Statistical magnitude analysis and distance determination of the nearby G0V stars

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ABSTRACT

The present paper has three folds: first, to provide some basic descriptive statistical parameters for the apparent and absolute magnitudes of the nearby stars of spectral type G0V stars Second, to establish the frequency functions $\Phi(M)$ *and* $\Psi(m)$ *of the absolute and apparent magnitudes for these stars. Third , to compute the distance r of these stars as a system assuming that they scatter around a mean absolute magnitude in a Gaussian distribution. The accuracy of the numerical results is satisfactory in that, the percentage error between r and the mean value is less than 3%.*

KEYWORDS: Distance determinations, spectral types, frequency functions, statistical magnitude analysis.

INTRODUCTION

G stars, of which the Sun is one, are yellow with temperatures around 5000 to 6000 K.Another example of the G class is Alpha Centauri A. The spectra of these stars ,are solar –type spectra, with CaII lines extremely strong, neutral metals prominent, ions weaker, band (CH) strong and H lines are weaker (Robinson,1985) .As for the late spectral type stars ,G class is less concentrated in the Milky Way than the early types(Scheffler and Elsässer,1988), as an example, the number of the G type stars per square degree up to the limiting magnitude $m_{pg} \approx 8.5$ is about 0.4 the corresponding number of the A type stars at the latitude 5^{o}.

G0 stars of luminosity class V, are denoted as G0V (main sequence) are the hottest with effective temperature about 5900K, bolometric correction -0.05 (Mihalas and Binney 1981),absolute magnitude +4.4,mass: $\log(M/M_{Sun}) = +0.04$,radius: $\log(R/R_{Sun}) = +0.02$, luminosity : $\log(L/L_{Sun}) = +0.1$ and mean density $(\mathrm{g\,cm^{-3}})$: $\log\overline{\rho} = 0.13$ (Zombeck,1990).

In the present paper we consider G0V stars and perform the following:

- First, to provide some basic descriptive statistics parameters for the apparent and absolute magnitudes of these stars.
- Second, to establish the frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitudes for these stars.
- Third , to compute the distance r of these stars as a system assuming that they scatter around a mean absolute magnitude in a Gaussian distribution.

The accuracy of the numerical results is satisfactory in that, the percentage error between r and the mean value is less than 3%.

BASIC MATERIALS

DATA

The data was taken from Hipparcos and Tycho Catalogues (*http://vizier.u-strasbg.fr/cats/Usage.htx*). In Table I of Appendix A, the data for some G0V stars are listed. The columns of the table have the following meaning:

- **Column 1:** Vmag (unit mag) :Apparent magnitude
- **Column 2:** B-V(unit mag):Color
- **Column 3:** U-B(unit mag):Color
- **Column 4:** plx (unit mas ≡ milli-second of arc): Parallax
- **Column 5:** M_V (unit mag): Absolute visual magnitude

THE DISTANCE EQUATION

The assumptions upon which the distance equation was derived were (Sharaf et al. 2003).

(**1**) All the members in a given cosmic group are at the same distance, r parsecs.

(**2**) The frequency function for the absolute magnitudes of the members is

$$\Phi(M) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(M-M_0)^2/2\sigma^2} \qquad (1)$$

That is the members scatter around a mean absolute magnitude M_0 in a Gaussian distribution with dispersion σ.

(**3**) The mean apparent magnitude $\overline{m}$ of the members of the cosmic group and the limiting apparent magnitude m_1 are related through the quantity α where,

$$\alpha = \frac{m_1 - \overline{m}}{\sigma}$$

According to the above assumptions ,the distance r and the frequency functions $\Psi(m)$ of the apparent magnitude are given respectively by

$$r = 10^{1+(m_1 - M_0 - \sigma y)/5} \tag{2}$$

$$\Psi(m) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(m+5-5\log r - M_0)^2/2\sigma^2} \tag{3}$$

where y is the solution of the transcendental equation :

$$\Lambda(y) = y + e^{-y^2/2}\left\{\sqrt{\frac{\pi}{2}}\left[1 + \operatorname{erf}\left(\frac{y}{\sqrt{2}}\right)\right]\right\}^{-1} - \alpha = 0, \tag{4}$$

and, erf (z) is the error function defined by the integral

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} \, dt . \tag{5}$$

We call Equation (4) the *distance equation*.

The relation between M_0 and σ is given by Bok(1937)

$$M_0 = \overline{M} + 1.382\ \sigma^2 \tag{6}$$

where $\overline{M}$ is the mean absolute magnitudes of the stellar group.

EMPIRICAL RELATION BETWEEN M_V VERSUS $(B-V)_0$

An empirical relation between the absolute visual magnitudes M_V versus $(B-V)_0$ is given (Mihalas and Binney1981) in a tabular form (Table II of Appendix A).
From this empirical relation ,we deduce an interpolating polynomial between $(B-V)_0$ and M_V in the form:

$$(B-V)_0 = \sum_{j=1}^{20} q_j M_V \tag{7}$$

where q's are listed in Table III of Appendix A.

NUMERICAL APPLICATION

PROCEDURE

1-Compute the parallax p in seconds of arc from

$p = \text{plx}/1000$

2-Compute for each star in the group $(B-V)_0$ from its given value of M_V using Equation (7).

3-Compute the absorption A_V for each star of the group from

$$A_V = 3\left[(B-V)-(B-V)_0\right]$$

4- Compute the average $\overline{A}_V$ of A_V

5-Correct the apparent magnitudes for absorption from

Vmag = Vmag - $\overline{A}_V$

6-Compute the average value $\overline{M}_V$ of the absolute magnitudes M_V
7-Compute the average value $\bar{r}$ of the individual distances $r_j (r_j = 1/p_j)$
8-Compute the median r_{median} of the individual distances
9-Select start and end values of σ ,say σ_s and σ_e respectively.[In the present application , σ_s=0.5 and σ_e=2]
10-Compute the optimum value of σ as follows

a-For $\sigma = \sigma_s (0.01)\ \sigma_e$ perform the following calculations

- $M_0 = \overline{M}_V + 1.382\sigma^2$
- From the values of Vmag, M_0 and σ compute the distance *r* (in parsec) of the stellar group as obtained from the statistical method using the algorithm of Sharaf et al. (2003).
- Compute the percentage error *f* in the mean from $f = |(r-\bar{r})/\bar{r}| \times 100$
- Compute the percentage error *g* in the median from $g = |(r - r_{median})/r_{median}| \times 100$

b-Compute the value σ_1 that gives the minimum of the *f's* values
c- Compute the value σ_2 that gives the minimum of the *g's* values
d-Compute the optimum value of σ as $\sigma = (\sigma_1 + \sigma_2)/2$.

11- Compute some basic descriptive statistics parameters for the apparent and absolute magnitudes of the stellar group.

12-Compute the measures of the central tendency for the individual distances $r_j = 1/p_j$ of the stellar group, of these measures are:

▲The Mean distance $\bar{r}$ (in parsec) of r_j

▲The Mode of r_j

▲The Median of r_j

▲The Harmonic Mean of r_j

▲The Geometric Mean of r_j

13-From the optimum values of M_0 and σ compute the frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitude, i.e., Equations (1) and (3) of the group.

NUMERICAL RESULTS

- Numerical results are listed in Appendix B covering the following points:
  - ∗ Statistics of the apparent and absolute magnitudes of the G0V group
  - ∗ Graphical representations:

    a- Histograms for the apparent and absolute magnitudes of the stellar group

    b- Graphical representations for the percentage errors for the mean and median
  - ∗ Optimum choice of σ
  - ∗ Distance analysis
  - ∗ The frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitudes for theGV0 group

CONCLUSION

In concluding we draw the attention that, the aim of the present paper is three folds: first, to provide some basic descriptive statistical parameters for the apparent and absolute magnitudes of the near by stars of spectral type G0V stars Second, to establish the frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitudes for these stars. Third , to compute the distance r of these stars as a system assuming that they scatter around a mean absolute magnitude in a Gaussian distribution. The accuracy of the numerical results is satisfactory in that, the percentage error between r and its mean value $\bar{r}$ is less than 2.5%. This percentage error indicates the applicability of the used method for distance determination in the range of the considered stellar group.

REFERENCES

Bok, B.J.(1937), The Distribution of Stars in Space, Chicago University Press Chicago,USA.

Mihalas,D. and Binney,J. (1981),Galactic Astronomy ,second edition, Freeman. and Company,USA

Robinson, R.,M (1985),The Cosmological Distance Ladder,W.H.Freeman.USA

Scheffler,H. and , Elsässer ,H,. (1988), Physics of the Galaxy and Interstellar Matter, Translated by Armstrong, A.H.,Springer-Verlag,Berlin,Germany

Sharaf,M.A, Issa,I.A. and Saad,A.S (2003), Method for the Determination of CosmicDistances, New Astronomy, **8**: 15-21

Zombeck,M.,V. (1990),Handbook of Space Astronomy &Astrophysics, second edition, Cambridge University Press, Cambridge, New York.

Appendix A: Input data

Table I: Data for some G0V near by stars

Vmag	B-V	U-B	plx	M_V
mag	mag	mag	mas	mag
6.15	0.61	0.1	50	4.64
8.47	0.56	0.08	42	6.6
5.88	0.58	0.01	31.7	3.39
8.02	0.56	-0.02	21	4.63
4.05	0.6	0.12	92.4	3.88
5.96	0.57	0.07	49	4.41
6.75	0.58	0.07	39.5	4.7
7.44	0.6	0.07	28	4.68
4.4	0.59	0.06	103.1	4.47
6.11	0.6	0.06	49	4.56
6.34	0.62	0.12	46	4.65
5.24	0.56	0.05	66.2	4.34
5.74	0.57	0	59	4.59
5.64	0.58	0.03	58	4.46
5.36	0.59	-0.06	62.5	4.34
5.05	0.61	0.13	71.9	4.33
6.41	0.61	0.1	44	4.63
6.2	0.57	0.07	41	4.26
6.45	0.57	-0.02	40	4.46
6.44	0.66	0.19	31.7	3.95
4.27	0.59	0.05	114.7	4.57
5.95	0.55	-0.03	63.5	4.96
4.26	0.57	0.07	119.8	4.65
5.2	0.58	0.1	74.2	4.55
4.43	0.6	0.11	83.9	4.05
5.41	0.6	0.08	60	4.3
7.67	0.61	0.12	23	4.48
4.91	0.55	0.04	92.4	4.74
6.13	0.65	0.13	51	4.67
5.88	0.58	0.05	40.6	3.92
6.53	0.58	-0.02	50	5.02
6.42	0.6	0.09	51.4	4.97
5.94	0.59	0.04	66.1	5.04
6.18	0.69	0.17	58	5

Table II: Empirical relation between M_V versus $(B-V)_0$

$(B-V)_0$	M_V	$(B-V)_0$	M_V
-0.30	-3.50	0.30	2.80
-0.25	-2.30	0.40	3.35
-0.20	-1.30	0.50	4.05
-0.15	-0.50	0.60	4.60
-0.10	0.30	0.70	5.20
-0.05	0.90	0.80	5.70
0.00	1.30	0.90	6.10
0.05	1.55	1.00	6.60
0.10	1.80	1.10	7.00
0.20	2.25	1.20	7.45
		1.30	7.90

Table III: The q's coefficients of Equation(7)

j	q_j	j	q_j
1	-0.786498	11	0.769777
2	3.37819	12	-0.421287
3	-1. 96742	13	0.0908398
4	-10.6822	14	0.000500399
5	20.8088	15	-0.00573177
6	- 11.6107	16	0.00172542
7	-5.11632	17	-0.000284527
8	10.3234	18	-0.0000296064
9	-5.29694	19	-1.94463×10^{-6}
10	0.467069	20	7.40906×10^{-8}

Appendix B: Output data

B.1: Statistics of the apparent and absolute magnitudes of the G0V stars

▲The average:
For Apparent Magnitudes = 5.92
For Absolute Magnitudes = 4.55559
▲The median (central value):
For Apparent Magnitudes = 5.955
For Absolute Magnitudes = 4.565
▲The mode:
For Apparent Magnitudes = 5.88
For Absolute Magnitudes= {4.34,4.46,4.63,4.65}
▲The geometric mean:
For Apparent Magnitudes = 5.831
For Absolute Magnitudes= 4.530
▲The harmonic mean:
For Apparent Magnitudes = 5.7412
For Absolute Magnitudes = 4.50574
▲The root mean square:
For Apparent Magnitudes = 6.00827
For Absolute Magnitudes = 4.58286
▲The Quartiles:
For Apparent Magnitudes = {5.24,5.955,6.42}
For Absolute Magnitudes = {4.34,4.565,4.68}
▲The range:
For Apparent Magnitudes = 4.42
For Absolute Magnitudes = 3.21
▲The unbiased estimate of the variance:
For Apparent Magnitudes = 1.08476
For Absolute Magnitudes= 0.256789
▲The maximum likelihood estimate of the variance:
For Apparent Magnitudes = 1.05286
For Absolute Magnitudes = 0.249236
▲The unbiased estimate of the variance of the sample mean:
For Apparent Magnitudes = 0.0319048
For Absolute Magnitudes = 0.00755262
▲The unbiased estimate of the standard deviation:
For Apparent Magnitudes = 1.04152
For Absolute Magnitudes = 0.506744
▲The maximum likelihood estimate of the standard deviation:

For Apparent Magnitudes = 1.02609
For Absolute Magnitudes= 0.499236

▲The unbiased estimate of the standard error (standard deviation)of the sample mean:
For Apparent Magnitudes = 0.178619
For Absolute Magnitudes= 0.0869058

▲The mean absolute deviation:
For Apparent Magnitudes = 0.769412
For Absolute Magnitudes= 0.31173

▲The median absolute deviation:
For Apparent Magnitudes = 0.52
For Absolute Magnitudes= 0.165

▲The interquartile range:
For Apparent Magnitudes = 1.18
For Absolute Magnitudes= 0.34

▲The Quartiles:
For Apparent Magnitudes = {5.24,5.955,6.42}
For Absolute Magnitudes= {4.34,4.565,4.68}

▲The coefficient of skewness:
For Apparent Magnitudes = 0.300792
For Absolute Magnitudes= 1.51746

▲Pearson's first coefficient of skewness:
For Apparent Magnitudes = 0.115216
For Absolute Magnitudes= {1.27632,0.565897,-0.440529,-0.558932}

▲Pearson's second coefficient of skewness:
For Apparent Magnitudes = -0.100814
For Absolute Magnitudes= -0.0557191

▲The quartile coefficient of skewness:
For Apparent Magnitudes = -0.211864
For Absolute Magnitudes= -0.323529

▲The kurtosis coefficient:
For Apparent Magnitudes = 3.10831
For Absolute Magnitudes= 9.51922

▲The kurtosis excess:
For Apparent Magnitudes = 0.108308
For Absolute Magnitudes= 6.51922

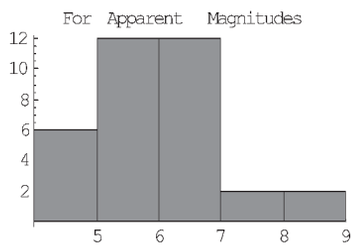
▲The confidence interval of the population mean based on the normal distribution:
For Apparent Magnitudes = {5.56991,6.27009}
For Absolute Magnitudes= {4.38526,4.72592}

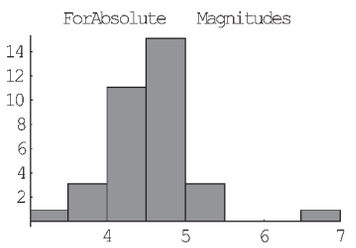
▲The confidence interval for the population variance based on the chi-square distribution :

For Apparent Magnitudes = {0.70571,1.87945}
For Absolute Magnitudes= {0.167058,0.444909}

B.2 :Graphical Representations

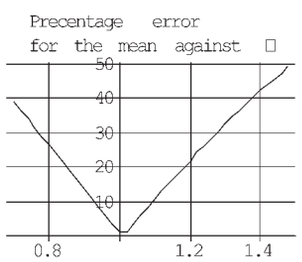
▲ The histograms of the apparent and absolute magnitudes of the stellar group

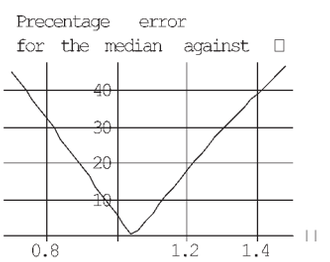




B.3: Optimum Choice of σ

▲Graphical representations of the percentage errors for the mean and median





▲The minimum value of the percentage error in the Mean is = 1.20438 occurs at σ = 1.02

▲The minimum value of the percentage error in the Median is = 0.884535 occurs at σ = 1.04

▲The optimum value of σ is = 1.03

B. 4: Distance Analysis

▲ The value of m_1 is = 10.0514

▲ The value of $\overline{m}$ is = 7.50139
▲The value of σ is = 1.03
▲ The value of M_0 is = 6.02175
▲The value of α is = 2.47573
▲The value of y is = 2.45604
▲The accuracy of the computed value of y is $= 4.62193 \times 10^{-7}$
▲The distance r of the stellar group as obtained from the statistical method is = 19.9518
parsec
▲The Median of the individual distances is = 19.5315 parsec
▲The Mode of the individual distances is = {10.8225,17.2414,20.,20.4082,31.5457}
parsec
▲ The geometric mean of the individual distances is = 18.7365 parsec
▲ The harmonic mean of the individual distances is = 17.2187 parsec
▲ The mean value of the individual distances is = 20.4468 parsec
▲ The percentage error between r and $\hat{r}$ is = 2.42074

B. 5:Frequency functions of the absolute and apparent magnitudes for the G0Vstars

$$\Phi(M) = 0.387323 \times e^{-0.490148\,(-6.02175+M)^2}$$

$$\Psi(m) = 0.387323 \times e^{-0.490148\ (-7.52167+m)^2}$$

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التحليل الأحصائى للقدر وتعين المسافة للنجوم القريبة ذات التصنيف الطيفيG0V

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الملخص:

يمثل هذة البحث ثلاث طيات ، أولا : حساب بعض معاملات الإحصاء الوصفى للأقدار الظاهرية والمطلقة وذلك للنجوم القريبة ذات التصنيف الطيفي G0V. ثانياً: تشيد دوال التواتر $\Phi(M)$, $\Psi(m)$ للأقدار الظاهرية والمطلقة لهذة النجوم. ثالثاً: إيجاد المسافة r لهذة النجوم على إنها نظاماً نجمياً يتوزع حول قدر مطلق متوسط في توزيع جاوس. وقد كانت النتائج مرضية للغاية حيث وصلت النسبة المئوية للخطأ بين r والقيمة المتوسطة حوالي 3%.