



Novelty Study of the Window Length Effects on the Adaptive Beam-forming Based-FEDS Approach

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Abstract: To date, there are very few researches on performance investigation or application of adaptive beam-forming that is based on the Fast-Euclidean Direction Search (FEDS) algorithm. Correspondingly, one of our primary goals in this paper will be to introduce a novelty study on the impact of selecting the iteration number and the window length (L) on the overall performance of FEDS-based beam-forming approach. The communication channel is implemented for multipath Rayleigh fading model with different paths numbers, delays, Doppler shift frequencies and gains. For comparison, this study considers different benchmark approaches which are Recursive Least Square (RLS), Least Mean Square (LMS), and Normalized LMS (NLMS). One of the important findings, based on the simulation results, indicates that the best window length (L) should be more than the number of the elements in the array. This is a necessary condition to obtain significant improvement in the performance of FEDS compared with both LMS and NLMS algorithms and slight improvement compared with RLS algorithm.

Keywords: Beam-forming, FEDS, RLS, LMS, NLMS.

1. INTRODUCTION

Adaptive Beam forming is an intelligent technique that consists of an array of multi-element antennas. Through estimating the signal arrived from the desired direction and cancelling other signals from other directions, the beam-forming can manage the main beam towards every user in the coverage area and acquire maximum reception in a particular direction. The Euclidean Direction Search (EDS) algorithm is a least squares algorithm that was applied to different adaptive systems applications [1, 2]. Both EDS and Recursive Least Square (RLS) algorithms have fast and comparable convergence rate, and small miss-adjustment compared to the traditional Least Means Square (LMS) and Normalized LMS (NLMS) algorithms [3, 4]. However, EDS and RLS have a disadvantage which was suffering from high computational complexity. In order to overcome this disadvantage of EDS, a new algorithm was developed and called Fast EDS (FEDS). This evolved algorithm was used for different adaptive filtering applications due to its better performance compared with traditional algorithms like LMS [5-7]. There has been a lot of work and performance studies on the adaptive beam-forming based on different algorithms, but very few consider the FEDS approach. [1-11]. We published a study in 2019 that sheds the light on this topic [11]. This

article, on the contrary, has been entirely rewritten to incorporate many various aspects and new results using different communication channels to extend and demonstrate the basic results in [11]. Accordingly, the aim of this paper is to investigate the effectiveness of utilizing the FEDS algorithm in the wireless communication systems with adaptive beam-forming techniques. It also provides a complete comparison between the considered FEDS algorithm and other classical algorithms, namely LMS, NLMS and RLS under different channel models. Afterward, an optimal window length, or L , for FEDS approach is identified. The remaining of the paper is organized as follows. The beam forming is presented in Section 2 and FEDS approach is in Section 3. Section 4 is dedicated to channel models while Section 5 is for numerical results. Conclusions are finally stated in Section 6.

2. FUNDAMENTAL CONCEPTS

Figure 1 illustrates diagram of an adaptive beam-forming system [12]. As shown, the error signal $\varepsilon(k)$ is going to be minimized through updating the array of weight coefficients $\bar{w}(k)$ in each iteration k .

For the LMS algorithm, the array output can be written as

$$y(k) = \bar{w}(k)^H \cdot \bar{x}(k) \quad (1)$$

where

$$\begin{aligned}\bar{x}(k) &= \bar{a}_0 s(k) + [\bar{a}_1 \bar{a}_2 \dots \bar{a}_N] \cdot \begin{bmatrix} i_1(k) \\ i_2(k) \\ \vdots \\ i_N(k) \end{bmatrix} + \bar{n}(k), \\ &= \bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k)\end{aligned}$$

where we used

$\bar{w} = [w_1 w_2 \dots w_M]^T$ to represent array weight coefficients;

$\bar{x}(k) = [x_1 x_2 \dots x_M]^T$ to represent input vector;

$\bar{x}_s(k)$ to represent desired vector;

$i_1(k), i_2(k), \dots, i_N(k)$ to represent interference signals;

$\bar{x}_i(k)$ to represent interference signals;

$\bar{n}(k)$ to represent Gaussian noise with zero variance;

\bar{a}_i to represent steering array signal;

$\varepsilon(k)$ to represent error signal such that:

$$\varepsilon(k) = d(k) - \bar{w}^H(k) \bar{x}(k). \quad (2)$$

For the LMS algorithm, the updating weight vector is [10]:

$$\bar{w}(k+1) = \bar{w}(k) + \mu \varepsilon(k) \bar{x}(k). \quad (3)$$

The convergence of the LMS is guaranteed if the convergence factor (μ) has the following bound condition

$$0 \leq \mu \leq \frac{1}{2\lambda_{max}} \quad (4)$$

where the estimation correlation matrix \hat{R}_{xx} has maximum eigenvalues λ_{max} . The matrix \hat{R}_{xx} can be instantaneous estimates as

$$\hat{R}_{xx}(k) \approx \bar{x}(k) \bar{x}^H(k) \quad (5)$$

The condition mentioned in Eq. (4) can be approximated as

$$0 \leq \mu \leq \frac{1}{2\text{trac}[\hat{R}_{xx}]} \quad (6)$$

while Eq. (3) for NLMS will be [10]

$$\bar{w}(k+1) = \bar{w}(k) + \frac{\mu_0}{\|\bar{x}(k)\|^2} \varepsilon(k) \bar{x}(k), \quad (7)$$

where μ_0 is small positive constant.

Recursive least squares (RLS) is an adaptive algorithm that supported the smallest amount squares method which tries to reduce a weighted linear least squares cost function. Initialize the weight vector and the inverse correlation matrix \hat{R}_{xx}^{-1} . The constants forgetting factor λ and regularization δ parameters are set by the user. The

value for λ is unity, and for δ is depend upon Signal-to-Noise (SNR) of the signals [13]. Initialize the weight vector and the inverse correlation matrix \hat{R}_{xx}^{-1} as

$$\bar{w}^H(0) = \bar{0} \quad (8)$$

$$\hat{R}_{xx}^{-1}(0) = \delta^{-1} \bar{I} \quad (9)$$

The vector π is used to compute the gain vector \bar{g} (also known as the search direction at iteration k). For each instance of time $k = 1, 2, 3 \dots$,

$$\pi(k+1) = \hat{R}_{xx}^{-1}(k) \bar{x}(k) \quad (10)$$

$$\bar{g}(k) = \frac{\pi(k)}{\lambda + \bar{x}^H(k) \pi(k)} \quad (11)$$

Update the weights:

$$\bar{w}(k+1) = \bar{w}(k) + \varepsilon(k) \bar{g}(k) \quad (12)$$

Given an initial estimate of $\bar{w}(k)$ and the search direction $\bar{g}(k)$, the process of minimizing the next objective function is called line direction. Then, the inverse matrix is re-calculated, and therefore the training start again with the new input values.

$$\hat{R}_{xx}^{-1}(k+1) = \lambda^{-1} \hat{R}_{xx}^{-1}(k) - \lambda^{-1} \bar{g}(k) \bar{x}^H(k) \hat{R}_{xx}^{-1}(k) \quad (13)$$

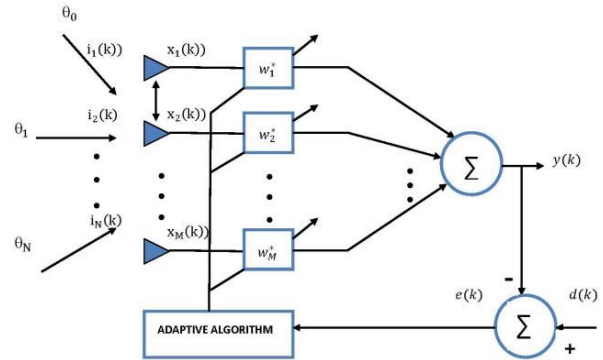


Figure.1 Block diagram of adaptive beam-forming system [12]

3. FEDS APPROACH FOR ADAPTIVE BEAM-FORMING

FEDS approach is a simplified or partial RLS algorithm [6, 7]. It combines the benefits of fast convergence of RLS algorithm and low computational complexity of LMS algorithm. FEDS approach updates the weight vector in sequential way. FEDS approach uses block exponential weighted least squares form instead of conventional exponentially one. The length of this block or window length is denoted by L , such that the weights will decrease exponentially with every block (L) of data. The error signal (Eq. (2)) can be re-written as

$$\varepsilon(k) = d(k) - \sum_{i=1}^M w_i(k) x_i(k) \quad (14)$$

Assuming the samples $k-L, k-L+1, k-L+2 \dots k$, where L is window (block) length and k is number of iterations as mentioned above. Equation (14) can be written in a vector form as



$$\bar{\varepsilon}(k) = \bar{d}(k) - \bar{X}(k)\bar{w}(k). \tag{15}$$

with

$$\bar{X}(k) = [\bar{x}_1(k), \bar{x}_2(k) \dots \bar{x}_M(k)]. \tag{16}$$

The column vector of $\bar{X}(k)$ are as following

$$\bar{x}_j(k) = [x_j(k), x_j(k-1) \dots x_j(k-L+1)]^T. \tag{17}$$

Also, the desired signal vector samples are

$$\bar{d}(k) = [d(k), d(k-1), d(k-2) \dots d(k-L+1)]^T \tag{18}$$

Furthermore we can define the error signal vector $\bar{\varepsilon}(k)$ in the same way. The prior approximation error $\bar{\varepsilon}_0$ at time k is given by:

$$\bar{\varepsilon}_0(k) = \bar{d}(k) - \bar{X}(k-1)\bar{w}(k-1). \tag{19}$$

Only one weight in past updating weight vector has a new error signal as [6]:

$$\bar{\varepsilon}_1(k) = \bar{d}(k) - [\bar{X}(k)\bar{w}(k-1) + \bar{X}(k) \bar{w}_{j_0(k)}^{update}(k)] \bar{F}_{j_0(k)} \tag{20}$$

The index of the weight to be update in the zero'th iteration at time k is $j_0(k)$ and $\bar{F}_{j_0(k)}$ is $M \times 1$ vector with 1 in position j and 0 in all other positions. Then, the updated weight $\bar{w}_{j_0(k)}(k)$ is given by

$$\bar{w}_{j_0(k)}(k) = \bar{w}_{j_0(k)}(k-1) + \bar{w}_{j_0(k)}^{update}(k) \tag{21}$$

where $\bar{w}_{j_0(k)}^{update}(k)$ is given as:

$$\bar{w}_{j_0(k)}^{update}(k) = \frac{\langle \bar{\varepsilon}_0(k), \bar{x}_{j_0(k)}(k) \rangle}{\|\bar{x}_{j_0(k)}(k)\|^2} \tag{22}$$

where $\langle \dots \rangle$ is inner product of two vectors. Thus, the update array weight vector:

$$\bar{w}^o(k) = \bar{w}(k-1) + \bar{w}_{j_0(k)}^{update}(k) \bar{F}_{j_0(k)} \tag{23}$$

Then a new parameter called the step size μ_{FEDS} will be inserted to possess stability and convergence rate of the FEDS algorithm as follows

$$\bar{w}^o(k) = \bar{w}(k-1) + \mu_{FEDS} \bar{w}_{j_0(k)}^{update}(k) \bar{F}_{j_0(k)} \tag{24}$$

Then, substituting Eq. (24) into Eq. (20), we get

$$\bar{\varepsilon}_1(k) = \bar{d}(k) - \bar{X}(k)\bar{w}^o(k) \tag{25}$$

Filter coefficient update equations updates only one element of the filter vector at a time. At each time instant, k , we can perform one or more such updates. The number of such single coefficient updates performed at each time instant is denoted by P . Only one element of \bar{w}^o is to be updated at a time, where P is the number of updates to perform at each sample time [10]. The FEDS approach was developed as modified conjugate gradient algorithm during which the seeking directions to minimum are guaranteed to the Euclidean directions. FEDS approach can find better estimation weights vector in duration of each direction by starting with initial estimate value of weights vector and then using linearly independent Euclidean direction set, such that FEDS approach performs one Euclidean direct search for every iteration. FEDS approach has better performance than traditional LMS and NLMS algorithms but comparable to the RLS. This is due to the fact that FEDS approach is regarded as partial or alternative of full RLS [6, 7].

4. CHANNEL MODELS

Two multipath channel (frequency-selective fading) models are used to simulate the system. The first one is called “channel 1”, as labelled in all subsequent results, which represents Rayleigh fading channel with four paths specified with discrete delays [0 2 4 6]* 1e-6 (seconds), and average path gains [0 -3 -6 -9] (dB). The impulse and frequency response of this channel are shown in Fig. 2 and 3 respectively. The maximum Doppler shift of all paths of a channel was equal to 80 Hz which corresponds to user movement (or mobility) equals to 27 m/s.

While the second channel, denoted as “channel 2” henceforth, is also a frequency-selective multipath Rayleigh fading channel. It consists of six paths with a specified delays of the discrete paths as [0 200 800 1200 2300 3700]*1e-9 (seconds) and the average gains of the discrete paths as [0 -0.9 -4.9 -8 -7.8 -23.9] (dB). Figures 4 and 5 illustrate its impulse and frequency response respectively. The maximum Doppler shift in this channel model was equal to 50 Hz which corresponds to user movement of 17 m/s.

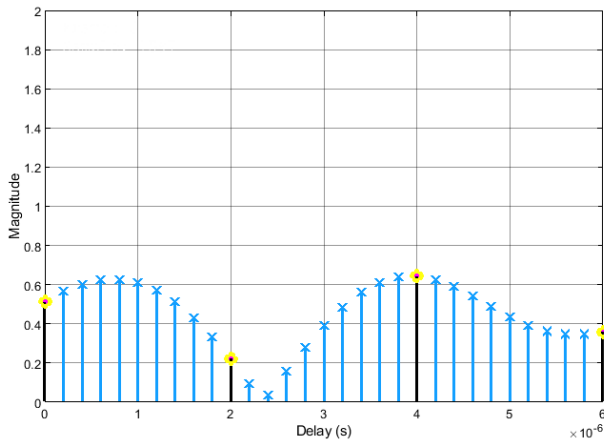


Fig. 2 Impulse response of channel 1

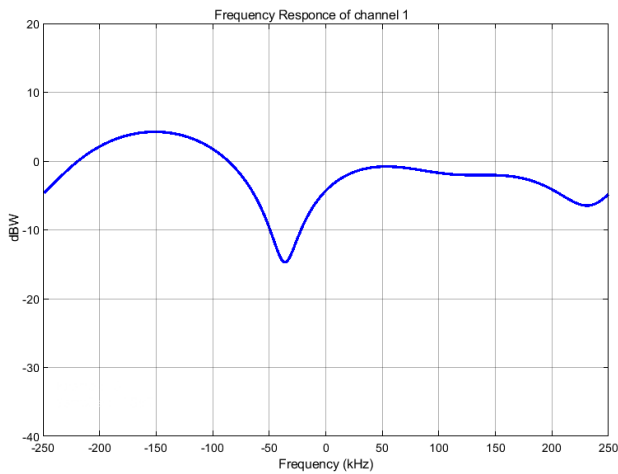


Figure. 3 Frequency response of channel 1

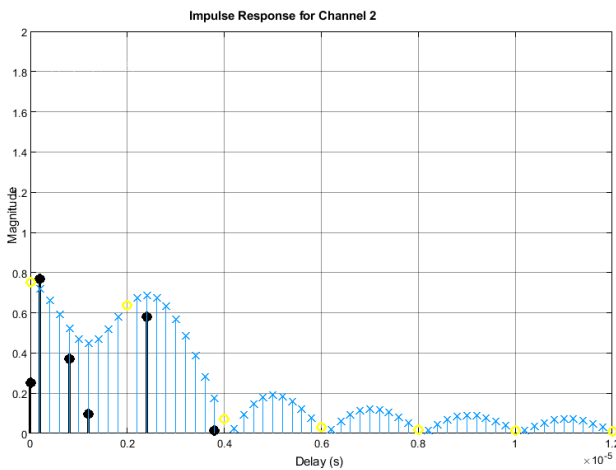


Figure. 4 Impulse response of channel 2

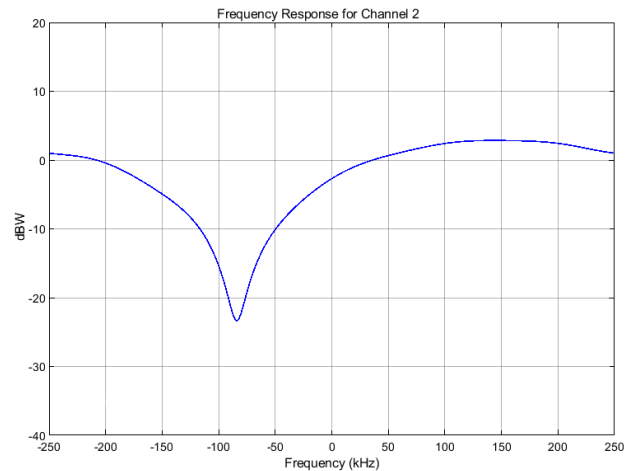


Figure. 5 Frequency response of channel 2

5. SIMULATION RESULTS

This section uses the adaptive beam-forming diagram shown in Figure 1 with the parameters listed below:-

- A linear array is formed of isotropic elements (M) equals 8 with element wideness (d) equals 0.5λ .
- Communication channel is multipath Rayleigh model.
- Desired AOA of $\theta_0 = 0^\circ$ and other undesired signals, with two AOA's, $\theta_1 = 20^\circ$, and $\theta_2 = -20^\circ$.
- Input desired signal $s(k) = \cos(2\pi f(k))$ with fundamental frequency equals 900 MHZ.
- An AWGN with zero mean and variance σ_n^2 equals to 0.001 is added.
- The SNR and Signal-to-Interference Ratio (SIR) are set at 30 dB and 10 dB respectively.
- All the step size used for all algorithms was set according to Eq. (5) and Eq. (6).
- Initial convergence factor μ_0 equals 1, and λ equals 0.9 for RLS.
- All weight vectors are initially set to zero.

A. Effect of setting number of samples (k ; number of iterations)

The number of samples or number of iterations for any adaptive filtering system depends upon the type of the adaptive filtering, applications used by the system and channel models. Therefore for our system we start evaluating the performance by choosing the number of iterations equals to 100, then increasing its upward.

Figure 6 shows the performance of FEDS algorithm in terms of array output signal and squared error respectively, when using different number of samples (i.e. $k = 100$, and 200, iterations) for two channels. It is clear that better output array signal estimation was

achieved for $k=200$ samples and it degrades when using small number of samples which is the case that most properly faced in the mobile communications due to the requirements of fast moving targets.

It is obvious that when number of samples was setting to 200, the FEDS approach gives better performance in terms of convergence rate. Therefore the number of iterations (k) used for all cases was set to k equals 200 samples.

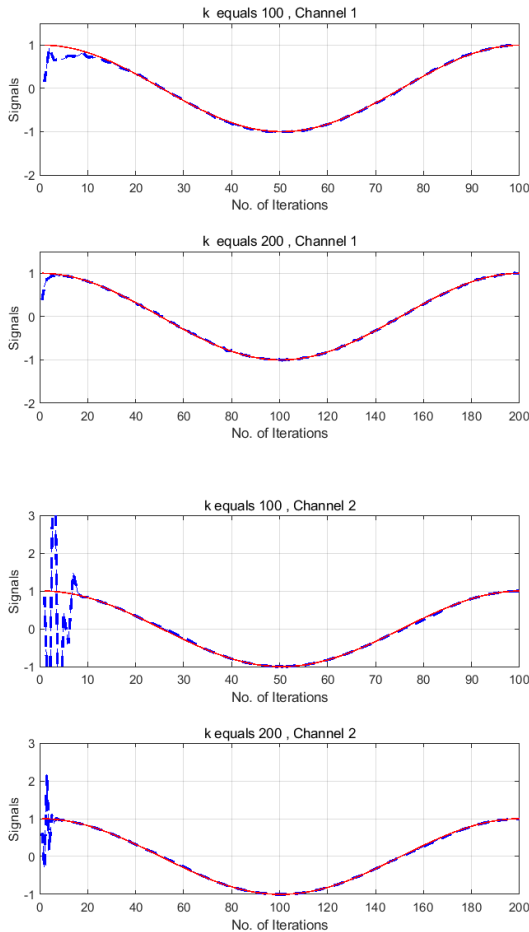


Figure. 6 Array output and desired signals for both channels.

B. Overall Performance of FEDS

Figure 7 and 8 shows the squared error (measured in dB) for all algorithms using channel 1 and 2. As shown in this figure, RLS algorithm has faster converge rate. Also, the results show that the FEDS algorithm has fast convergence rate compared to the both LMS and NLMS. Each algorithm has the following convergence rate; 50, 40 and 20 iterations for LMS, NLMS, and FEDS algorithm respectively.

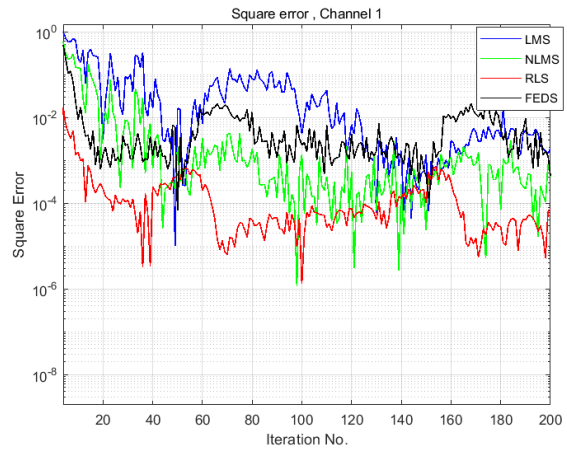


Figure. 7 Squared error for all algorithms using channel 1

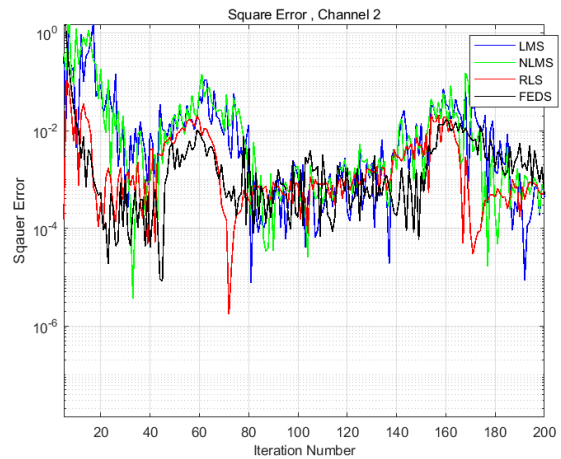


Figure. 8 Squared error for all algorithms using channel 2.

C. Choosing optimal window lengths (L)

In this section, an optimal window length (L) of the FEDS algorithm is evaluated to show its effect. Figure 9 shows the linear plot of the radiation diagram of FEDS approach with L equals 06, 07, 08, and 10 respectively.

It is obvious that, the interference suppression ability of FEDS10 is the best one at undesirable angles 20^0 and -20^0 . While this above mentioned feature is degraded for other window length. Moreover, for increasing value of L above 10 the performance will come down.

Figure 10 and 11 also show that the square error (in dB) plots for FEDS10 is better than the others window lengths. Figures 12 and 13 confirms previous results when plotting the radiation diagram for all algorithms.

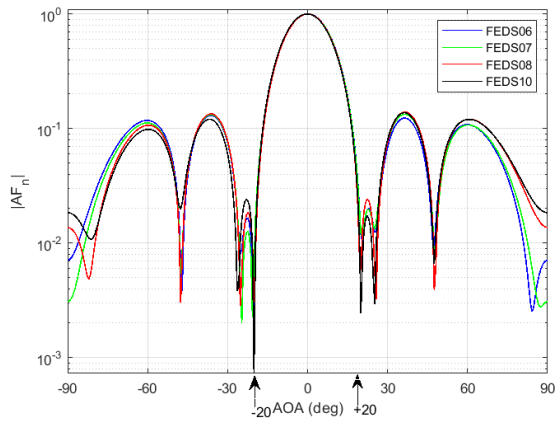


Figure. 9 Linear radiation diagram for different L parameter.

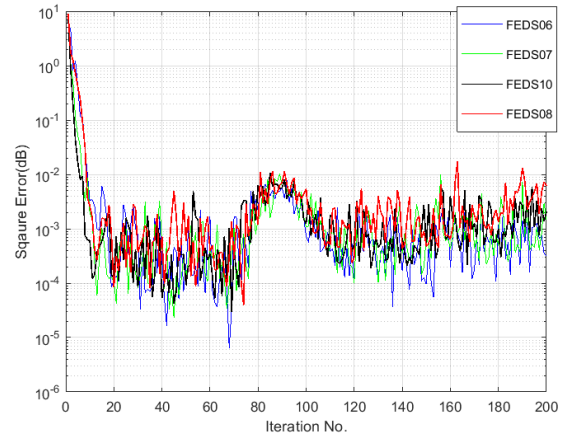


Figure. 11 Square error plot of FEDS for different L parameter (channel 2).

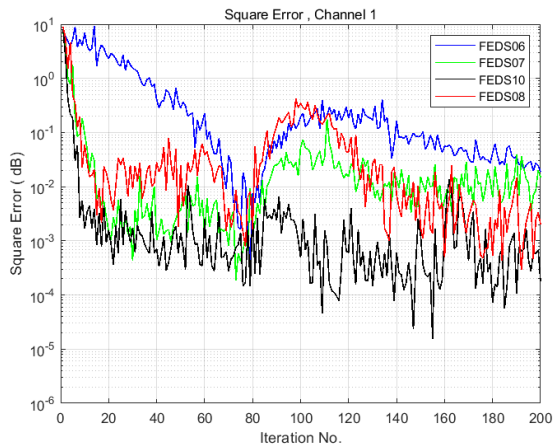


Figure. 10 Square error plot of FEDS for different L parameter (channel 1).

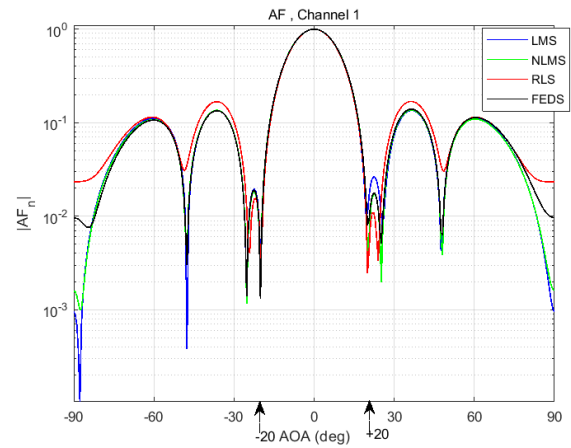


Figure. 12 Linear radiation diagram (channel 1).

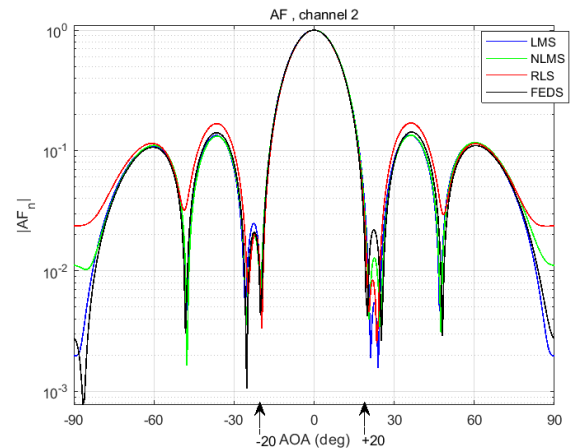


Figure. 13 Linear radiation diagram (channel 2).

In addition, fast convergence rate could be achieved using RLS and FEDS10 in contrast with other algorithms. We can observed that the optimum window length parameter (L) is ten and thus the FEDS's performance starts to decline when another value of L is employed. This optimum window length (L equals 10) confirm our previous results in [11].

6. CONCLUSIONS

This paper make an investigation of the overall performance of adaptive beam-forming by leveraging FEDS approach over Rayleigh fading models. It also tried to seek out the effect of choosing window length on the overall performance. From the achieved results, the FEDS approach with optimal selection of window length has exceeded the performance of other algorithms. The preferable value for the window length should be slightly beyond the number of array elements so as to realize good performance than conventional algorithms, but close to that of the RLS algorithm.

REFERENCES

- [1] Mathurasai, T., Bose, T., Etter, D.M. : Decision Feedback Equalization Using an Euclidean Direction Based Adaptive Algorithm ; In IEEE proceeding of Signals, Systems, and Computers; 24-27 October 1999; Pacific Grove, CA, USA, USA: Volume: 1 , p.519-524.
- [2] Cheng Zhao, and Abeyssekera, S.S. : Investigation of Euclidean Direction Search Based Laguerre Adaptive Filter for System Identification :In IEEE International Conference on Communications, Circuits and Systems ; 27-30 May 2005; Hong Kong, China, China : IEEE ; p.691-696.
- [3] Xie, B. Partial Update Adaptive Filtering [thesis]. Virginia Polytechnic Institute and State University, Virginia, December 2011.
- [4] Guo-Fang Xu and Tamal Bose : Analysis of the Euclidean Direction Set Adaptive Algorithm: In Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing; 15-15 May 1998 ; Seattle, WA, USA, USA : IEEE ; vol.3 p. 1689 – 1692.
- [5] Zhang, Z., Bose, T., and Gunther, J. : Optimal Order EDS and FEDS Algorithms: In IEEE Thirty-Eighth Asilomar Conference on Signals, Systems and Computers ; 7-10 Nov. 2004; Pacific Grove, CA, USA, USA: IEEE ; p.1559-1564.
- [6] Seyed A. Hosseini , Sayed A. Hadei , Mohammad B. Menhaj and Paeiz Azami : Fast Euclidean Direction Search Algorithm: Adaptive Noise Cancellation and System Identification: In International Journal of Innovative Computing , Information and Control , 2013 ; 9(1) 191-206 .
- [7] Sayed. A. Hadei, and M. lotfzad: A Family of Adaptive Filter Algorithms in Noise Cancellation for Speech Enhancement; In International Journal of Computer and Electrical Engineering; 2010 ; 307-315 .
- [8] J.M.Th., and Mansoor B. : Application of Fast Euclidean Direction Search (FEDS) method For Smart Antennas in Mobile Communications Systems. In Intelligent Signal Processing (ISP) Conference, 2 - 3 December 2013 ; Strand Palace Hotel, London, UK, 2013.
- [9] .M.Th. : Performance Evaluation of the Fast Euclidean Direction Search Algorithm for Adaptive Beam forming Applications . In Al Sadiq International Conference on Multidisciplinary In Information and Communication Technologies 2016 AIC -MITC 2016 ; IEEE Iraq Section ; 9-10 May 2016, Baghdad. Iraq.
- [10] M. Shams Esfand Abadi, John H. Husøy, and A. Mahlooji Far: Interpretation and Convergence Analysis of Two Recently Introduced Adaptive Filter Algorithms (FEDS/RAMP), in Iranian Journal of Electrical and Computer Engineering, Vol. 7, No. 1, Winter-Spring 2008.
- [11] Th. M. Jamel: Optimal Window Length and Performance Analysis of FEDS Based Adaptive Beam Forming Approach over Rayleigh Fading Channels? International Journal of Computing and Digital System, ISSN (2210-142X) Int. J. Com. Dig. Sys. 8, No.4 (July-2019).
- [12] Gross, F. B. Smart Antenna for Wireless Communication. Graw-Hill, Inc, USA, 2005.



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