A Periodic Review Deterministic Inventory Model with Exponential Rate of Demand for Deteriorating Items and Partial Backlogging

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Abstract: In this paper, a Periodic review deterministic inventory model for deteriorating items with Exponential rate of demand is considered. The model is developed on the basis of constant rate of deteriorating item with shortages and the demand is partially backlogged. The aim of this paper is to find the optimal time to order by minimizing the total inventory cost. The model is illustrated numerically and the sensitivity analysis is also carried out with percentage changes in the parameters.

1. INTRODUCTION

In the traditional inventory models, one of the basic assumptions is that the items preserved their physical characteristics while they are stored in the inventory. This assumption is evidently true for most items, but not for all. However, the deteriorating items are subject to a continuous loss in their masses or utility throughout their lifetime due to decay, damage, spoilage or of other reasons. Owing to this fact, controlling and maintaining the inventory of deteriorating items becomes a challenging problem for decision makers.

Harris (1915) has developed the first inventory model on Economic Order Quantity, which was generalized by Wilson (1934). Later, Whitin (1957) has considered an inventory model for the deterioration of the fashion goods at the end of the prescribed shortage period. Ghare and Schrader (1963) have developed a model for an exponentially decaying inventory. Dave and Patel (1981) have studied a deteriorating inventory model with linearly increasing demand and no shortages. Wee (1995) has studied an deterministic inventory model for declining market with shortages. Later, Chang and Dye (1999) have developed an inventory model with time-varying demand and partial backlogging. In the 21st century, Goyal and Giri (2001) have classified the inventory models on the basis of demand variations and various other conditions or constraints related to it. Dye et al. (2007) have developed a deterministic inventory model for lot size with a varying rate of deterioration and exponential partial backlogging to find an optimal selling price. They have also assumed that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases.

Recently Roy(2008) has developed a time dependent deterministic inventory model where the deteriorating rate is time proportional, the demand rate is a function of selling price and holding cost is time dependent. Also, Liao (2008) gave an economic order quantity (EOQ) model with non instantaneous receipt for exponentially deteriorating item under two level of trade credits There are several models developed using Weibull rate of deterioration instead of constant rate and one such model is give by Skouri et al. (2009). He has also assumed a ramp type of demand rate with partial backlogging to find the optimal number of orders.

Another model is given by Mandal (2010) for an EOQ inventory model with Weibull-distributed deteriorating items under ramp-type demand and shortages. Mishra and Singh (2011a, b) have considered an inventory model for

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ramp-type demand, time-dependent deteriorating items with salvage value and time dependent holding cost. Hung (2011) gave an inventory model with generalized-type of demand, deterioration, and backorder rates.

In classical inventory models, the demand rate and holding cost is assumed to be constant but in reality the demand and holding cost for physical goods are time dependent. Time also plays an important role in the inventory system and therefore there are models available in the literature for time dependent rates. The work of Roy (2008) is more realistic by considering demand rate and holding cost as linear functions of time and developed an inventory model where shortages are allowed and partially backlogged

In this paper, a periodic review deterministic inventory model for constant rate deteriorating items with shortages is considered. The model assumes that the demand rate is exponential land partially backlogged. The aim of this paper is to develop an optimal policy by minimizing the total inventory cost and illustrated numerically. The sensitivity analysis is also carried out for the model with percentage change in the parameters.

2.   ASSUMPTION AND NOTATIONS

The following assumptions are used to develop mathematical model

1. Deterioration rate is constant
2. The rate of deterioration; $0 < \theta < 1$ constant rate of deterioration
3. Demand is time dependent and exponential $D(t) = e^{-\alpha t}$, $\alpha > 0$
4. The inventory level is depleted due to the deterioration and demand of item
5. Shortages are allowed
6. There is no repair or replenishment of deteriorating item during the period under consideration
7. Replenishment is instantaneous and the lead time is assumed to be zero

The following notations are used to develop mathematical model

T: The length of the cycle
I_0: The maximum inventory level during (0, T)
I_1: Inventory level during the positive stock period
I_2: Inventory level during shortage period
C_1 : Holding cost per unit/per unit time
C_2 : Shortage cost per unit/per unit time
C_3 : Ordering cost per unit/per unit time

3.   DESCRIPTION OF THE MODEL

In this model, a deterministic periodic review inventory model is considered with time dependent demand rate which is assumed to be exponential. The items are said to deteriorate at a constant rate.
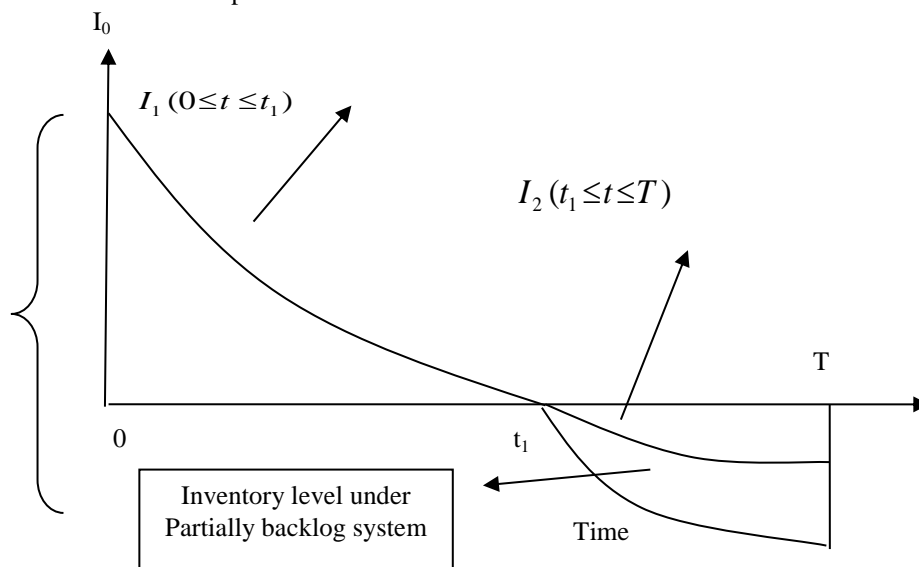

Figure 1. Inventory Model with shortages

From Fig. 1 it is seen that the amount of inventory during time $(0,t_1)$ the inventory level is depleted due to deterioration and demand. Let $I_1(t)$ be the instantaneous inventory level at time $t(0 \leq t < t_1)$ then the inventory level is governed by the following differential equation

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -e^{-\alpha t} \tag{1}$$

The solution of the above equation (1) and by using the initial condition $t = 0$, $I_1(0) = I_0$ to get $c = I_0$ and the inventory level is given by

$$I_1 = \int_0^{t_1} I_1(t)dt \tag{2}$$

Thus,

$$I_1 = \frac{\theta t_1^3}{6} + \frac{\alpha t_1^3}{6} - \frac{t_1^2}{2} - \frac{\alpha \theta t_1^4}{8} - \frac{I_0 \theta t_1^2}{2} \tag{3}$$

The inventory during the shortage period (t_1, T) is governed by the following differential equation

$$\frac{dI_2(t)}{dt} = -e^{-\alpha t} dt \tag{4}$$

The solution of the above equation (4) using the boundary condition $t = 0, I_2(0) = 0$ to get $c = 0$ and the inventory level is given by

$$I_2 = \int_{t_1}^{T} I_2(t)dt \tag{5}$$

Thus,

$$I_2 = \left[\frac{-1}{2}[T^2 - t_1^2] + \frac{\alpha}{6}[T^3 - t_1^3] \right] \tag{6}$$

Therefore,

Holding cost is given as $C_1 \left[\dfrac{\theta t_1^3}{6} + \dfrac{\alpha t_1^3}{6} - \dfrac{t_1^2}{2} - \dfrac{\alpha \theta t_1^4}{8} - \dfrac{I_0 \theta t_1^2}{2} \right]$ (7)

Shortage cost is given as $C_2 \left[\dfrac{-1}{2}[T^2 - t_1^2] + \dfrac{\alpha}{6}[T^3 - t_1^3] \right]$ (8)

and Ordering cost$= C_3$ (9)

The average total cost is the sum of the holding cost, shortage cost and ordering cost and is given by
Total cost TC=Holding cost + Shortage cost + ordering cost / T

$$TC = \frac{C_1 \theta t_1^2}{6T} + \frac{C_1 \alpha t_1^3}{6T} - \frac{C_1 t_1^2}{2T} - \frac{C_1 \alpha \theta t_1^4}{8T} - \frac{C_1 I_0 \theta t_1^2}{2T} - \frac{C_2 T}{2} + \frac{C_2 t_1^2}{2T} + \frac{C_2 \alpha T^2}{6} - \frac{C_2 \alpha t_1^3}{6T} + \frac{C_3}{T}. \tag{10}$$

By Using calculus, to minimize the average total cost for finding the optimal values of t_1 and T are the solutions of the equation (10) gives

$$\frac{\partial TC}{\partial t_1} = 0 \quad and \quad \frac{\partial TC}{\partial T} = 0$$

With the sufficient conditions as

$$\frac{\partial^2 TC}{\partial t_1^2} > 0, \ \frac{\partial^2 TC}{\partial T^2} > 0 \ and \ \left(\frac{\partial^2 TC}{\partial t_1^2} \right) \left(\frac{\partial^2 TC}{\partial T^2} \right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T} \right)^2 > 0$$

Therefore,

$$\frac{\partial TC}{\partial t_1} = \frac{2C_1 \theta t_1}{6T} + \frac{3C_1 \alpha t_1^2}{6T} - \frac{2C_1 t_1}{2T} - \frac{4C_1 \alpha \theta t_1^3}{8T} - \frac{2C_1 I_0 \theta t_1}{2T} + \frac{2C_2 t_1}{2T} - \frac{3C_2 \alpha t_1^2}{6T} = 0 \tag{11}$$

$$\frac{\partial TC}{\partial T} = \frac{-C_1\theta t_1^2}{6T^2} - \frac{C_1\alpha t_1^3}{6T^2} + \frac{Ct_1^2}{2T^2} + \frac{C_1\alpha\theta t_1^4}{8T^2} + \frac{C_1 I_0\theta t_1^2}{2T^2} - \frac{C_2}{2} - \frac{C_1 t_1^2}{2T^2}$$
$$+ \frac{2C_2\alpha T}{6} + \frac{C_2\alpha t_1^3}{6T^2} - \frac{C_3}{T^2}.=0 \tag{12}$$

Solving the non-linear equations (11) and (12) by using R- Program, the optimal time periods t_1^* and T^* are found. Using the values of t_1^* and T^* the optimal total cost is obtained.

4. NUMERICAL ILLUSTRATION

The following numerical values of the parameter in proper units were considered as input for numerical analysis of the model.

$C_1 = 2$, $C_2=0.5$, $C_3=3$, $I_0 =10$, alpha=3, theta=0.01

With the above values the model is solved using the R-program.

Thus, the optimal values of $t_1$and Tare obtained as optimal time period t_1^* is 28.5598 and optimal cycle time T^* is 67.3926

The optimal average total cost as TC* is Rs. 1293.597

5. SENSITIVITY ANALYSIS

The effects of changes in the system parameters α and θ for the optimal time period for inventory level t_1^* and optimal cycle time T^* and optimal total cost TC^* are studied in the model. The sensitivity analysis is performed by changing each of the parameter by +50%, +25%, +10%, -10%, -25%, -50% and the results are shown in Table-1. The graphical representation for percentage changes in the parameters of the optimal time period for inventory level t_1^* is shown in Fig. 2.The optimal cycle time T^* is shown in Fig. 3 and for the optimal total cost is shown in Fig. 4

TABLE I   SENSITIVITY ANALYSIS FOR THE RATE OF CHANGE OF DEMAND AND DETERIORATION

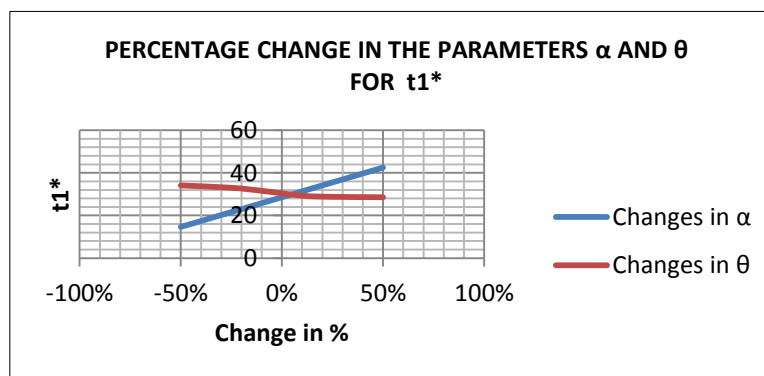| Percentage Changes	Changes in α			Changes in θ		
	t_1^*	T^*	TC^*	t_1^*	T^*	TC^*
+50%	42.4798	65.1654	1529.413	28.5597	66.4521	1461.3332
+25%	35.5198	66.2790	1392.567	28.7598	67.2238	1435.7673
+10%	31.3438	66.9471	1328.207	29.3021	69.4697	1400.9148
-10%	25.7757	67.8380	1265.82	31.5849	70.0025	1366.8842
-25%	21.5997	68.5062	1236.921	32.9998	72.6985	1301.6765
-50%	14.6397	69.6198	1220.613	34.1106	73.2687	1293.595

Figure 2. Percentage change in the parameter α and θ for t_1^*
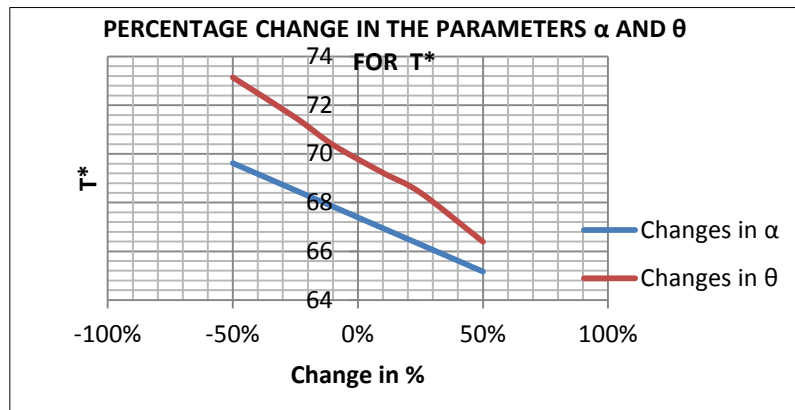
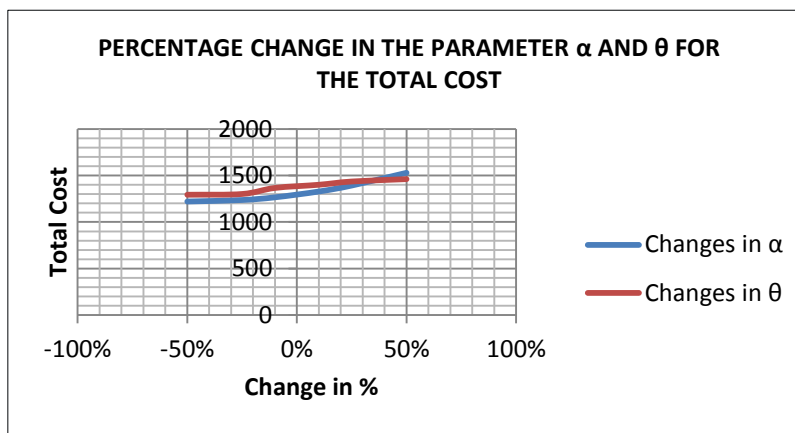**Figure 3. Percentage change in the parameter** α **and** θ **for t***

Figure 4. Percentage change in the parameter α **and** θ **for the total cost**

The following observations are made from the Tables and Figures

1. Table 1, indicates that as the parameter α increases the optimal time period for inventory level t_1^* increases, the optimal cycle time T^* is decreases, and the total cost is increases. It is also seen that as the parameter θ increases, the optimal time period for inventory level t_1^* and the optimal cycle time T^* decreases and the total cost is increases.

2. From Fig. 2, it is seen that the parameter α is highly sensitive whereas the parameter θ is moderately sensitive to percentage changes in the optimal time period for inventory level t_1^*.

3. From Fig. 3, it is observed that the parameter α is highly sensitive and also the parameter θ is highly sensitive to percentage changes in the optimal cycle time T^*.

4. From Fig. 4, it is seen that the parameter α is slightly sensitive and also the parameter θ is slightly sensitive to percentage changes in the optimal total cost.

6. CONCLUSION

This paper presents an inventory model of direct application to the business enterprise that considers the fact that the storage item is deteriorated and time dependent demand rate. In this paper, a deterministic inventory model with time-dependent exponential rate of demand and constant holding cost for deteriorating items with shortages are considered. The optimal time period to order and optimal cycle time are found by minimizing the total inventory cost and it is illustrated numerically. The sensitivity analysis is also carried out for the model with percentage change in the parameters.

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