International Journal of Computational and Theoretical Statistics
ISSN (2210-1519)
Int. J. Comp. Theo. Stat. 7, No. 1 (May-2020)

http://dx.doi.org/10.12785/ijcts/070103

Power Prakaamy Distribution and its Applications

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Received May 20, 2019, Revised Oct 20, 2019 , Accepted February 13, 2020, Published May 1, 2020

Abstract: This In this paper, a two-parameter power Prakaamy distribution (PPKD) has been suggested. The statistical properties including behavior of the density function, moments and moments based measures have been discussed. The hazard rate function and the stochastic order relations of the proposed distribution have been studied. The maximum likelihood estimation has been discussed for estimating its parameters. The usefulness of the proposed distribution among other distributions has been explained by fitting three lifetime datasets and using some goodness-of-fit measures.
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Keywords: Prakaamy distribution, Moments, Hazard rate function, Stochastic order relations, Maximum likelihood, Goodness of fit.

1. Introduction

The Prakaamy distribution introduced by [1] is defined by its probability density function (pdf)

$$f_1(y;\theta)=\frac{\theta^6}{\theta^5+120}\left(1+y^5\right)e^{-\theta y}\quad ;y>0,\ \theta>0 \tag{1.1}$$

$$= p\,g_1(y;\theta)+(1-p)\,g_2(y;\theta) \tag{1.2}$$

where,

$$p=\frac{\theta^5}{\theta^5+120}$$

$$g_1(y;\theta)=\theta e^{-\theta y}\ ;y>0,\theta>0$$

$$g_2(y;\theta)=\frac{\theta^6}{\Gamma(6)}e^{-\theta y}y^{6-1}\ ;y>0,\theta>0$$

The pdf in (1.1) reveals that the Prakaamy distribution is a two - component mixture of an exponential distribution (with scale parameter θ) and a gamma distribution (with shape parameter 6 and scale parameter θ), with mixing proportion $p=\frac{\theta^5}{\theta^5+120}$. [1] has discussed behavior and properties of Prakaamy distribution and its applications for lifetime data from engineering and medical sciences. Its superiority has also been shown over Pranav distribution suggested by [2], Akash distribution proposed by[3], Ishita distribution suggested by [4], Sujatha distribution introduced by [5], Lindley distribution proposed by[6] and exponential distribution. [7] have comparative study on modeling of lifetime data from engineering and biomedical sciences using one parameter Akash, Lindley and exponential distributions. It has been observed that there are some situations where Prakaamy distribution may not be suitable from either theoretical or applied point of view.

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The corresponding cumulative distribution function (cdf) of (1.1) is given by

$$F_1(y;\theta)=1-\left[1+\frac{\theta y\left(\theta^4 y^4+5\theta^3 y^3+20\theta^2 y^2+60\theta y+120\right)}{\theta^5+120}\right]e^{-\theta y}\quad ;y>0,\theta>0 \tag{1.3}$$

Some of important one parameter lifetime distributions along with their pdf and introducers (year) are given in table 1.

TABLE I. THE PDF AND THE CDF OF ONE PARAMETER LIFETIME DISTRIBUTIONS AND THEIR INTRODUCER (YEAR)

Name of the Distribution	**Probability density function (pdf)**	**Introducers (years)**
Pranav	$f(x;\theta)=\frac{\theta^4}{\theta^4+6}\left(\theta+x^3\right)e^{-\theta x};x>0,\theta>0$	Shukla (2019 a) [2]
Ishita	$f(x;\theta)=\frac{\theta^3}{\theta^3+2}\left(\theta+x^2\right)e^{-\theta x}\;;x>0,\theta>0$	Shanker and Shukla (2017 a)[4]
Akash	$f(x;\theta)=\frac{\theta^3}{\theta^2+2}\left(1+x^2\right)e^{-\theta x};x>0,\theta>0$	Shanker (2015)[3]
Lindley	$f(x;\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x};x>0,\theta>0$	Lindley(1958)[6]
Exponential	$f(x;\theta)=\theta e^{-\theta x};x>0,\theta>0$	

. The detailed study regarding statistical properties, estimation of parameter and applications of these one parameter lifetime distributions are available in the respective research papers.

The pdf and the cdf of some two-parameter Power distributions along with their introducers (years) for $x>0,\theta>0,\alpha>0$ are presented in table 2.

TABLE II. THE PDF AND THE CDF OF TWO –PARAMETER DISTRIBUTIONS AND THEIR INTRODUCER (YEAR)

Name of Distribution	**pdf/cdf**	**Introducers(years)**
Power Pranav Distribution (PPD)	$f(x;\theta,\alpha)=\frac{\alpha\theta^4}{\theta^4+6}\left(\theta+x^{3\alpha}\right)x^{\alpha-1}e^{-\theta x^\alpha}$	Shukla (2019 b) [8]
	$F(x;\theta,\alpha)=1-\left[1+\frac{\theta x^\alpha\left(\theta^2 x^{2\alpha}+3\theta x^\alpha+6\right)}{\theta^4+6}\right]e^{-\theta x^\alpha}$	
Power Ishita distribution (PID)	$f(x;\theta,\alpha)=\frac{\alpha\theta^3}{\theta^3+2}\left(\theta+x^{2\alpha}\right)x^{\alpha-1}e^{-\theta x^\alpha}$	Shukla and Shanker (2018) [9]
	$F(x;\theta,\alpha)=1-\left[1+\frac{\theta x^\alpha\left(\theta x^\alpha+2\right)}{\theta^3+2}\right]e^{-\theta x^\alpha}$	

Power Akash distribution (PAD)	$f(x;\theta,\alpha)=\frac{\alpha\theta^3}{\theta^2+2}\left(1+x^{2\alpha}\right)x^{\alpha-1}e^{-\theta x^\alpha}$ $F(x;\theta,\alpha)=1-\left[1+\frac{\theta x^\alpha\left(\theta x^\alpha+2\right)}{\theta^2+2}\right]e^{-\theta x^\alpha}$	Shanker and Shukla (2017 b)[10]
Power Lindley distribution (PLD)	$f(x;\theta,\alpha)=\frac{\alpha\theta^2}{(\theta+1)}\left(1+x^{\alpha}\right)x^{\alpha-1}e^{-\theta x^\alpha}$ $F(x;\theta,\alpha)=1-\left[1+\frac{\theta x^\alpha}{\theta+1}\right]e^{-\theta x^\alpha}$	Ghitany *et al* (2013)[11]
Weibull distribution (WD)	$f(x;\theta,\alpha)=\theta\alpha x^{\alpha-1}e^{-\theta x^\alpha}$ $F(x;\theta,\alpha)=1-e^{-\theta x^\alpha}$	Weibull (1951) [12]

2. Power Prakaamy Distribution

Taking $X=Y^{1/\alpha}$ in (1.1), pdf of the random variable X can be obtained as

$$f_2(x;\theta,\alpha)=\frac{\alpha\theta^6}{\theta^5+120}\left(1+x^{5\alpha}\right)x^{\alpha-1}e^{-\theta x^\alpha}\quad ;x>0,\theta>0,\alpha>0 \tag{2.1}$$

$$=p\,g_3(x;\theta,\alpha)+(1-p)\,g_4(x;\theta,\alpha) \tag{2.2}$$

where ,

$$p=\frac{\theta^5}{\theta^5+120}$$

$$g_3(x;\theta,\alpha)=\alpha\theta x^{\alpha-1}e^{-\theta x^\alpha}\quad ;x>0,\alpha>0,\theta>0$$

$$g_4(x;\theta,\alpha)=\frac{\alpha\theta^6x^{6\alpha-1}e^{-\theta x^\alpha}}{120};x>0,\alpha>0,\theta>0$$

Since (1.1) is a particular case of (2.1) at $\alpha=1$, and the pdf has been obtained as the power transformation of Prakaamy distribution, we would call the density in (2.1) "Power Prakaamy distribution (PPKD)". Clearly PPKD is also a two-component mixture of WD with parameters (θ,α), and a generalized gamma distribution(GGD) with parameters $(\theta,\alpha,6)$ suggested by Stacy (1962) with their mixing proportion $p=\frac{\theta^5}{\theta^5+120}$. The corresponding cdf of (2.1) for $x>0,\theta>0,\alpha>0$ can be expressed as

$$F_2(x;\theta,\alpha)=1-\left[1+\frac{\theta x^\alpha\left(\theta^4x^{4\alpha}+5\theta^3x^{3\alpha}+20\theta^2x^{2\alpha}+60\theta x^\alpha+120\right)}{\theta^5+120}\right]e^{-\theta x^\alpha} \tag{2.3}$$

The natures of the pdf and the cdf of PPKD for varying values of the parameters have been shown in figures 1 and 2 respectively. The pdf of PPKD is monotonically decreasing for increasing values of the parameter θ. But for $\alpha>1$ and increasing values of the parameter θ, the shapes of the pdf of PPKD becomes negatively skewed, positively skewed, symmetrical, platykurtic and mesokurtic.

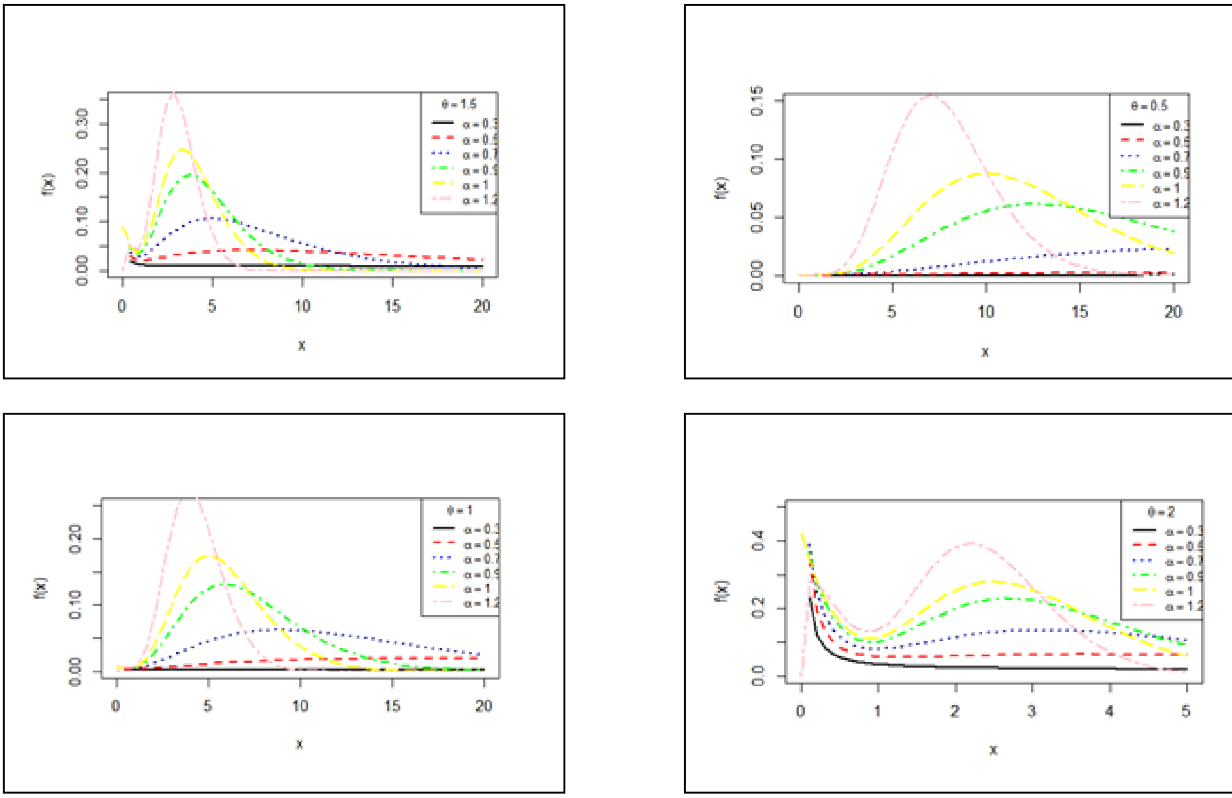

Figure1. The pdf of PPKD for values of and

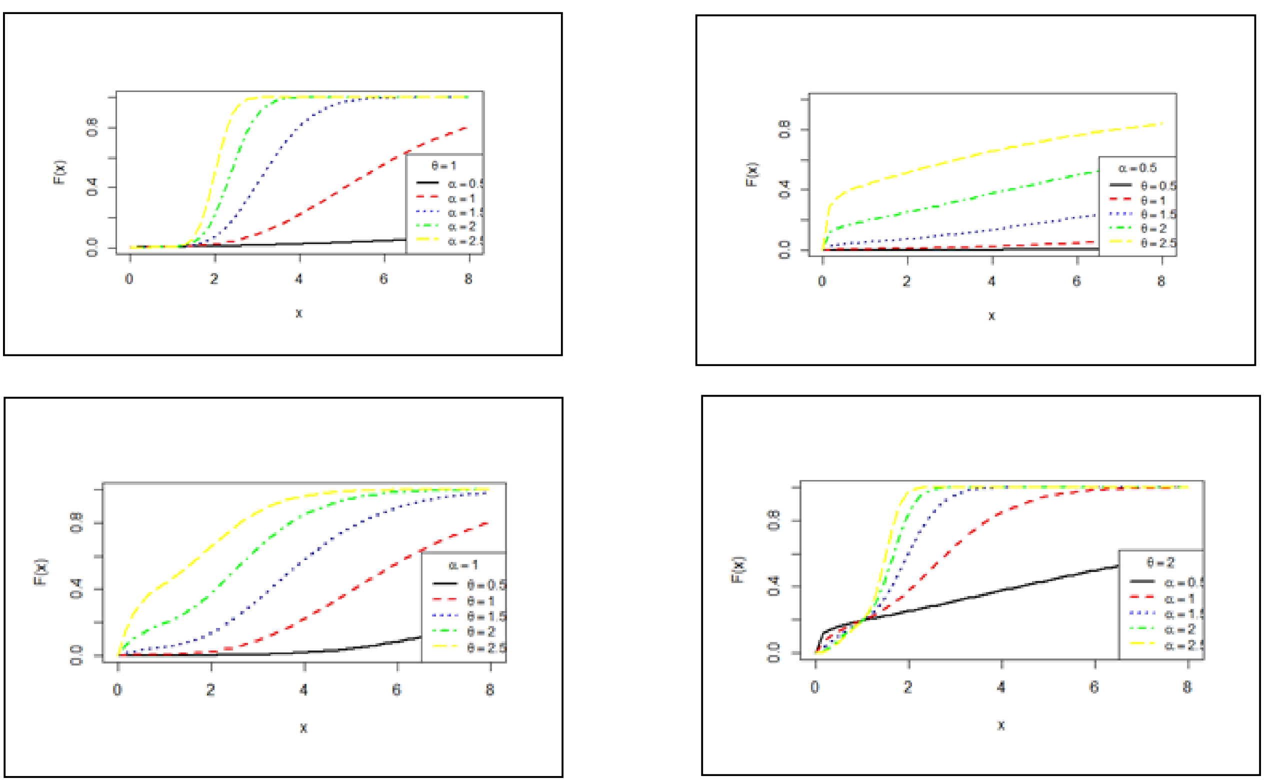

Figure2. The cdf of PPKD for values of θ and α

3. Prepare Hazard Rate Function

The survival function, $S(x)$ of PPKD for $x>0,\theta>0,\alpha>0$ can be obtained a

$$S(x;\theta,\alpha)=1-F_2(x;\theta,\alpha)$$

$$=\left[1+\frac{\theta x^{\alpha}\left(\theta^4x^{4\alpha}+5\theta^3x^{3\alpha}+20\theta^2x^{2\alpha}+60\theta x^{\alpha}+120\right)}{\theta^5+120}\right]e^{-\theta x^{\alpha}} \quad (3.1)$$

Thus the hazard rate function, $h(x)$ of PPKD can be expressed as

$$h(x;\theta,\alpha)=\frac{f_2(x;\theta,\alpha)}{S(x;\theta,\alpha)}$$

$$=\frac{\alpha\theta^6x^{\alpha-1}\left(\theta+x^{5\alpha}\right)}{\theta x^{\alpha}\left(\theta^4x^{4\alpha}+5\theta^3x^{3\alpha}+20\theta^2x^{2\alpha}+60\theta x^{\alpha}+120\right)+\left(\theta^5+120\right)} \quad (3.2)$$

The nature of $h(x)$ of the PPKD for parameters θ and α are shown graphically in figure 3. Clearly $h(x)$ takes different shapes for varying values of the parameters. Abbreviations and Acronyms

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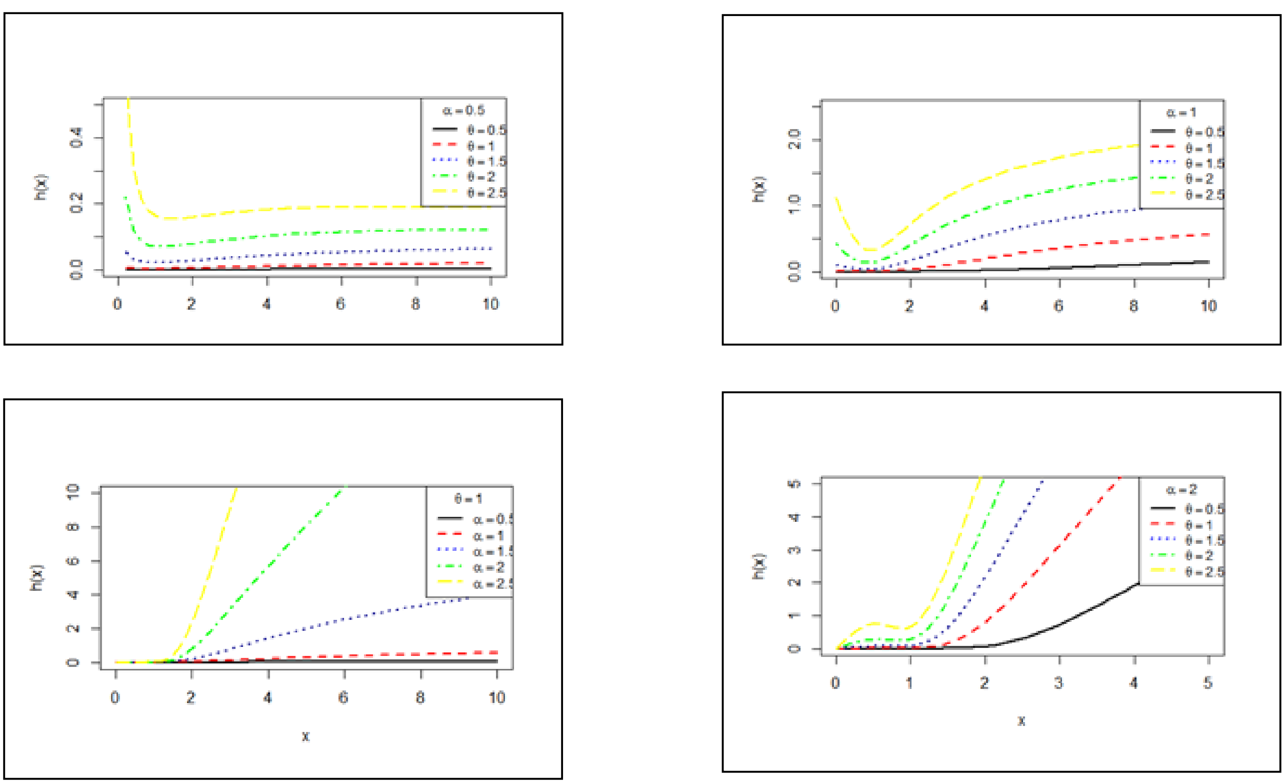

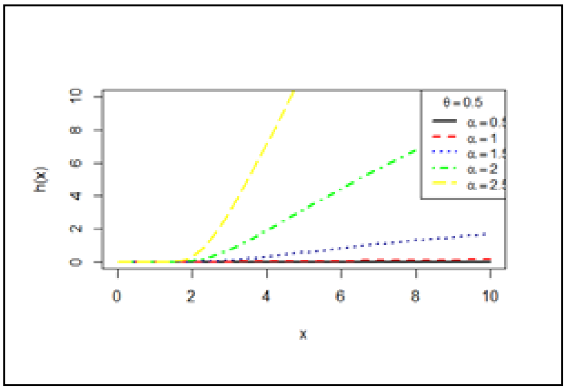

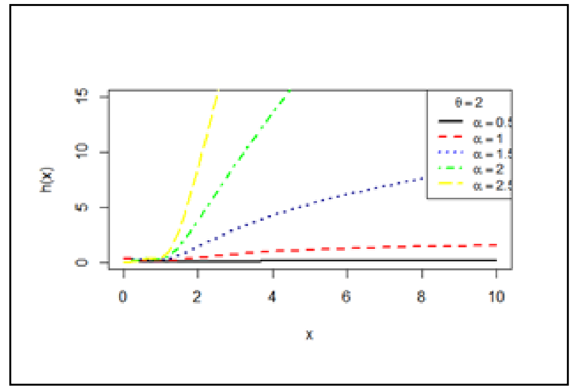

Figure 3. The $h(x)$ of PPKD for values of θ and α

4. MOMENTS AND RELATED MEASURES

Use Using (2.2), the r th moment about origin of the PPKD can be obtained as

$$\mu_r' = E\left(X^r\right) = p\int_0^\infty x^r g_3\left(x;\theta,\alpha\right)dx + (1-p)\int_0^\infty x^r g_4\left(x;\theta,\alpha\right)dx$$

$$= \frac{\left(\frac{r}{\alpha}\right)!\left[\alpha^5\theta^5 + (r+\alpha)(r+2\alpha)(r+3\alpha)(r+4\alpha)(r+5\alpha)\right]}{\alpha^5\theta^{r/\alpha}\left(\theta^5+120\right)};r=1,2,3,.... \tag{4.1}$$

At $\alpha = 1$, the above expression will reduce to the r th moment about origin of Prakaamy distribution and is given by

$$\mu_r' = \frac{r!\left[\theta^5 + (r+1)(r+2)(r+3)(r+4)(r+5)\right]}{\theta^r\left(\theta^5+120\right)};r=1,2,3,...$$

From equation (4.1), the first four raw moments can be obtained as

$$\mu_1' = \frac{\left(\frac{1}{\alpha}\right)!\left[\alpha^5\theta^5 + (\alpha+1)(2\alpha+1)(3\alpha+1)(4\alpha+1)(5\alpha+1)\right]}{\alpha^5\theta^{1/\alpha}\left(\theta^5+120\right)}$$

$$\mu_2' = \frac{\left(\frac{2}{\alpha}\right)!\left[\alpha^5\theta^5 + 4(\alpha+1)(\alpha+2)(3\alpha+2)(2\alpha+1)(5\alpha+2)\right]}{\alpha^5\theta^{2/\alpha}\left(\theta^5+120\right)}$$

$$\mu_3' = \frac{\left(\frac{3}{\alpha}\right)!\left[\alpha^5\theta^5 + 3(\alpha+1)(\alpha+3)(2\alpha+3)(4\alpha+3)(5\alpha+3)\right]}{\alpha^5\theta^{3/\alpha}\left(\theta^5+120\right)}$$

$$\mu_4' = \frac{\left(\frac{4}{\alpha}\right)!\left[\alpha^5\theta^5 + 8(\alpha+1)(\alpha+2)(\alpha+4)(3\alpha+4)(5\alpha+4)\right]}{\alpha^5\theta^{4/\alpha}\left(\theta^5+120\right)}$$

Therefore, the variance of the PPKD can be expressed as

$$\sigma^2 = \frac{\begin{bmatrix} \left(\frac{2}{\alpha}\right)!\left[\alpha^5\theta^5 + 4(\alpha+1)(\alpha+2)(3\alpha+2)(2\alpha+1)(5\alpha+2)\right]\alpha^5\left(\theta^5+120\right) - \\ \left(\left(\frac{1}{\alpha}\right)!\right)^2\left[\alpha^5\theta^5 + (\alpha+1)(2\alpha+1)(3\alpha+1)(4\alpha+1)(5\alpha+1)\right]^2 \end{bmatrix}}{\alpha^{10}\theta^{2/\alpha}\left(\theta^5+120\right)^2}$$

Coefficient of variation, coefficient of skewness, coefficient of kurtosis and Index of dispersion of PPKD can be obtained, upon substituting of the above obtained raw moments, which are as follows:

$$\text{C.V} = \frac{\mu_2^{1/2}}{\mu_1'}\ ,\quad \text{Skewness} = \frac{\mu_3' - 3\mu_2' + 2\mu^3}{\sigma^3}\ ,\ \text{Kurtosis}\ \frac{\mu_4' - 4\mu_3'\mu + 6\mu_2'\mu^2 - 3\mu^4}{\sigma^4}\ \text{and}$$

$$\text{Index of dispersion} = \frac{\mu_2}{\mu_1'}$$

Graphs of the corresponding measures are given in figure 4.

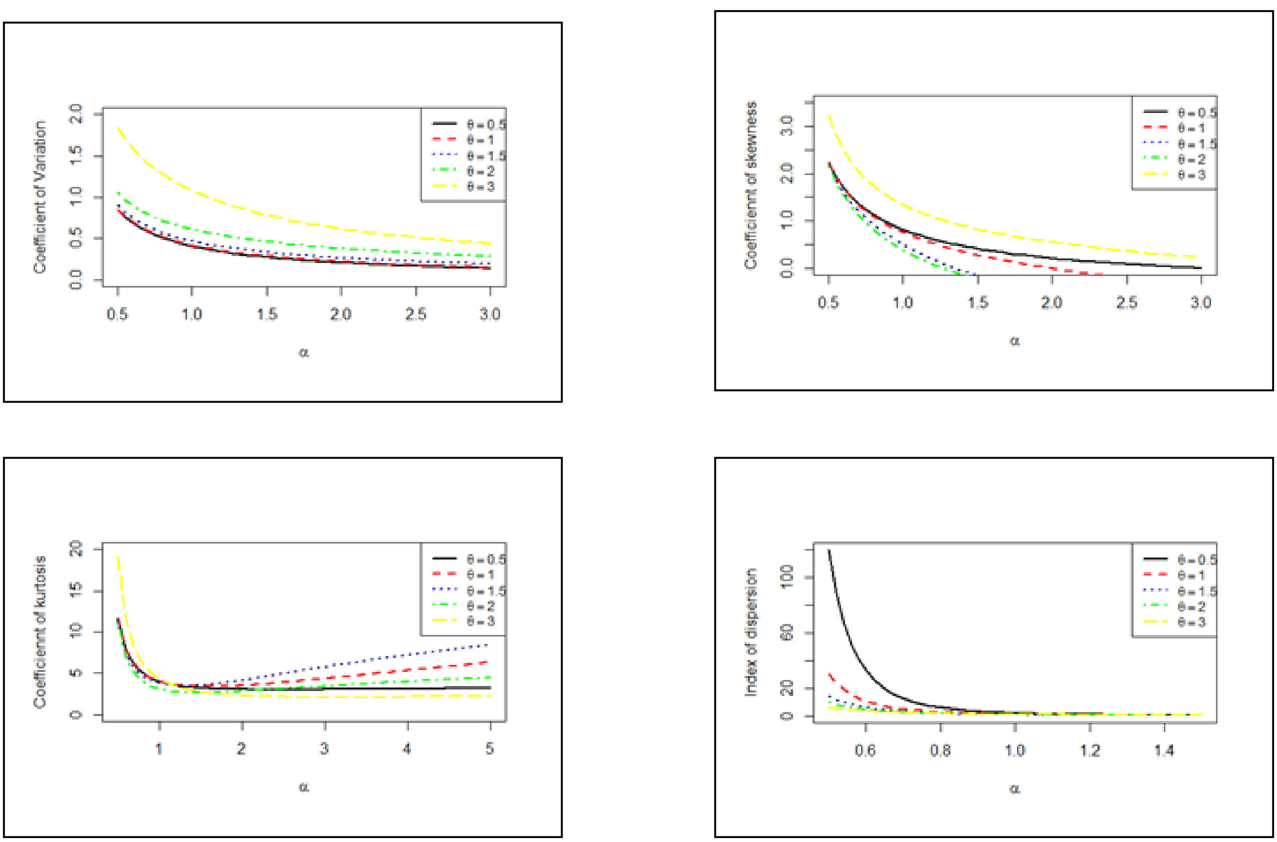

Figure 4. Coefficient of variation, coefficient of skewness, coefficient of kurtosis and Index of dispersion of PPKD for values of θ and α

5. Stochastic Ordering

Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteps. This often leads to confusion because A random variable X is said to be smaller than a random variable Y in the

(i) stochastic order $\left(X \le_{st} Y\right)$ if $F_X\left(x\right) \ge F_Y\left(x\right)$ for all x

(ii) hazard rate order $\left(X \le_{hr} Y\right)$ if $h_X\left(x\right) \ge h_Y\left(x\right)$ for all x

(iii) mean residual life order $\left(X \le_{mrl} Y\right)$ if $m_X\left(x\right) \le m_Y\left(x\right)$ for all x

(iv) likelihood ratio order $\left(X \le_{lr} Y\right)$ if $\dfrac{f_X\left(x\right)}{f_Y\left(x\right)}$ decreases in x.

These stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. The following important interrelationships due to [13] are well known for establishing stochastic ordering of distributions

$$\begin{array}{c} X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{mrl} Y \\ \Downarrow \\ X \le_{st} Y \end{array}$$

The PPKD is ordered with respect to the strongest 'likelihood ratio ordering' as shown in the following theorem:

Theorem: Let X ~ PPKD $\left(\theta_1,\alpha_1\right)$ and Y ~ PPKD $\left(\theta_2,\alpha_2\right)$. If $\theta_1 > \theta_2$ and $\alpha_1 = \alpha_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 = \theta_2$) then $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

Proof: From the pdf of PPKD (2.1), we have

$$\frac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} = \left(\frac{\alpha_1\theta_1^{\,6}\left(\theta_2^{\,5}+120\right)}{\alpha_2\theta_2^{\,6}\left(\theta_1^{\,5}+120\right)}\right)\left(\frac{1+x^{5\alpha_1}}{1+x^{5\alpha_2}}\right)\left(x^{\alpha_1-\alpha_2}\right)e^{-\left(\theta_1 x^{\alpha_1}-\theta_2 x^{\alpha_2}\right)} \quad ;\ x>0$$

Now

$$\ln\frac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} = \ln\left(\frac{\alpha_1\theta_1^{\,6}\left(\theta_2^{\,5}+120\right)}{\alpha_2\theta_2^{\,6}\left(\theta_1^{\,5}+120\right)}\right)+\ln\left(\frac{1+x^{5\alpha_1}}{1+x^{5\alpha_2}}\right)+\left(\alpha_1-\alpha_2\right)\ln x-\left(\theta_1 x^{\alpha_1}-\theta_2 x^{\alpha_2}\right)$$

.

This gives

$$\frac{d}{dx}\left\{\ln\frac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)}\right\} = \frac{5\left(\alpha_1x^{5\alpha_1-1}-\alpha_2x^{5\alpha_2-1}\right)+5\left(\alpha_1-\alpha_2\right)x^{5\left(\alpha_1+\alpha_2\right)-1}}{\left(1+x^{5\alpha_1}\right)\left(1+x^{5\alpha_2}\right)}+\frac{\alpha_1-\alpha_2}{x}-\left(\alpha_1\theta_1x^{\alpha_1-1}-\alpha_2\theta_2x^{\alpha_2-1}\right)$$

Clearly for $\theta_1 > \theta_2$ and $\alpha_1 = \alpha_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 = \theta_2$), $\dfrac{d}{dx}\left\{\ln\frac{f_X\left(x;\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)}\right\} < 0$. This means that $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

6. Maximum Likelihood Estimation

Do Let $\left(x_1, x_2, \dots, x_n\right)$ be a sample values corresponding to random sample $\left(X_1, X_2, \dots, X_n\right)$ from PPKD $\left(\theta, \alpha\right)$. Then log-likelihood function is

$$\ln L = \sum_{i=1}^{n} \ln f_{10}\left(x_i; \theta, \alpha\right)$$

$$= n\left[\ln\alpha + 6\ln\theta - \ln\left(\theta^5 + 120\right)\right] + \sum_{i=1}^{n} \ln\left(1 + x_i^{5\alpha}\right) + \left(\alpha - 1\right)\sum_{i=1}^{n} \ln\left(x_i\right) - \theta\sum_{i=1}^{n} x_i^{\alpha}$$

The maximum likelihood estimate (MLE) $\left(\hat{\theta}, \hat{\alpha}\right)$ of $\left(\theta, \alpha\right)$ of PPKD are the solutions of the following log likelihood equations

$$\frac{\partial \ln L}{\partial \theta} = \frac{6n}{\theta} - \frac{5n\theta^4}{(\theta^5 + 120)} - \sum_{i=1}^{n} x_i^{\alpha} = 0$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \frac{5x_i^{\alpha} \ln\left(x_i\right)}{(1 + x^{5\alpha})} + \sum_{i=1}^{n} \ln\left(x_i\right) - \theta\sum_{i=1}^{n} x_i^{\alpha} \ln\left(x_i\right) = 0$$

These two log likelihood equations do not seem to be solved directly because these cannot be expressed in closed form. The (MLE's) $(\hat{\theta}, \hat{\alpha})$ of (θ, α) can be computed directly by solving the natural log likelihood equation using Newton-Raphson iteration available in R-software till sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained Equations

7. Applications

The In this section, the goodness of fit of PPKD using maximum likelihood estimates of parameters to three real dataset have been presented and compared with PPD, PID, PAD, PLD, WD and Prakaamy distribution (PD). The following real lifetime data have been considered for the goodness of fit of the considered distributions.

Data Set 1: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England , available in [14]

0.55 0.93 1.25 1.36 1.49 1.52 1.58 1.61 1.64 1.68 1.73 1.81 2.00 0.74 1.04
1.27 1.39 1.49 1.53 1.59 1.61 1.66 1.68 1.76 1.82 2.01 0.77
1.11 1.28 1.42 1.50 1.54 1.60 1.62 1.66 1.69 1.76 1.84 2.24 0.81
1.13 1.29 1.48 1.50 1.55 1.61 1.62 1.66 1.70 1.77 1.84 0.84 1.24
1.30 1.48 1.51 1.55 1.61 1.63 1.67 1.70 1.78 1.89

Data Set 2: The data is related with behavioral sciences, collected by [15] The scale "General Rating of Affective Symptoms for Preschoolers (GRASP)" measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis, which are:

19(16) 20(15) 21(14) 22(9) 23(12) 24(10) 25(6) 26(9) 27(8) 28(5) 29(6) 30(4)
31(3) 32(4) 33 34 35(4) 36(2) 37(2) 39 42 44

Data Set 3: The data set reported by [16] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

12.20 23.56 23.74 25.87 31.98 37 41.35 47.38 55.46 58.36 63.47 68.46 78.26
74.47 81.43 84 92 94 110 112 119 127 130 133 140 146 155

159 173 179 194 195 209 249 281 319 339 432 469 519
633 725 817 1776

The values of $-2\ln L$, AIC (Akaike Information Criterion), K-S Statistic (Kolmogorov-Smirnov Statistic) for the real dataset have been computed using maximum likelihood estimates for comparing the considered distributions, and presented in table 3. The AIC and K-S Statistic is computed as follows:

$AIC = -2\ln L + 2k$ and $\text{K-S} = \sup_x \left|F_n(x) - F_0(x)\right|$, where k = the number of parameters, n = the sample size, $F_n(x)$ is the empirical (sample) cumulative distribution function and $F_0(x)$ is the theoretical cumulative distribution function. The best distribution is the distribution corresponding to lower values of $-2\ln L$, AIC, and K-S statistic.

TABLE-IV: THE PDF AND THE CDF OF TWO –PARAMETER DISTRIBUTIONS AND THEIR INTRODUCER (YEAR)

Data	Model	ML Estimates	-2ln L	AIC	K-S	p-value
1	PPKD	$\hat{\theta} = 1.6086$ $\hat{\alpha} = 2.7882$	25.98	29.98	0.777	0.000
	PPD	$\hat{\theta} = 0.877574$ $\hat{\alpha} = 3.204218$	26.61	30.61	0.788	0.000
	PID	$\hat{\theta} = 0.575355$ $\hat{\alpha} = 3.492621$	28.97	32.97	0.798	0.000
	PAD	$\hat{\theta} = 0.50105$ $\hat{\alpha} = 3.714359$	27.36	31.36	0.797	0.000
	PLD	$\hat{\theta} = 0.222441$ $\hat{\alpha} = 4.45818$	29.37	33.37	0.804	0.000
	WD	$\hat{\theta} = 0.0598$ $\hat{\alpha} = 5.7795$	30.41	34.41	0.803	0.000
	Prakaamy	$\hat{\theta} = 1.56075$	180.96	182.96	0.488	0.000
2	PPKD	$\hat{\theta} = 0.01150$ $\hat{\alpha} = 1.93220$	819.12	823.12	0.129	0.000
	PPD	$\hat{\theta} = 0.004914$ $\hat{\alpha} = 2.068030$	827.09	833.09	0.174	0.000
	PID	$\hat{\theta} = 0.0032$ $\hat{\alpha} = 2.1120$	840.41	844.41	0.218	0.000
	PAD	$\hat{\theta} = 0.003784$ $\hat{\alpha} = 2.061716$	843.09	847.09	0.226	0.000
	PLD	$\hat{\theta} = 0.0020$ $\hat{\alpha} = 2.1311$	871.37	875.37	0.287	0.000

	Prakaamy	$\hat{\theta}=0.16022$	945.03	947.03	0.362	0.000
3	PPKD	$\hat{\theta}=0.8217$ $\hat{\alpha}=0.3921$	557.29	561.29	0.0958	0.778
	PPD	$\hat{\theta}=0.349318$ $\hat{\alpha}=0.476545$	557.95	561.95	0.101	0.715
	PID	$\hat{\theta}=0.177744$ $\hat{\alpha}=0.548088$	558.65	562.65	0.107	0.660
	PAD	$\hat{\theta}=0.167518$ $\hat{\alpha}=0.557642$	559.10	563.10	0.109	0.636
	PLD	$\hat{\theta}=0.053079$ $\hat{\alpha}=0.68875$	560.78	564.78	0.118	0.531
	WD	$\hat{\theta}=0.00647$ $\hat{\alpha}=0.938187$	563.68	567.68	0.129	0.416
	Prakaamy	$\hat{\theta}=0.01791$	646.17	648.17	0.327	0.0001

For comparing two-parameter lifetime distributions with one parameter lifetime distributions, the likelihood of selecting a distribution as the best model will be increased in terms of lower values of $-2\ln L$ and AIC. In this case the goodness of fit is measured using Kolmogorov-Smirnov test statistic or Anderson Darling test or Crammer-Von Mises test to validate the superiority of a model for the considered dataset. Table 3 shows that the PPKD has the least value of $-2\ln L$, AIC and K-S, which indicates that the PPKD demonstrates superiority over other considered distributions under study.The profile of likelihood estimates of $\hat{\theta}$ and $\hat{\alpha}$ of PPKD for the first and third datasets are shown in figures 5 and 6, respectively.

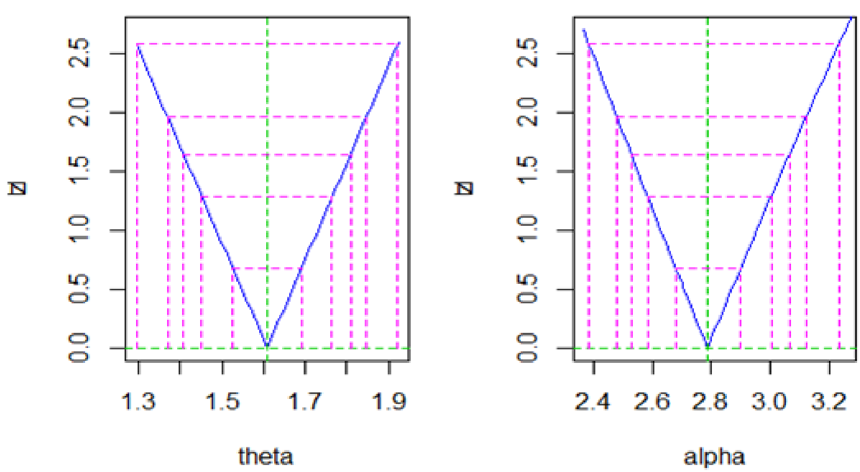

Figure 5. Profile of the likelihood estimates $\hat{\theta}$ and $\hat{\alpha}$ of PPKD for the first dataset

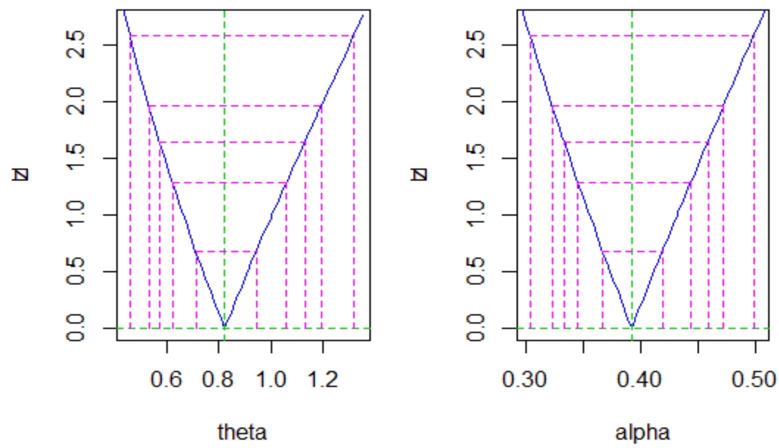

Figure 6. Profile of the likelihood estimates $\hat{\theta}$ and $\hat{\alpha}$ of PPKD for the third dataset

8. Conclusion

Power Prakaamy distribution (PPKD) has been introduced. Shapes of the density, moments, skewness and kurtosis measures, hazard rate function, stochastic ordering of PPKD have been discussed. The maximum likelihood estimation has been discussed Goodness of fit of PPKD has been discussed with three real lifetime datasets and the fit has been found quite satisfactory over PPD, PID, PAD, PLD, WD and Prakaamy distribution.

Acknowledgment

Authors are thankful to the Chief editor and anonymous reviewers for given fruitful comments to improve the quality of paper.

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