$.

As we can observe the vector $\boldsymbol{c}$ is composed of the elements $c_i \in e^{j\frac{2l\pi}{p}}, l = 1 \cdots p$. Globally we can easily estimate the relative error probability of the ambiguity resolving of the use of an $M-QAM$ compared to the use of a $M'-QAM$. This relative error probability is inverse proportional to the ratio of the minimum distance of the elements of the $M-QAM$ to the minimum distance of the elements of the $M'-QAM$. Therefore, we can conclude that for p = 4, the error probability of the ambiguity $e^{j\frac{l\pi}{2}}$ resolving will be 3dB less than the error probability of the ambiguity $e^{j\frac{l\pi}{4}}$ in the case of p = 8. In another hand computing 4th-root is less complex than computing 8th-root or 12th-root. In addition of the ambiguity resolving power error, taking in consideration the low complexity of channel estimation algorithm, 4th order moment is more suitable than the higher moments.

B. Simulation results of the 4th order moment estimation algorithms versus some well-known algorithms

Before threating of performance results algorithms obtained by simulation of the 4th order moment estimation algorithm and some well-known other blind estimation algorithms, we have simulated the error probability versus NMSE of the $\frac{\pi}{2}$ ambiguity resolving algorithm discussed in section A. the simulations are performed in the case of use of the 4th order moment algorithm. Figure 5 shows this error probability in the case of an OFDM waveform transmission with a guard interval length *L=4* and *L = 8*.

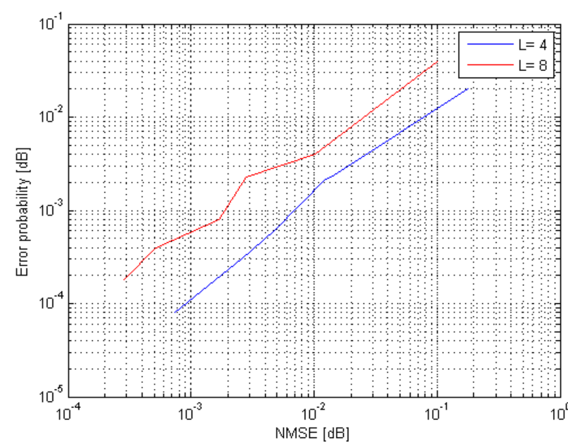

Figure 5. Error probability of $\frac{\pi}{2}$ ambiguity resolving versus NMSE.

The results showed by figures 6 and 7 are obtained by simulating the 4th order moment algorithm, the subspace based algorithm [6] and the autocorrelation matrix based algorithm for 4-QAM and 16-QAM modulation consecutively in the propagation conditions described by Hilly terrain channel [20]. The generation of the propagation channel is done according to the Jakes Model [21]. The number of averaged OFDM symbols is $N = 200$. We have obtained approximatively the same performance for our algorithm and the subspace based algorithm.

$$\left[\left|\varepsilon_{h_k}\right|^2\right] \cong \frac{1}{N\pi\left(p\mu^{(0)}_{x,(p)}\right)^2}\frac{\sum_{m=1}^{p}\left[\binom{p}{m}^2 E[|x^{p-m}|^2]\left(\sum_{l=0}^{p-m}\binom{p-m}{l}\Gamma\left(\frac{2l+1}{2}\right)\Gamma\left(\frac{2(p-m-l)+1}{2}\right)\right)\left(\sum_{l=0}^{m}\binom{m}{l}\sigma_n^{2m}\Gamma\left(\frac{2l+1}{2}\right)\Gamma\left(\frac{2(m-l)+1}{2}\right)\right)\right]}{\sum_{l=0}^{p-1}\binom{p-1}{l}\Gamma\left(\frac{2l+1}{2}\right)\Gamma\left(\frac{2(p-1-l)+1}{2}\right)}. \quad (20)$$

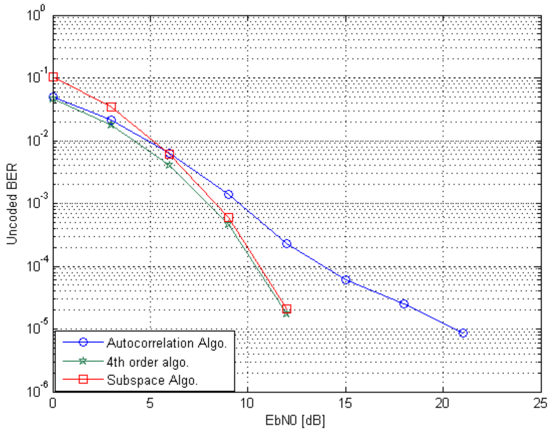

Figure 6. BER comparison vs EbN0 of blind estimation algorithms in the case of 4-QAM and HT propagation channel

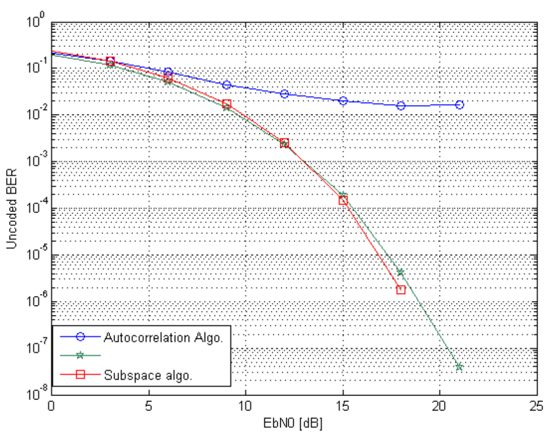

Figure 7. BER comparison vs EbN0 of blind estimation algorithms in the case of 16-QAM and HT propagation channel

4. Conclusion

We proposed in this paper a comparison between different orders of moment used for high order moments blind channel estimation. We derived a theoretical expression of estimation error power which is function of the QAM modulation order, the moment order and the number of OFDM symbols used to estimate the channel. We notice that this expression can be applied also to the FBMC waveform which structurally well accepts the high order moment estimation as showed in [18].

This theoretical value was compared to simulation results with different Eb/N0 and shows a good agreement. Furthermore, obtained results show that 4^{th}, 8^{th} order moment estimation algorithms exhibit outperformance of the 4^{th} order moments performance in term of BER for the 4-QAM and better performance at high Eb/N0 for the 8^{th} order moments and 8-QAM configuration. These results comfirm the theritical expression of the estimation error given by (20). A slightly different performance on NMSE is observed. This latter is too small to prefer an order to another one. However based on the complexity of the algorithm it seems clear that forth order is the good trade-off in blind channel estimation.

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Dr. Rabah Maoudj He received the Eng. Degree, the Master degree from CNAM and the Ph.D. in Signal Processing for Communications from EDITE Paris in 2004, 2010 and 2015, respectively. Between 2005 and 2010, he works as an R&D senior engineer in different industrial companies such as SAGEM and EADS. His research interests include statistical signal processing, MIMO receiver architecture and interference cancellation algorithms.

MSc. Juan Medeiros He received the Degree in Mathematics from the Federal University of Paraíba (2001) and MA in mathematics from the Federal University of Paraíba (2001) in Brazil. He was a professor at the Federal Institute of Education, Science and Technology of Paraíba (2005-2008, Cajazeiras - PB, Brazil). Since 2008 he is professor at the Federal University of Ceará linked to the Course of Electrical Engineering (Sobral - CE, Brazil). Has experience in the area of Mathematics (higher education and graduate-level specialization). He was a PhD student of the Programme Postgraduate in Teleinformatics Engineering, UFC / CT / PPGETI, having realized in 2014 doctoral sandwich (PDSE / CAPES) and co-tutelle at the Conservatoire National des Arts et Métiers - Le CNAM, Paris - France, with emphasis on the study of the use of quaternions and manifolds (Grassmannianas and Stiefel) in theory bias signals to digital transmission.

Dr. Michel Terré He received the engineering degree from the Institut National des Telécommunications, Evry, France, in 1987, the Ph.D. degree in signal processing from the Conservatoire National des Arts et Métiers - Le CNAM, Paris, France, in 1995, and the Habilitation à Diriger des Recherches (HDR) from the University Paris XIII, Villetaneuse, France in 2004. He first had an industrial career in R&D, mostly in the field of radiocommunications working successively for TRT Philips, Thalés Communications and Alcatel. He is currently professor at the Conservatoire National des Arts et Métiers, Paris. Pr. TERRE is a fellow member of SEE (Société de l'électricité, de l'électronique et des technologies de l'information et de la communication), he was associate editor of the Annals of Telecommunications in 2008 and Technical Program Chairman of the European Wireless Conference in 2007.

E-mail: rabah.maoudj@univ-paris13.fr, juca@ufc.br, terre@cnam.fr